

Genuine Fortuitousness, Relational Blockworld, Realism, and Time

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Abstract

Quantum mechanics, which has no completely accepted interpretation but many seemingly strange physical results, has been interpreted in a number of bizarre and fascinating way over the years. The two interpretations examined in this paper, Bohr and Ulfbeck's "Genuine Fortuitousness" and Stuckey, Silberstein, and Cifone's "Relational Blockworld" seem to be two such strange interpretations; Genuine Fortuitousness posits that causality is not fundamental to the universe, and Relational Blockworld suggests that time does not act as we perceive it to act. In this paper, I analyze these two interpretations of quantum mechanics, examining the predecessors whose works they drew from, both interpretations' physical and mathematical derivations, and their physical and philosophical consequences. After this intensive review of each interpretation, I determine that, despite the fact that Genuine Fortuitousness and Relational Blockworld make claims that run contrary to common sense, both interpretations are important and interesting ways of looking at the quantum world that may proffer solutions to some of the toughest problems plaguing physics today.

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1 Introduction

It has become cliché in contemporary theoretical physics or philosophy of physics to begin one’s paper by noting that physics today is in a state of turmoil; however, the reason that such commentary on the state of modern physics is so common is that, in this case, the commentary happens to be correct. There are at least two problems in modern physics, that of quantum gravity (QG) and the fine-tuning problem, that no modern physical theory has satisfactorily resolved. The first of these problems, QG, is especially insidious since it has held physics back from its aspirations to the grand unified “theory of everything” for over half a century now. How did physics come to be in such a state?

There are some who claim that the failure of modern attempts to describe QG results from some error in interpreting physics along the way; that is, perhaps mistakes in the history of physics have propagated through to leave us with an intractable problem. There are some physicists and philosophers who pin their hopes for the solution to modern physical problems on a reinterpretation of extant physical theories. The question is, then, which physical theory ought to be reinterpreted?

An obvious candidate is quantum mechanics (QM), which has long posed conceptual difficulties due to problems like the measurement problem and the EPR paradox that philosophers and physicists are still trying to resolve. There is hope that these conceptual difficulties in QM are a result of some error in interpretation that, when corrected, will lead to the resolution of many of the worst problems plaguing contemporary physics.

Two such promising reinterpretations of QM have emerged in the last decade: the theory of Genuine Fortuitousness (GF), which was proposed in two papers by Bohr and Ulfbeck[16][17] and one more recent paper by Bohr, Mottelson, and Ulfbeck; and Relational Blockworld (RBW) proposed more recently in a series of papers by Stuckey, Silberstein, and Cifone[52][53][54][55][56][57][58]. These two interpretations are related in that RBW cites some of GF’s argumentation and both argue that special relativity (SR) provides the basis for QM; however, these are fundamentally different interpretations of QM and SR that draw different conclusions.

One thing that unites GF and RBW, however, is that both GF and RBW make claims that some might find philosophically outrageous. GF, for instance, does away with the idea of any fundamentally “real” particles and concludes that causation is a myth since the world is fundamentally lawless or “genuinely fortuitous”. RBW pushes the bounds of common sense even farther by asserting that our macroscopic notion of time is flawed as well and that the past, present, and future should all be regarded as equally real. It is these radical departures from traditional thinking about the world, however, that allow GF and RBW to reconcile SR and QM to suggest new ways of looking at the quantum world and its relationship to relativistic space-time.

The purpose of this paper is to explicate the physical reasoning behind the quantum interpretations of GF and RBW and to evaluate their promise as both explanatorily sufficient and pragmatically useful interpretations of QM. I will show that both theories are founded on sound physics and that while their radical claims about the nature of reality may be unconventional, they seem to be supported by good physical reasoning. Both of these theories will be shown to hold great promise for future work in both

theoretical and experimental physics, and with contributions from these theories, it may be possible for us solve some of the most troubling problems that plague physicists today

The paper is organized in the following manner: first, in section 2 I present the background to my discussion of GF and RBW. I discuss the interpretation of QM that has been the most popular for the past eighty years or so, the Copenhagen interpretation, along with its conceptual difficulties so as to set the stage for the solutions proposed by GF and RBW.

In section 3, I examine the physical reasoning of GF and explain this theory in detail through a close analysis of Bohr, Mottelson, and Ulfbeck's papers. I discuss their derivation of an important fundamental quantum mechanical relation, the canonical commutation relation (CCR), from the Poincaré group as well as their novel derivation of the quantum mechanical density matrix from basic facts about abstract algebra with appropriate physical assumptions. I also attempt to explain GF's use of matrix variables and thus motivate GF's assertions about the nature of reality and randomness.

In section 4, I investigate other modern reinterpretations of QM, such as Mermin's "Ithaca Interpretation" and Jeeva Anandan's non-dynamical approach to QM. Mermin's interpretation emphasizes the same relationality that is present in both GF and RBW, and by examining the interpretations of Mermin and his intellectual predecessors, I reveal the similarities and differences between GF's treatment of relationality and other physical theories dealing with the same topic. Anandan's interpretation is utilized by RBW, so the discussion of Anandan's interpretation preempts the discussion of RBW soon to come while pointing out how Anandan's approach differs from that of GF. In short, this section's purpose is to "fill in the gaps" between GF and RBW while providing the appropriate context of other physical theories from which to view both GF and RBW.

In section 5, I present RBW and examine its key features. I begin with a new discussion of Stuckey, Silberstein, and Cifone's argument for the role of the relativity of simultaneity (RoS) in supporting the view of space-time as a four-dimensional block-world. I then show how RBW synthesizes the eternalist perspective on space-time and the relationality of GF to form a new interpretation of QM, SR, and time.

In section 6, I address a phenomenon that one might use to argue against the position of RBW: that of the quantum liar paradox (QLP) as proposed by Elitzur and Dolev[25]. Elitzur and Dolev argue that only their "backwards-causal" theory of time can account for the QLP. Following Stuckey, Silberstein, and Cifone's discussion of the QLP[57], however, along with a novel argument of my own design, I will show that RBW is, in fact, capable of explaining the QLP and thus argue that QLP actually supports RBW instead of refuting it.

In section 7, I examine some of the philosophical perspectives in GF and RBW, particularly the stance of GF and RBW on the nature of reality and realism, the nature of time, and the issue of fundamental indeterminacy in space-time. I attempt to sort out what each of these interpretations brings to the table philosophically after which I propose an amalgamation of GF and RBW that I call the "Fortuitous Relational Blockworld" (FRBW) to solve the problem of the asymmetry of time.

In section 8, I examine some of the modern criticism that has arisen concerning GF, specifically the response of Ulrich Mohrhoff[40] to GF. After investigating Mohrhoff's

criticisms, I propose my own critique of GF and RBW; however, despite the criticism, I hold that both GF and RBW are promising and well-motivated physical theories.

In section 9, I inquire into the possibility of experimental verification of GF and RBW, specifically drawing upon the work of Jerzy Czyz[21] to see if GF is experimentally verifiable. I hold that the true rubric that will be used to judge both of these interpretations will be how successful both are at providing the appropriate background perspective from which a solution to the problems of modern physics can be found. I then examine the potential GF and RBW hold for proposing solutions to the fine-tuning problem and the problem of QG. Finally, in section 10, I draw my final conclusions about GF and RBW.

2 Background

Before beginning a discussion of Bohr and Ulfbeck’s “principle of Genuine Fortuitousness” or Stuckey, Silberstein, and Cifone’s (SSC’s) “Relational Blockworld”, it is first important to understand the context surrounding these two quantum mechanical theories/interpretations. This requires a return to the origins of quantum theory in the early 20th century to examine the work of physicists like Niels Bohr, Werner Heisenberg, and Max Born, and the Copenhagen interpretation of QM that has survived, for the most part, as the dominant interpretation of QM among most physicists to this day¹. Not only will I explain what the Copenhagen interpretation entails in this section, but I will also discuss some of the issues that have plagued the Copenhagen interpretation from its inception, thus suggesting why authors like Aage Bohr, Mottelson, Ulfbeck, Stuckey, Silberstein, and Cifone might feel an urge to inquire into other interpretations of QM. Finally, I will end this section with a brief look at some of the early pioneers of quantum theory whose work anticipated that of Bohr and Ulfbeck as well as SSC.

2.1 The Copenhagen Interpretation

In the late 1920’s, due to the experimental success of quantum theory, many physicists who were now used to the accuracy of what was a phenomenologically descriptive theory of the quantum scale began to question how QM fit in with the rest of physics as whole. In short, physicists asked the question, “what does quantum mechanics tell us about nature?” as they clamored for an interpretation of the new quantum mechanical “laws”. The most historically significant interpretation to emerge from this time period was primarily fathered by Niels Bohr with some help from Max Born and, to a larger degree, Werner Heisenberg, among others[28], around 1927 or so. This interpretation was dubbed the “Copenhagen interpretation”, and since the middle of the 20th century, this interpretation of QM has been the most widely referred to by physicists and laypeople alike².

The Copenhagen interpretation of QM, unlike many other philosophical positions whose views are expressed explicitly in manifestos, is somewhat vaguely defined. Even Bohr and Heisenberg were said to have argued over several issues now commonly cited as part of the dogma of Copenhagen[28]. Despite the generally (and perhaps intentionally) non-specific and inexact phrasing of the Copenhagen interpretation as it is commonly referred to in contemporary times, I will do my best to present the tenets

¹Recent work in the history and philosophy of physics (specifically that of Don Howard[32]) has suggested that the Copenhagen interpretation is essentially a myth. There may, in fact, never have been a unified view called “the Copenhagen interpretation” that any one person ascribed to. However, myth though it may be, the “Copenhagen interpretation” as many physicists conceive of it is the most popular interpretation of QM available today. For this reason, though no “Copenhagen” may actually exist (or have existed), I shall define a set of beliefs about QM that I will lump under the header of “Copenhagen” since it seems that many throughout history have done the same.

²There have, of course, always been those who have argued against the Copenhagen interpretation from its inception (most notably Einstein[20]), especially among philosophers of physics, but, among the general public, the Copenhagen interpretation is certainly the most prominent interpretation of QM to this day

of the Copenhagen interpretation in as clear and concise a manner as possible.³

One of the first tenets of the Copenhagen interpretation is that quantum uncertainty is both metaphysical and fundamental. That is, the uncertainty inherent in quantum phenomena such as radiation and measurement does not reflect some sort of epistemological or subjective uncertainty about the world but rather reveals the world as it truly is. The uncertainty of certain physical variables under various conditions is an objective fact about the universe and must be treated as such. Copenhagen's treatment of uncertainty stands in stark contrast to "hidden variable" theories, a series of other QM theories like Bohmian mechanics which treat uncertainty as the result of some unknown variable that has not been accounted for in traditional QM.

The objective uncertainty of QM in Copenhagen suggests the existence of objective constraints on all knowledge of the physical world due to fundamental, objective uncertainty. Another aspect of the Copenhagen interpretation closely related to the reality of uncertainty is the idea that only certain qualities of a quantum system can exist at a given time. For instance, non-commuting observables like position and momentum cannot both assume values simultaneously. Measurement of one such observable precludes the existence of a value for the other observable, and thus by measuring a system, one can effectively determine which properties of a quantum system have definite values and which ones are "unreal".

Another effect of the ontology of uncertainty in Copenhagen may be a bit surprising given my characterization of Copenhagen's treatment of uncertainty. Though there are specific situations wherein the uncertainty of results is a fundamental, ontological feature of a given quantum process or system, there are still laws that govern these processes and systems. These laws, however, are necessarily probabilistic, meaning that though one cannot determine with certainty the result of one measurement, one may determine the distribution of measurement results over a series of system observations. The Born rule opens the door for the predictive power of QM and requires that the only laws that can describe QM are necessarily probabilistic. The stance of Copenhagen towards probabilistic laws seems somewhat confusing given that the results of one trial are entirely random while a series of trials are required to follow certain statistical laws, and Neils Bohr's son Aage Bohr, along with Ole Ulfbeck, later attempt to better define the nature of uncertainty in their idea of genuine fortuitousness.

Another feature of the Copenhagen interpretation is suggested by the contrast between Copenhagen and hidden variable theories of QM. Unlike hidden variable theories, Copenhagen holds that the laws of QM, as they currently stand (and, for the most part, have stood since the early part of the century) are complete. There are no hidden variables in the Copenhagen interpretation, nothing more to be described in quantum systems apart from what the theory is already able to describe. This stance, like many of the others held by Copenhagen, follows as a consequence of the ontological status of uncertainty in the Copenhagen interpretation since any uncertainty in the physical theory as it currently stands is to be embraced as a "real" feature of the system instead of explained away as a result of some hidden variable.

Copenhagen also holds that the primary entity of QM, the wave function, is a

³The characterization of the Copenhagen interpretation that follows is primarily built up from source material such as Bohr's "Quantum Mechanics and Physical Reality"[14], the commentaries of Bub[20] and Faye[28] on the early development of QM, and from various conversations with Professor John Boccio.

complete description of a quantum system. All that one needs to know about what QM is saying about a given system can be gleaned from the formulation of the wave function. The completeness of the wave function as an explanatory tool seems to suggest some sort of ontological priority afforded to the wave function, but it is not common for those who accept the Copenhagen interpretation to refer to the wave function as being “real” in itself.

The ontological status of the wave function is perhaps the strangest aspect of the Copenhagen interpretation, for instead of taking the wave function, the complete description of a quantum system, as fundamentally real, the Copenhagen interpretation suggests that the atoms and particles we “see” in the world around us are what is truly real. Bohr explains this seeming contradiction between the treatment of QM and a belief in the reality of a macroscopic world by suggesting that QM is only meant as a descriptive, instrumentalist theory of reality. QM ought to be used to make predictions about the world and nothing more. No ontological status should be afforded to the wave function precisely because it is nothing more than a descriptive mathematical object. According to Copenhagen, then, it is not the job of QM or physics in general to explain or elucidate reality or “the way the world really works” to anyone but rather to make useful and correct predictions about events in the natural world. What is real, then, is what we see around us, and about the quantum world, nothing can be said.

Copenhagen clearly treats the quantum and macroscopic worlds differently, especially when it comes to metaphysics; however, another tenet of the Copenhagen interpretation is that the laws that govern the macroscopic world ought to emerge from the quantum world. Even though it is not yet understood exactly how macroscopic phenomena arise from quantum phenomena, the Copenhagen interpretation asserts that the laws of QM are somehow fundamental to the laws of classical physics that govern the world on a large scale.

An interaction between the macroscopic world and the quantum world can be seen in the process of quantum measurement. Measurement is viewed as the interaction between a large-scale (macroscopic) and a quantum (microscopic) system. According to Bohr in his response to the EPR paradox⁴, this interaction results in a shifting of the initial conditions of the quantum system. Thus, macroscopic objects can exert a direct power and influence over microscopic objects, though once again the description of how interactions between these two scales work is rather unclear.

The issue of macro-quantum interactions is of great interest to one of the fundamental problems that arises from QM, the measurement problem. The Copenhagen interpretation’s treatment of this problem, along with an explication of the measurement problem, will be addressed in the next section. Before moving on from a more general statement of the Copenhagen interpretation to discussing how Copenhagen deals with certain “quantum paradoxes”, I would first like to state that the views of the Copenhagen interpretation as described in the previous paragraphs are meant as a general outline of what is and has been the most widely-accepted interpretation of QM; thus, many of QM’s conceptual difficulties can be traced back to problems with the description of quantum phenomena according to the Copenhagen interpretation. Whether views other than Copenhagen suggest a better resolution to these difficulties

⁴More on the EPR paradox in Section 2.3

than Copenhagen affords is an open question, but it is important to understand that it is the Copenhagen interpretation, not the experimentally verified facts of quantum physics, that underlies both the measurement problem and the EPR paradox I will now turn to.

2.2 The Measurement Problem

The problem of measurement is one of the major conceptual difficulties with one peculiar aspect of the physical and mathematical description of quantum system in QM⁵. The problem, stated most simply, is that there are two distinct mathematical equations for the dynamical evolution of a quantum system. These distinct evolutionary laws are split by the act of measurement.

The following is a brief example of what measurement of a quantum system does to the state vector, the Copenhagen interpretation’s “complete” description of a quantum system. Imagine that an electron starts in an eigenstate of y-spin. The electron, a 1/2 spin particle, can be defined by the following initial state vector in terms of x-spins:

$$|electron\rangle = \frac{1}{\sqrt{2}} |spin_x \uparrow\rangle + \frac{1}{\sqrt{2}} |spin_x \downarrow\rangle \quad (1)$$

The above state vector $|electron\rangle$ describes the state of an electron prior to measurement. However, we now send the electron through a x-spin detector, meaning that either the up or down spin must be selected as the spin of the electron. Thus, the final state after measurement must be one of the following:

$$|electron\rangle = |spin_x \uparrow\rangle \text{ or } |electron\rangle = |spin_x \downarrow\rangle \quad (2)$$

The Born rule requires that the probability of either of these results is 1/2, the square of either of the coefficients from the original state vector prior to measurement.

Thus, the measurement problem appears in the previous example as the following: first, why should the electron evolve differently before measurement (when it remained in the superposition of x-spin values) and after measurement (when it collapsed an eigenstate of x-spin)? This is an important dualism that measurement imposes on quantum systems. A second problem is that the “collapse” of the wave function from a superposition to an eigenfunction of an observable is instantaneous, meaning that it is either impossible or extremely, extremely difficult for one to explain the dynamical process by which this “collapse” occurs. Finally, the measurement process is completely random; though we know that, over millions of experiments, the number of spin-up outcomes will be approximately equal to the number of spin-down outcomes, for any given measurement it is impossible to determine what the outcome will be. These three concerns together comprise the measurement problem⁶.

The Copenhagen interpretation of the measurement problem as described above follows from the characterization of Copenhagen in the previous section. As may be

⁵The majority of the characterization of the measurement problem in this section comes from Wigner’s paper “The Problem of Measurement” [66].

⁶Though traditionally only one or two of these issues will be addressed directly by someone attempting to answer the measurement problem.

expected, Copenhagen's answers to each of the three issues raised by the measurement problem are not answered to everyone's satisfaction. First, Copenhagen takes it as a fact of the world that the measurement process occurs. Measurement, according to Copenhagen, is a discontinuous event involving the interaction of the quantum world and an observer. This event changes the wave function, and thus the dualism of pre- and post-measurement state vectors is a real feature of the quantum world⁷.

This assertion of measurement "as it is" forces the Copenhagen interpretation to accept the first and second conceptual issues in the measurement problem as inherent in a correct description of the quantum phenomenon of measurement. The third conceptual issue raised by measurement, that of the randomness of experimental outcomes, falls in line with Copenhagen's emphasis on the fundamental ontological nature of uncertainty. If uncertainty is a brute fact about the quantum world, why should anyone worry about the fact that experimental outcomes are uncertain? All three conceptual difficulties with QM in the measurement problem are incorporated into the Copenhagen interpretation. The dichotomy, discontinuity, and uncertainty that define the measurement problem are all essential facets of the Copenhagen interpretation.

The problem of measurement for Copenhagen, then, is a question of observers. Who counts as an observer for a measurement to take place? And where, exactly does the phenomenon of "collapse" occur? These are the questions of measurement that can be meaningfully discussed in the context of Copenhagen. However, once one starts to question the fundamental nature of the three conceptual issues raised above, one leaves the Copenhagen interpretation behind in favor of other (perhaps more conceptually favorable) interpretations of QM.

There are several reasons why one might prefer a different account of measurement to Copenhagen's. First, the dichotomy of pre- and post-measurement state vectors is not explained in the Copenhagen interpretation. There is no mechanism posited by Copenhagen QM by which the state vector should shift from evolution in accordance to one law to evolution in accordance with another. What is more, Copenhagen does not explain why such a mechanism should be discontinuous. In common experience with other laws of physics (for example, in classical dynamics or even in relativity), there is some mechanism by which change occurs, and these mechanisms are always carried out over some interval of time, so what reason does Copenhagen supply for why the quantum mechanism should be either so radically different or non-existent aside from the fact that their interpretation fits the experimental results? The Copenhagen interpretation provides no good motivation for why collapse should not have a mechanism.

Another unsettling quality of Copenhagen for some is the fundamental nature of uncertainty. In scientific inquiries before QM, uncertainty was typically regarded as a reflection of some misunderstood or overlooked factor in a description of a physical system. Thus, the uncertainty was subjective since the poor predictive power of a model or equation resulted from human ignorance instead of the objective world. However, the Copenhagen interpretation moves one form of scientific uncertainty from the subjective world to the objective world in treating uncertainty as a "fact of the matter". Since such a move has no historical precedent in the sciences, one might reasonably be led

⁷As real, of course, as anything in the quantum world can be in the Copenhagen interpretation

to doubt it.

These concerns with the Copenhagen interpretation do not, of course, disprove it; the Copenhagen interpretation takes pains to align itself neatly with the experimental results of QM, takes the results of QM as fundamentally real, and leaves very little room for metaphysical postulating, thus making itself difficult to disprove. It would take new scientific data that would conflict with the current formulation of QM to displace the Copenhagen interpretation conclusively, and such data has certainly not arisen in the 80 years since Copenhagen was formulated. However, these concerns do call the metaphysical stance taken by Copenhagen on issues like the fundamental nature of uncertainty into doubt since Copenhagen's stance tends to fly in the face of our common understanding of the natural world. Thus, many physicists and philosophers find the Copenhagen interpretation unsatisfactory and seek alternative interpretations of QM.

Another reason to seek such alternative interpretations of QM might be found in another conceptually difficult problem in QM which draws upon the measurement problem. This problem, the EPR paradox, will be addressed in the next section.

2.3 The EPR Paradox

The EPR paradox was first proposed in 1935 in a paper by Einstein, Podolsky, and Rosen[26]. This paper suggests that QM is an incomplete theory due to conflicting predictions from special relativity (SR) and QM. The argument was reformulated by Bell in a later paper[11], and eventually the paradox led to an experiment carried out by Aspect which confirmed the QM prediction[10]. I will give a basic outline of the paradox and explain the problem it poses to the Copenhagen interpretation.

The EPR paradox, most simply put by Einstein, Podolsky, and Rosen in their original paper, is that a measurement of a system A should not impact the measurement of a system B if the two systems are separated[26]. The reason why such measurements should not affect each other is that, if the two systems are space-like separated, an effect of one measurement on the other would constitute some sort of superluminal causation since the effect would be instantaneous across a potentially enormous distance of space.

A more mathematical formulation of the problem goes as follows: imagine two electrons whose total spin is measured. The possibilities for the spin of the system are:

$$|S = 1\rangle = |s_1 = +1/2\rangle |s_2 = +1/2\rangle \quad (3)$$

$$|S = 0\rangle = \frac{1}{\sqrt{2}} |s_1 = +1/2\rangle |s_2 = -1/2\rangle + \frac{1}{\sqrt{2}} |s_1 = -1/2\rangle |s_2 = +1/2\rangle \quad (4)$$

$$|S = -1\rangle = |s_1 = -1/2\rangle |s_2 = -1/2\rangle \quad (5)$$

where $|s_1 = \pm 1/2\rangle$ is the individual spin of the first electron and $|s_2 = \pm 1/2\rangle$ is the individual spin of the second electron.

Assume, then, that two experimenters, Alice and Bob, each pick up one of the two electrons that have passed through the spin detector to yield a system eigenstate of $|S = 0\rangle$. Alice takes her electron, the first one, and jumps into a spaceship which drops her off on Mars. Bob, however, remains on Earth with his electron. Now, if

Alice decides to measure her electron's spin, she will get a definite result; however, this act of measurement, as per the Copenhagen interpretation's understanding of "collapse", will immediately force Bob's electron to change its evolution from a pre-measurement superposition into a post-measurement eigenstate. This collapse happens instantaneously, but Bob and Alice are separated by a great distance; thus, it seems that there must be some nonlocal connection between the two systems since they seem able to "communicate" faster than the speed of light⁸.

Thus, the EPR paradox suggests that there is some sort of "superluminal causation" that seems to violate SR's postulate that nothing can travel faster than the speed of light. The Copenhagen interpretation holds that their version of this story holds despite the contradiction with SR, and Aspect's previously alluded-to experiment shows that the Copenhagen story is, in fact, a good one for describing the EPR paradox. Despite this empirical verification, however, something still seems strange in the Copenhagen story of the EPR paradox since nothing built into the Copenhagen interpretation explains non-local connections. Much like with the measurement problem, Copenhagen simply builds in non-locality as a "fact of the matter" about the physical world, and nothing more needs to be said about it. However, non-locality, like many other aspects of the quantum world, seems strangely incoherent with many beliefs we hold about the macroscopic world, and as such, there are many physicists and philosophers who feel that the correct interpretation of QM should account for non-locality in a more satisfying way.

As this section and the two previous sections have shown, there are several conceptual points on which the Copenhagen interpretation of QM makes assertions that are not explained or even supported. For this reason, many physicists and philosophers have sought a deeper understanding of QM in order to formulate a more conceptually appealing explanation of how the quantum world is integrated into the rest of physics. These new interpretations of QM tend to carry with them some sort of metaphysical risk. Almost always, something has to be abandoned or taken for granted. For Bohm, hidden variables were taken for granted, while for Everett, the existence of "other worlds" provided a way out of Copenhagen. Relational Blockworld and Genuine Fortuitousness are two other interpretations of QM for which this true as well.

In the next section, I will discuss the insight of two physicists/philosophers explicitly: Wheeler and Weyl. I will also explain, briefly, one of the major problems facing physics today and how a radical innovation along the lines of Wheeler and Weyl

⁸On the issue of superluminal communication via entangled states, it should be noted that it is not possible to send signals using entangled states. Imagine the following: Alice makes no observations on her electron, and thus Bob's measurements of his electron must cause it to collapse (by the Copenhagen account) and thus, no matter how many entangled electrons he has, he will, for a large sample, have about the same number of $spin = +1/2$ and $spin = -1/2$ electrons. However, if the situation were different and Alice was measuring her electrons' spin before Bob, Bob's electrons' spins would be determined by the results of Alice's measurements; still, the fact of the matter is that these measurements produce random results, meaning that Bob's electron spin results are still, for many samples, that the number of $spin = +1/2$ and $spin = -1/2$ electrons is about equal. Thus, there is nothing Alice can do to decidedly send Bob a signal, and so, even though a comparison between Bob and Alice at a later (and relativistically allowed) time will reveal what may have been a signal, no measurement Alice can make can change Bob's electrons in such a deterministic way as to convey information.

suggested may, in fact, provide an opportunity to solve it.

2.4 Life After Copenhagen

For several decades now, theoretical physics has been dealing with one particularly difficult problem that seems to have no solution in sight: the problem of quantum gravity. Physicists have succeeded in unifying physical explanations of the strong and weak nuclear forces as well as the electromagnetic force, but the final force, gravity, has been left out of this picture. This leads many physicists and philosophers feeling that they must have missed something along the way since physics should be able to integrate gravity into some “grand theory of everything” in a meaningful way. To date, however, no experimentally verified theory has been able to successfully unite all four major forces.

The problem of gravity has motivated different physicists in different ways. Some have attempted a top-down approach, creating involved, unified theories which they then twist and contort to fit the data. Others, however, prefer to take a bottom-up approach by re-evaluating the physics that has preceded contemporary work on the problem of gravity to see if any of the assumptions are faulty or misleading. The bottom-up approach seems a more modest and fruitful approach for an attack on the problem of gravity for the time being since string theory, the most popular top-down theory, is at best several decades away from the experimental considerations that will be necessary to provide it with confirming or disconfirming evidence. The bottom-up approach, however, promises to cast the way physicists and philosophers think about the natural world in a new light, and it may be from such a revolution in thought that the “Eureka!” moment for quantum gravity arises.

Given some of the questionable assertions of the Copenhagen interpretation, it seems no surprise, then, that many physicists and philosophers are examining the foundations of QM to determine whether or not the Copenhagen interpretation is omitting some important fact(s) about the world that could possibly provide contemporary thinkers with the necessary framework for a solution to quantum gravity. Both of the interpretations that I will be concentrating on foremost in this paper, Bohr and Ulfbeck’s Genuine Fortuitousness and Stuckey, Silberstein, and Cifone’s Relational Blockworld, provide such radical suggestions of new ways to look at and evaluate the predictions of QM⁹. However, before examining and evaluating the merits of these two interpretations of QM, it is first important to discuss two thinkers who predate Bohr, Ulfbeck, Stuckey, Silberstein, and Cifone. These are Hermann Weyl and John Archibald Wheeler, and these two physicists’ work seems to point in the direction that both Genuine Fortuitousness and Relational Blockworld eventually go.

Hermann Weyl was a German mathematician and physicist who was especially interested in questions of symmetry and geometry in physics. Most of Weyl’s writing dates from the 1930’s, which places him around the time when the debate between

⁹Stuckey, Silberstein, and Cifone in particular have noticed the link between their theory and its implications for quantum gravity and are currently investigating ways in which their theory might aid in reconceptualizing the problem of gravity. Bohr and Ulfbeck have not drawn such a link between their work and quantum gravity, but I believe that any new theory or interpretation that casts QM in a new light must, down the line, affect the assumptions that have led to the gravity problem.

Einstein and Bohr over the nature of reality (and of how to interpret QM) was raging. Weyl's beliefs center on SR and the idea of a real 4-dimensional world rather than QM. One of Weyl's central beliefs was that there is a special explanatory role given to the geometry of space-time, as is evident from the following quote from his book *Symmetry*[60]: "...before one studies geometric forms in space with regard to their symmetry one must examine the structure of space itself under the same aspect." (127). Thus, for Weyl, one of the most important considerations about physics is that the *structure* of space-time is in some way fundamental to the *content* of space-time; this "structural realism" forms the foundation for Kaiser's, Bohr and Ulfbeck's, and Stuckey, Silberstein, and Cifone's work which reveals an intrinsic link between the geometry of space-time a la SR and the theory of matter that is QM. Weyl's contribution is even more relevant when one considers Weyl's interest in abstract algebra, the mathematics which, as I will soon show, allows for Kaiser's famous derivation of the canonical commutation relation from space-time symmetries, thus establishing SR as fundamental to QM and emphasizing the importance of Weyl's beloved symmetry.

Another one of Weyl's beliefs that paved the way for his more modern philosophical descendants is the idea that the universe must be 4-dimensional, once again due to the nature of SR and Einstein's interpretation of it. Weyl writes, "In fact physics shows that cosmic time and physical form cannot be dissociated from one another. The new solution of the problem of amalgamating space and time offered by the theory of relativity brings with it a deeper insight into the harmony of action in the world." (*Space, Matter, Time*[61], 6). Thus, Weyl clearly envisioned time as an element of the symmetry he perceived in the universe. Others of Weyl's works ([62], [63]) convey more of Weyl's beliefs connecting the 4-dimensional universe with symmetry and unity, thus anticipating Stuckey, Silberstein, and Cifone's Relational Blockworld interpretation rather impressively.

If Weyl is the grandfather of Relational Blockworld, then J. A. Wheeler would seem to be the grandfather of Genuine Fortuitousness. In his essay "Law without Law"[65], Wheeler plays up the fundamental uncertainty of QM:

...when we view each of the laws of physics—and no laws are more magnificent in scope or better tested—as at bottom statistical in character, then we are at last able to forego the idea of a law that endures from everlasting to everlasting. Individual events. Events beyond law. (204-5)

Wheeler's idea of "events beyond law", while not fully developed at this point in time, is certainly reflected in much of Bohr and Ulfbeck's discussion of the principle of genuine fortuitousness. Though Wheeler may not have proposed as radical an indeterminacy as Bohr and Ulfbeck advocate, his phrasing and the ideas behind it certainly provide the foundation for much of Bohr and Ulfbeck's Genuine Fortuitousness.

Weyl and Wheeler may not have had the physical arguments and proofs to back up their reasoning, but their intellectual descendants do. It is now, in the context of Copenhagen and from the tradition of Weyl and Wheeler, that I turn to Bohr and Ulfbeck and their theory of Genuine Fortuitousness.

3 The Bohr/Ulfbeck Principle of Genuine Fortuitousness

Over a series of three papers[16][17][18] spanning the years from 1995 to 2004, Aage Bohr and Ole Ulfbeck presented and explained their “principle of genuine fortuitousness” (GF). GF exploits the group symmetrical nature of the universe as per the Poincaré group and SR to provide the foundations for non-relativistic quantum mechanics. Bohr and Ulfbeck presented the mathematical foundations of this theory first[16], then proceeded to write two follow-up papers[17][18] explaining a proper interpretation of the mathematics they presented. I will follow Bohr and Ulfbeck’s lead, beginning with the mathematical foundations of the theory and then proceeding to how these results are to be interpreted.

3.1 Derivations of the Canonical Commutation Relation (CCR) From Special Relativity

The first mathematical result that paved the way for GF was the derivation of the canonical commutation relation (CCR) from the relativistic Poincaré group. The CCR is the commutation relation between the position operator x and momentum operator p .

$$[x, p] = i\hbar \tag{6}$$

The non-commutivity of these operators can be shown[12] to directly lead to the differential representation of momentum in x-space and thus to the Schrodinger equation. The differential representation of momentum in position space arises due to the fact that:

$$\left[x, \frac{d}{dx}\right] = -1 \tag{7}$$

A solution to the operator p in Equation 6 could be $p = -i\hbar \frac{d}{dx}$, and since any physical solution must be unique, the differential form of momentum in position space has been proven. When one begins the study of quantum mechanics, one usually starts with either the the differential representation of momentum in position space or the CCR since either one of these results can yield a derivation of the other as well as the Schrodinger equation, yet none of the three are derived from first principles. Thus, one traditional way to view the CCR is as a postulate of quantum mechanics.

However, as Kaiser[35][36] first showed, the CCR can be derived explicitly from the Lie algebra of the Poincaré group when one defines the position and momentum operators in terms of the generators of Lorentz boosts and spatial translations. The second section below will explain Kaiser’s derivation after an introduction to group theory and the Poincaré group. Bohr and Ulfbeck utilize a different method from Kaiser but come to the same result. Both of these derivations provide ample evidence that the CCR can be viewed not a premise of quantum mechanics but rather as a conclusion of relativistic spacetime symmetries as embodied by the Poincaré group.

3.1.1 Introduction to the Poincaré Group

Before I address the Poincaré group specifically, I will address group theory more generally. A group can be loosely defined[34] as “a set of *distinct* elements” which are “endowed with a law of composition”, such as multiplication or addition (2). The elements of a group must satisfy the following four properties[34] for the collection to be considered a group¹⁰:

1. The composition of two elements A and B of a group results in another member of a group. Thus, if the symbol \circ is used to denote the composition, then any group containing A and B as elements must also contain the element $A \circ B$.
2. There exists an element E in the group such that, for any element A , one can write: $E \circ A = A \circ E = A$. This element E is called the identity.
3. There exists a unique element B for any element A such that $A \circ B = B \circ A = E$. This element B is called the inverse of A and is denoted by A^{-1} .
4. The composition under which the group is defined must be associative; that is, it must be the case that $(A \circ B) \circ C = A \circ (B \circ C)$ for any A , B and C which are elements of the same group.

The elements of a given group may be numbers or matrices of any rank¹¹, but it is required that all elements of a given group be the same kind of mathematical object. Since a group can be represented by matrices, one can talk about the group’s Lie algebra, which characterizes the commutation relations among the individual elements of the group. The Lie algebra of the Poincaré group in particular will be important for Kaiser’s derivation of the CCR.

Physicists in general tend to be primarily interested with groups of transformations (such as rotations, permutations, reflections, translations, etc.) of physical systems[34] (4). Such transformations are called symmetry transformations if they leave the physical system unchanged. The Poincaré group is such a group of transformations, specifically a group that leaves Minkowski space-time invariant. Minkowski space-time (or M4 hereafter) is a 4-dimensional space-time defined by the following metric of special relativity[37][41]:

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{8}$$

As this metric already shows, the behavior of time (dt) in this equation is fundamentally different from the behavior of the other three dimensions (dx , dy , dz) in that it differs by a sign. This will lead to important consequences for the Lie algebra of the Poincaré group.

How many elements does the Poincaré group need? It certainly needs enough to form a closed algebra, but it should also be shown that the Poincaré group is suitably

¹⁰These properties are outlined in more mathematically rigorous language in Joshi [34] pages 2-3

¹¹In general, the elements of a group can be any type of mathematical object.

general; that is, if the Poincaré group indeed represents *all* of the possible space-time permutations, it must be shown not only that all of its elements form a closed group algebra but also that the Poincaré group is not a subgroup of some other group. If all of the possible permutations of M4 (e.g. translations and rotations) are elements of the Poincaré group, then the following method for determining the number of elements should suffice: first, since there are four coordinates in M4 (t, x, y, and z, which we shall relabel as $x_0, x_1, x_2,$ and x_3), a permutation could consist of any of the following:

1. A change in none of the coordinates ($\frac{4!}{0!4!} = 1$ way of doing this)
2. A change in one of the coordinates ($\frac{4!}{1!3!} = 4$ ways of doing this)
3. A change in two of the coordinates ($\frac{4!}{2!2!} = 6$ ways of doing this)
4. A change in three of the coordinates ($\frac{4!}{3!1!} = 4$ ways of doing this)
5. A change in all four of the coordinates ($\frac{4!}{4!0!} = 1$ way of doing this)

Thus, there should be a total of 16 different elements in the Poincaré group. However, there is a constraint upon these coordinate changes: they must leave the space-time metric specified in Equation 8 unchanged. The changes listed above must also operate with only a single, unique variable condition (such as θ in a rotation, for instance); otherwise, the change in 2 coordinates could be enacted by simply changing one coordinate and then changing the other, and such a change would not be unique as we require group elements to be.

To leave space-time invariant, the transformation must leave the ds term in Equation 8 unchanged. I will assume, then, that this quantity is 0 for a given system and attempt to determine whether the changes described above preserve the quantity ds . For case 1, in which none of the coordinates is changed, the result is trivially true; this is simply an identity transformation. As I shall show shortly, however, the changes in 1 or 2 coordinates (which will correspond to translations and rotations) become identity transformations when their variables are zero, and thus to add an identity element would be redundant.

For case 2, a change in one coordinate, the transformation could simply correspond to a change by a constant. For instance, if dx were to increase or decrease by 2 and cdt to increase or decrease by 2, the quantity ds^2 would remain unchanged. Another way of thinking about this situation is envisioning a 4D hypersphere with radius ds^2 . The transformations of the Poincaré group are transformations that allow motion along the surface of this hypersphere without leaving the hypersphere. Translations are an example of such transformations, and thus the four types of translations (one temporal and three spatial) are included in the Poincaré group.

Likewise, one could change two variables by incorporating sines and cosine or hyperbolic sines and cosines (due to the change in sign for ct in Equation 8) and stay on the surface of the hypersphere as well. These transformations require only a single, unique variable (the angle of rotation) to transform both coordinates, and thus all possible 2-coordinate transformations are elements of the Poincaré group. There are 3 of one character, however, and 3 of a separate character since 3 involve only spatial coordinates (and sines and cosines) while the other 3 involve one spatial coordinate and time (and hyperbolic sines and cosines).

Finally, the only functions that change 3 or 4 coordinates and leave space-time invariant are combinations of the translations and rotations already described. There is no set of three functions that take the same argument and, when squared and added together, yield a constant so that space-time can be left invariant, and there are certainly no transformations that change 4 coordinates that can do it (again, excepting sines, cosines, hyperbolic sines, and hyperbolic cosines). Thus, since cases 4 and 5 above add no new symmetry to the Poincaré group that it did not previously possess via translations and rotations, the only elements of the Poincaré group can be the 4 translations, 3 rotations in space, and 3 rotations in time and space. These elements form a complete algebra, thus supporting the logic of this exercise. The Poincaré group then consists of 10 elements: 1 time translation, 3 spatial translations, 3 spatial rotations, and 3 space-time rotations which are called Lorentz boosts. These Lorentz boosts derive from the Lorentz transformations and represent changes in velocity.

Calling the Lorentz transformations “rotations” may seem a bit counter-intuitive, but Bohr and Ulfbeck[16] provide a simple definition for the Lorentz boosts (L) which shows their rotational nature. They define the boosts as a rotation through an angle l_u such that:

$$\tanh l_u = \frac{u}{c} \tag{9}$$

This definition allows one to write the Lorentz transformations as:

$$x' = x \cosh l_u + ct \sinh l_u \text{ and } ct' = x \sinh l_u + ct \cosh l_u \tag{10}$$

Thus, the Lorentz boosts can be written in a form similar to rotations with hyperbolic sine and cosine functions instead of the sine and cosine functions used for rotations. The quantity l_u in the above equations is commonly known as the rapidity. Thus, using the rapidity, the hyperbolic rotation previously predicted is shown to occur via Lorentz transformations.

For each of these three kinds of elements in the Poincaré group (translations, Lorentz boosts, and rotations) there exists a generator. Generators are defined such that all spatial rotations, for example, can be expressed as a function of the generator [59]. The typical form of the generator is, for example:

$$R(\theta) = e^{iJ\theta} \tag{11}$$

Where $R(\theta)$ is a rotation, J is the generator of rotations, and θ is the angle of the rotation generated. This relation between the transformation and its generator arises from the fact that:

$$R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2) \tag{12}$$

Equation 11 satisfies this relation, and thus it provides the proper relationship between the transformation and its generator. The generator of each of these subgroups, then, provides a wealth of information concerning the subgroup it generates, and the Poincaré group’s behavior can be described by the Lie algebra of the generators of the Poincaré group’s elements. In fact, all of the information about the symmetries of the transformation is embodied by the generator; for instance, a rotation is a rotation only

by virtue of its generator J which contains all of the essential mathematical structure of a rotation. Rotations are rotations *about an angle* θ , however, both because of the generator J and the given angle θ . Thus, all of the information about the space-time symmetries of a given transformation is present in the generator of that transformation.

The generators for the four subgroups of the Poincaré group are T^0 , the time translation generator; T_m , the spatial translation generator for x_m where $m \neq 0$; J_n , the spatial rotation generator for $n = 1, 2, 3$; and K_n , the spatial Lorentz boost generator for $n = 1, 2, 3$. The Lie algebra for the Poincaré group has been previously derived[59][35][36] and can be written as the following commutation relations:

$$\begin{aligned} [T^0, T_n] &= 0 \\ [T^0, J_n] &= 0 \\ [T^0, K_n] &= iT_n \\ [T_n, T_m] &= 0 \\ [T_m, J_n] &= i\epsilon^{mnl}T_l \\ [T_m, K_n] &= \frac{-i}{c^2}\delta_{mn}T^0 \\ [J_m, J_n] &= i\epsilon^{mnl}J_l \\ [K_m, J_n] &= i\epsilon^{mnl}K_l \\ [K_m, K_n] &= \frac{-i}{c^2}\epsilon^{mnl}J_l^{12} \end{aligned}$$

It is the Lie algebra above that Stuckey, Silberstein, and Cifone (SSC)[53][54][55][57] and Kaiser[35][36] invoke to derive the CCR. The c^2 term in these commutators accounts for the proper units. The units of a generator are the inverse units of the physical variable change generated. For instance, the units of the time translation generator are inverse seconds, the units of the spatial translation generators are inverse meters, and so on. This requirement of units makes sense given the form of Equation 11 since the exponential quantity must be unitless.

Before leaving the topic, one might be skeptical of the commutator $[T_m, K_n] = \frac{-i}{c^2}\delta_{mn}T^0$ since this result seems to come from nowhere (and since belief in this particular commutator will be essential to believing the derivation of the CCR to follow). The following is a derivation of this relation:

First, define $L(\epsilon)$ as Lorentz boosts through an infinitesimal velocity ϵ , K as the Lorentz boost generator, $N(\epsilon)$ as a spatial translation of infinitesimal distance, and T as the spatial translation generator. It is obvious that, if N is in a different spatial direction than L , the two quantities will commute since they will be affecting different variables. Thus, I will simply assume that N , L , K , and T are all in the same spatial direction and introduce a Kronecker delta of form δ_{mn} to the commutator $[T_m, K_n]$.

Next, I will examine the quantity $N(\epsilon)L(\epsilon)N^{-1}(\epsilon)L^{-1}(\epsilon)$ in two different ways: first, I will examine how it acts on the coordinates (x, t) and then use Equation 11 to determine how this quantity is related to the commutator of K and T . I define the action of $N(\epsilon)$ and $L(\epsilon)$ on (x, t) as follows:

¹²Most derivations like Kaiser (160) use P instead of T since momentum (usually symbolized as P) turns out to be directly proportional to the generator of translations; however, I have changed it to T here so that future arguments based on momentum will be less confusing.

$$N(\varepsilon)(x, t) = (x + \varepsilon, t) \quad (13)$$

$$L(\varepsilon)(x, t) = (x + \varepsilon t, t + \frac{\varepsilon x}{c^2}) \quad (14)$$

Where the action of $L(\varepsilon)$ is the Lorentz transformations of coordinates to a first approximation. The action of the entire quantity $N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)L^{-1}(\varepsilon)$ on (x, t) is as follows:

$$N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)L^{-1}(\varepsilon)(x, t) = N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)(x - \varepsilon t, t - \frac{\varepsilon x}{c^2}) \quad (15)$$

$$= N(\varepsilon)L(\varepsilon)(x - \varepsilon t - \varepsilon, t - \frac{\varepsilon x}{c^2}) \quad (16)$$

$$= N(\varepsilon)(x - \varepsilon t - \varepsilon + \varepsilon(t - \frac{\varepsilon x}{c^2}), t - \frac{\varepsilon x}{c^2} + \varepsilon(\frac{x - \varepsilon t - \varepsilon}{c^2})) \quad (17)$$

$$= N(\varepsilon)(x - \varepsilon - \frac{\varepsilon^2 x}{c^2}, t + \frac{\varepsilon^2 t - \varepsilon \varepsilon}{c^2}) \quad (18)$$

$$= (x + \frac{\varepsilon^2 x}{c^2}, t - \frac{\varepsilon \varepsilon}{c^2} - \frac{\varepsilon^2 t}{c^2}) \quad (19)$$

$$\approx (x, t - \frac{\varepsilon \varepsilon}{c^2}) \text{ in the non-relativistic regime} \quad (20)$$

This shows that $N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)L^{-1}(\varepsilon)$ simply results in an infinitesimal time translation N_0 , and thus

$$N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)L^{-1}(\varepsilon) = N_0(-\frac{\varepsilon \varepsilon}{c^2}) \quad (21)$$

Now, using the generators, the quantity $N(\varepsilon)L(\varepsilon)N^{-1}(\varepsilon)L^{-1}(\varepsilon)$ can be written as $e^{iT\varepsilon}e^{iK\varepsilon}e^{-iT\varepsilon}e^{-iK\varepsilon}$. This quantity can be approximated as follows, to first order:

$$e^{iT\varepsilon}e^{iK\varepsilon}e^{-iT\varepsilon}e^{-iK\varepsilon} \approx (I + i\varepsilon T)(I + i\varepsilon K)(I - i\varepsilon T)(I - i\varepsilon K) \quad (22)$$

$$= I + \varepsilon \varepsilon [T, K] + O(\varepsilon^2 \varepsilon) + O(\varepsilon^2 \varepsilon) \quad (23)$$

$$\approx I + \varepsilon \varepsilon [T, K] \quad (24)$$

Setting the two quantities equal to each other yields:

$$N_0(-\frac{\varepsilon \varepsilon}{c^2}) = I + \varepsilon \varepsilon [T, K] \quad (25)$$

$$e^{-iT^0 \frac{\varepsilon \varepsilon}{c^2}} = I + \varepsilon \varepsilon [T, K] \quad (26)$$

$$I - iT^0 \frac{\varepsilon \varepsilon}{c^2} \approx I + \varepsilon \varepsilon [T, K] \quad (27)$$

$$-\frac{i}{c^2} T^0 = [T, K] \quad (28)$$

Thus, $[T_m, K_n] = \frac{-i}{c^2} \delta_{mn} T^0$ once the Kronecker delta, which allows T_m and K_n to commute when $n \neq m$, has been incorporated.

3.1.2 Kaiser's Derivation of the CCR from the Poincaré Group

Kaiser's original results[35][36] are reproduced in a much simpler and easier to follow format by SSC[53][54][55][57], and as such, I will be reproducing Kaiser's result in the same form as SSC. SSC begin by rewriting the Lie algebra for the unrestricted Poincaré group described above while neglecting any relations between commuting generators. He lists the Lie algebra as follows:

$$\begin{aligned} [T^0, K_n] &= iT_n \\ [T_m, J_n] &= i\epsilon^{mnl}T_l \\ [T_m, K_n] &= \frac{-i}{c^2}\delta_{mn}T^0 \\ [J_m, J_n] &= i\epsilon^{mnl}J_l \\ [K_m, J_n] &= i\epsilon^{mnl}K_l \\ [K_m, K_n] &= \frac{-i}{c^2}\epsilon^{mnl}J_l \end{aligned}$$

Taking the low-velocity limit of these commutators reduces $[T_0, K_n]$ and $[K_m, K_n]$ to zero, leaving the non-zero commutators as follows:

$$\begin{aligned} [T_m, J_n] &= i\epsilon^{mnl}T_l \\ [T_m, K_n] &= \frac{-i}{c^2}\delta_{mn}T^0 \\ [J_m, J_n] &= i\epsilon^{mnl}J_l \\ [K_m, J_n] &= i\epsilon^{mnl}K_l \end{aligned}$$

At this point, Kaiser's[36] "Postulate Q" (160) should be invoked. This postulate states that quantum mechanics is based on unitary representations of the Poincaré group which appear as quantum mechanical operators. Thus, the generators of certain transformations can be seen to directly correspond with quantum mechanical observables. For instance, the generator of a rotation should be a physical quantity that characterizes a rotating object, and thus angular momentum is related to the generator J .

By direct analogy, generalized linear 4-momentum is related to the generator of translations. This result is particularly important, since $P_n \propto T_n$ where P_n is the generalized linear 4-momentum. To relate these two values, a proportionality constant that accounts for the differing dimensions of the two quantities should be incorporated. Since T_n is in units of $\frac{1}{m}$ for $n = 1, 2, 3$ and units of $\frac{1}{t}$ for $n = 0$ and since P_n is in units of $\frac{kgm}{s}$ for $n = 1, 2, 3$ and units of joules for $n = 0$, a constant with units of Joule-seconds is needed for equality to hold instead of proportionality. \hbar is such a constant. As such, the relation $P_n = \hbar T_n$ obtains.

Now, for $n = 0$, the 4-momentum P_0 is the energy, meaning that the relativistic invariant can be invoked:

$$P_0^2 = P_{spatial}^2 c^2 + (Mc^2)^2 \quad (29)$$

Where $P_{spatial}$ is the momentum operator in 3-space and M is the mass operator whose value is mI where m is the mass and I is the identity operator. Taking the low-velocity limit of this equation yields:

$$\frac{P_0^2}{c^4} = \frac{P_{spatial}^2}{c^2} + M^2 \quad (30)$$

$$\frac{P_0}{c^2} = \left(\frac{P_{spatial}^2}{c^2} + M^2 \right)^{\frac{1}{2}} \quad (31)$$

$$\frac{P_0}{c^2} = M \left(1 + \frac{P_{spatial}^2}{M^2 c^2} \right)^{\frac{1}{2}} \quad (32)$$

$$\frac{P_0}{c^2} \approx M \left(1 + \frac{P_{spatial}^2}{2M^2 c^2} \right) \quad (33)$$

$$\frac{\hbar T_0}{c^2} = M + \frac{P_{spatial}^2}{2Mc^2} \quad (34)$$

$$\frac{\hbar T_0}{c^2} \approx M \quad (35)$$

Thus, the mass operator M can be substituted for T_0 in the Lie algebra of the Poincaré group. Replacing T_0 with M in the Lie algebra of the Poincaré group yields:

$$\begin{aligned} [T_m, J_n] &= i\epsilon^{mnl} T_l \\ [T_m, K_n] &= \frac{-i}{\hbar} \delta_{mn} M \\ [J_m, J_n] &= i\epsilon^{mnl} J_l \\ [K_m, J_n] &= i\epsilon^{mnl} K_l \end{aligned}$$

The commutator of interest in this Lie algebra is $[T_m, K_n] = \frac{-i}{\hbar} \delta_{mn} M$, the commutator between the spatial translation generator and the Lorentz boost generator. These two generators can be related to the momentum operator P and the position operator Q by the following relations:

$$P_s \equiv \hbar T_s \quad (36)$$

$$Q_n \equiv \frac{\hbar}{m} K_n \quad (37)$$

The first relation between P_s and T_s has already been shown. The position operator form, however, seems to come out of nowhere. This relation makes more sense, however, if one considers the form a rotation generator can take in terms of other generators:

$$J_k = x_i P_j - x_j P_i \quad (38)$$

This representation of J follows from the fact that J represents the angular momentum of the system and angular momentum is (classically) the cross-product of position and momentum. As a direct analogy, the Lorentz boost, also a rotation, must also be of the form

$$K_k = x_0 P_k - \frac{x_k P_0}{c^2} \quad (39)$$

where the extra constant c^2 emerges only to keep the units consistent. In the low-velocity limit, again, $P_0 = Mc^2$, which reduces the equation greatly:

$$K_k = x_0 P_k - x_k M \quad (40)$$

$$-\frac{K_k}{M} = x_k - x_0 \frac{P_k}{M} \quad (41)$$

However, $\frac{P_k}{M}$ is just the traditional definition of the velocity operator V and x_0 is the time t , thus reducing the equation to:

$$-\frac{K_k}{M} = x_k - Vt \quad (42)$$

This quantity on the right-hand side is just $Q(t)$, the position as a function of time, and thus the relation of $Q_n \equiv \frac{\hbar}{m} K_n$, which incorporates the constant \hbar to correct the dimensions, is well supported by classical analogues and physical intuition. The commutator between the two quantities Q and P , then, can be written as:

$$[P_s, Q_n] = \frac{\hbar^2}{m} [T_s, K_n] = \frac{-\hbar^2}{m} \frac{i}{\hbar} \delta_{mn} m I = -i\hbar \delta_{mn} I \quad (43)$$

This result is simply the generalization of Equation 6 to multiple dimensions!

What Stuckey et. al. and Kaiser have done in this proof is, essentially, to derive the CCR from the non-commutivity of spatial translations and Lorentz boosts. It is important to note that Galilean boosts of the form

$$x' = x - vt \text{ and } t' = t \quad (44)$$

would not yield the same result since such Galilean boosts commute with spatial translations. SSC[57] cite Bohr and Ulfbeck's[16] observation that the Lorentz boosts used in this derivation are not strictly Lorentz boosts. Rather, they change the coordinates such that:

$$x' = x - vt \text{ and } t' = t - \frac{vx}{c^2} \quad (45)$$

This transformation lacks the factor of $\gamma = (1 - \frac{v^2}{c^2})^{\frac{1}{2}}$ contained in the true form of the Lorentz boosts. Thus, the regime of this derivation is only weakly relativistic. SSC call this a ‘‘Kaiser boost’’ (11) and the space-time in which occurs ‘‘K4’’ in honor of Kaiser’s derivation. The new space K4 is necessitated since the classical space (referred to as ‘‘G4’’ in honor of Galileo and his eponymous velocity transformations) allows for a commuting spatial translation generator and boost generator while the M4 requirement of the extra γ factor in the boost coordinate transformation is not met.

This result suggests that the majority of physicists, who tend to treat non-relativistic quantum mechanics as residing in G4, are wrong; one cannot separate quantum mechanics from relativistic effects if the CCR really originates in space-time symmetries. The Lie algebra of the Poincaré group is all that is necessary for Kaiser and SSC to derive the foundation of non-relativistic quantum mechanics. A weakly relativistic

space-time background is necessary even when doing non-relativistic quantum mechanics.

What is more, the CCR result makes no mention of any evolution in time. As opposed to others who might view space-time as itself evolving¹³, SSC's and Kaiser's formalism does not require any sort of dynamism. This will be a key point for all of SSC's and Bohr and Ulfbeck's derivations: both groups derive already-known physical results using none of the dynamical framework traditionally utilized in such derivations.

3.1.3 Bohr and Ulfbeck's Derivation of the CCR

Bohr and Ulfbeck[16] also derive the CCR from symmetry groups, but their more detailed derivation does not appeal to the Poincaré group. Though the mathematics behind Bohr and Ulfbeck's proof is not made as explicit as Stuckey et. al.'s and Kaiser's derivations cited in the previous section, I will go through the reasoning behind Bohr and Ulfbeck's derivation in much more detail than they use so that their method will be more obvious.¹⁴

The derivation begins with some facts about rotations. In general, for all rotations $R(a)$, the following is true:

$$R(a_1) + R(a_2) = R(a_1 + a_2) \quad (46)$$

Since Equations 9 and 10 in the previous section evince the Lorentz boost's behavior as a rotation, one can therefore conclude that likewise:

$$L(l_{u1} + l_{u2}) = L(l_{u1}) + L(l_{u2}) \quad (47)$$

Where $L(l_u)$ is the Lorentz boost through an angle of l_u .¹⁵

There are also two important relations between the Lorentz boosts and the translation operator (both spatial and temporal) $T(x, t)$ ¹⁶. These relations state that:

$$L(l_u; x, t) = T(x, t)L(l_u)T^{-1}(x, t) \text{ and } T(x', t') = L(l_u)T(x, t)L^{-1}(l_u) \quad (48)$$

From these two equations it follows that

$$L(l'_u; x', t') = L(l_u)L(l'_u; x, t)L^{-1}(l_u) \quad (49)$$

This equation allows Bohr and Ulfbeck to derive the proper relation between the operator $L(l_u; x, t)$ and its generator. This relation states that

$$L(l_u; x, t) = e^{il_u \epsilon(x, t)} \quad (50)$$

¹³See [25] for more on this position

¹⁴I am grateful to John Boccio for his help in working out some of the trickier steps that Bohr and Ulfbeck omitted in their original paper.

¹⁵In this derivation, the notation $L(l_u)$ will denote a Lorentz boost through an angle of l_u at the point $x = t = 0$ while the notation $L(l_u; x, t)$ will denote a Lorentz boost through an angle of l_u at the point $\{x, t\}$

¹⁶I return to SSC's[53][54][55][57] original formalism here for translation operators. In Bohr and Ulfbeck's original paper, the symbol $J(x, t)$ was used for translation operators, but since $J(x, t)$ signifies rotation operators elsewhere in this paper, it is less confusing to continue with SSC's formalism.

Where $\epsilon(x, t)$ is the generator of the Lorentz boost. The generator for space-time translations, on the other hand, is written

$$T(x, t) = e^{i(\omega t - kx)} \quad (51)$$

Where ω is the generator of temporal translations while k is the generator of spatial translations. The sign difference between the k and ω terms represents the sign difference in the M4 space-time metric between position and time. Bohr and Ulfbeck also give the following relations between the generators:

$$\omega^2 - c^2 k^2 = \omega_0^2 \text{ and } \omega_0 = ck_0 \quad (52)$$

Where ω_0 and k_0 are the frequency and wave number respectively. This relationship draws upon the parallel between Equation 51 and the equation of motion for a free particle in one dimension.

Bohr and Ulfbeck now turn their attention to the Lie algebra of the three generators ϵ , ω , and k . To begin with, Bohr and Ulfbeck make several approximations that assume that the system being measured does not involve relativistic velocities. For position, they simplify the Lorentz transformation as:

$$x' = \gamma(x - ut) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}(x - ut) \approx (x - ut)\left(1 + \frac{u^2}{2c^2}\right) \approx x - ut + \frac{u^2 x}{2c^2} \quad (53)$$

Likewise, the Lorentz transformation of time is simplified to:

$$t' = \gamma\left(t - \frac{ux}{c^2}\right) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\left(t - \frac{ux}{c^2}\right) \approx \left(t - \frac{ux}{c^2}\right)\left(1 + \frac{u^2}{2c^2}\right) \approx t - \frac{ux}{c^2} + \frac{u^2 t}{2c^2} \quad (54)$$

The final approximation Bohr and Ulfbeck make before using the relations between the translation and Lorentz boost operators to determine the Lie algebra of the generators is an approximation for the angle of the Lorentz boost l_u .

$$\tanh l_u = \frac{u}{c}, \text{ so } l_u = \tanh^{-1} \frac{u}{c} = \frac{u}{c} + \frac{u^3}{3c^3} + \dots \approx \frac{u}{c} \quad (55)$$

Again, the higher order terms in this approximation are ruled out by virtue of the fact that the velocities represented by u are non-relativistic and, as such, are much smaller than c .

Returning to the second part of equation 48, Bohr and Ulfbeck are now able to more explicitly calculate the quantity $J(x', t')$ on the left hand side of this equation. They write:

$$J(x', t') = e^{i(\omega t' - kx')} \quad (56)$$

$$= e^{i\left(-k\left(x - ut + u^2 \frac{x}{2c^2}\right) + \omega\left(t - u \frac{x}{c^2} + u^2 \frac{t}{2c^2}\right)\right)} \quad (57)$$

$$= e^{-ikx + i\omega t + (ikt - i\omega \frac{x}{c^2})u + (-ik \frac{x}{2c^2} + i\omega \frac{t}{2c^2})u^2} \quad (58)$$

An approximation of the Taylor series of this function yields

$$J(x', t') = 1 + (-ikx + i\omega t) + (ikt - i\omega \frac{x}{c^2} + (-ikx + i\omega t)(ikt - \frac{i\omega x}{c^2})) \quad (59)$$

$$\begin{aligned} & + (ikx - i\omega t)(-ikt + \frac{i\omega x}{c^2})u + \frac{(-ikx + i\omega t)^2}{2} + \dots \\ & = e^{i(\omega t - kx)} + (ikt - \frac{i\omega x}{c^2} + \frac{1}{2}(kxkt - \frac{kx\omega x}{c^2} - \omega tkt + \frac{\omega t\omega x}{c^2})) \\ & \quad + \frac{1}{2}(ktkx - kt\omega t - \frac{\omega xkx}{c^2} + \frac{\omega x\omega t}{c^2})u \end{aligned} \quad (60)$$

Now the right hand side of the second part of Equation 48 must be reduced as well. Written in terms of the generators, this side is:

$$L(l_u)T(x, t)L^{-1}(l_u) = e^{il_u \epsilon(x, t)} e^{i(\omega t - kx)} e^{-il_u \epsilon(x, t)} \quad (61)$$

Using the previously-produced approximations reduces this side of the equation to:

$$L(l_u)T(x, t)L^{-1}(l_u) = (1 + \frac{i u}{c} \epsilon) e^{i(\omega t - kx)} (1 - \frac{i u}{c} \epsilon) \quad (62)$$

$$\approx e^{i(\omega t - kx)} + \frac{i u}{c} (\epsilon e^{i(\omega t - kx)} - e^{i(\omega t - kx)} \epsilon) \quad (63)$$

Setting the two side of the equation equal to each other allows for some cancellation of terms, and once higher order terms are neglected and exponentials are approximated, the commutation relations $[k, \epsilon]$ and $[\omega, \epsilon]$ emerge:

$$\begin{aligned} & e^{i(\omega t - kx)} + (ikt - \frac{i\omega x}{c^2} + \frac{1}{2}(kxkt - \frac{kx\omega x}{c^2} - \omega tkt + \frac{\omega t\omega x}{c^2}) + \frac{1}{2}(ktkx - \\ & \quad kt\omega t - \frac{\omega xkx}{c^2} + \frac{\omega x\omega t}{c^2}))u = e^{i(\omega t - kx)} + \frac{i u}{c} (\epsilon e^{i(\omega t - kx)} - e^{i(\omega t - kx)} \epsilon) \end{aligned} \quad (64)$$

$$ikt - \frac{i\omega x}{c^2} + \dots = \frac{i}{c} (-ix\epsilon k - it\epsilon\omega + ixk\epsilon + it\omega\epsilon + \dots) \quad (65)$$

$$ikt - \frac{i\omega x}{c^2} \approx \frac{[k, \epsilon]x - [\omega, \epsilon]t}{c} \quad (66)$$

Thus, the commutation relations $[k, \epsilon]$ and $[\omega, \epsilon]$ are:

$$[k, \epsilon] = -\frac{i\omega}{c} \quad (67)$$

$$[\omega, \epsilon] = -ikc \quad (68)$$

These commutation relations are important for determining the analogue to the position operator in terms of the k and ω operators as well as for the final calculation of the CCR.

Now, seeing as the operator $\epsilon(x, t)$ is a function of both position and time with no explicit dependence on either, the next step in the derivation is to determine the relationship between $\epsilon(0, 0)$ and $\epsilon(x, t)$ so as to determine a definition for the position operator x . The relationship between $\epsilon(0, 0)$ and $\epsilon(x, t)$ once again involves the spatial and temporal translation operators:

$$\epsilon(x, t) = T(x, t)\epsilon(0, 0)T^{-1}(x, t) \quad (69)$$

To determine the time and position dependence of ϵ explicitly, the following two relations between operators are needed[12]:

$$e^{xA}Be^{-xA} = B + [A, B]x + [A, [A, B]]\frac{x^2}{2} + \dots \quad (70)$$

$$e^{A+B} = e^Ae^Be^{-\frac{1}{2}[A, B]} \quad (71)$$

For any operators A and B. Likewise, the unstated commutation relation between k and ω is needed:

$$[k, \omega] = 0 \quad (72)$$

This commutation relation makes sense since translations in position and time have no affect on one another in the absence of a Lorentz boost, and thus the generators of these two translations should commute. Using the commutation relations previously derived as well as these two identities allows for the calculation of the $\epsilon(x, t)$:

$$\epsilon(x, t) = e^{i(\omega t - kx)}\epsilon(0, 0)e^{-i(\omega t - kx)} \quad (73)$$

$$= e^{i\omega t}e^{-ikx}\epsilon(0, 0)e^{-i\omega t}e^{ikx} \quad (74)$$

$$= e^{i\omega t}e^{-ikx}\epsilon(0, 0)e^{ikx}e^{-i\omega t} \quad (75)$$

$$= e^{i\omega t}(\epsilon(0, 0) - [k, \epsilon(0, 0)]ix + [k, [k, \epsilon(0, 0)]]\frac{(-ix)^2}{2} + \dots)e^{-i\omega t} \quad (76)$$

$$= e^{i\omega t}(\epsilon(0, 0) - (-\frac{i\omega}{c})ix + [k, (-\frac{i\omega}{c})]\frac{(-ix)^2}{2} + \dots)e^{-i\omega t} \quad (77)$$

$$= e^{i\omega t}(\epsilon(0, 0) - \frac{x\omega}{c})e^{-i\omega t} \quad (78)$$

$$= e^{i\omega t}(\epsilon(0, 0))e^{-i\omega t} - e^{i\omega t}(\frac{x\omega}{c})e^{-i\omega t} \quad (79)$$

$$= (\epsilon(0, 0) + [\omega, \epsilon(0, 0)]it + [\omega, [\omega, \epsilon(0, 0)]]\frac{(it)^2}{2} + \dots) \quad (80)$$

$$- (\frac{x\omega}{c} + [\omega, \frac{x\omega}{c}]it + [\omega, [\omega, \frac{x\omega}{c}]]\frac{(it)^2}{2} + \dots)$$

$$= \epsilon(0, 0) + (-ikc)it - \frac{x\omega}{c} \quad (81)$$

$$= \epsilon(0, 0) + kct - \frac{\omega x}{c} \quad (82)$$

The position dependence of this equation for $\epsilon(x, t)$ can be made more explicit by taking the lower limit on spatial resolution¹⁷. In this limit, $|k| \ll k_0$ and $\omega \approx \omega_0$. This reduces $\epsilon(x, t)$ to:

$$\epsilon(x, t) = \epsilon(0, 0) + kct - \frac{\omega_0 x}{c} = \epsilon(0, 0) + kct - k_0 x = \epsilon(0, t) - k_0 x \quad (83)$$

Now, since the generator of Lorentz boosts centered at a position x should leave the position invariant with respect to time, the time operator can be written as

$$x(t) = \frac{\epsilon(x, t)}{k_0} + x = \frac{\epsilon(0, t)}{k_0} \quad (84)$$

This definition of $x(t)$ is in terms of a generator ϵ whose Lie algebra is already known. Thus, The commutator $[k, x(t)]$ can be calculated simply:

$$[k, x(t)] = \frac{1}{k_0} [k, \epsilon(0, t)] = -\frac{1}{k_0} \frac{i\omega_0}{c} = -i \quad (85)$$

This commutator will be of more interest once the momentum operator p is fully specified. Using an analogue to classical physics again, one would expect the momentum to be approximately equal to

$$p = \frac{k}{k_0} mc \quad (86)$$

Where the m , c , and k_0 have been used to modify the generator k_0 to represent the appropriate scaling for momentum. Finally, then, the commutator $[p, x(t)]$ can be calculated:

$$[p, x(t)] = \frac{mc}{k_0} [k, x(t)] = -i \frac{mc}{k_0} = -i\hbar \quad (87)$$

Where the constant \hbar has been introduced as $\frac{mc}{k_0}$. This final equation provides the familiar result of the canonical commutation relation, as expected.

As was the case in SSC's and Kaiser's derivations of the CCR, this derivation also resides in a weakly relativistic regime in that it takes advantage of the structure of Lorentz boosts while making simplifying assumptions that keep $u \ll c$. Like SSC's, then, Bohr and Ulfbeck's proof necessitates K4 instead of G4 as the structure of space-time for non-relativistic QM.

What both Kaiser's proof and Bohr and Ulfbeck's proof suggest, then, is that it is possible to derive non-relativistic QM from SR but that the security gained from such a derivation comes with a price. That price is the relegation of non-relativistic QM to the weakly relativistic space-time of K4. The other price concerns ontological priority; SR (as it appears in K4) is necessarily ontologically prior to QM since it is from SR's relations and geometry that QM laws come. The problem of the dependence of a traditionally dynamical theory of matter like non-relativistic QM on a kinematical theory of space-time like special relativity will be addressed later on; for now, the focus will shift to another quantum mechanical result which can also be derived *a priori* from the geometries of K4 space-time: the density matrix.

¹⁷This spatial resolution will become important later in the discussion of coarse-graining and fine-graining as they pertain to the laws of QM

3.2 Bohr and Ulfbeck's Derivation of the Density Matrix

Having derived the CCR from space-time symmetries in K4 as opposed to QM's traditional Hilbert space, the question of whether the same space-time considerations can describe other aspects of a quantum system in accordance with non-relativistic QM arises. To show that space-time geometries do, in fact, yield the same quantum mechanical results, Bohr and Ulfbeck[18] derive many of the properties of the density matrix from the algebra of irreducible representations of a symmetry group. This derivation is also referred to by SSC[57] and is reproduced by SSC in more detail.

The derivation of the density operator, according to an addition to Bohr and Ulfbeck's proof by Stuckey et. al.[57], begins with an equation from Georgi[30] which appears on page 18 of his book:

$$\sum_{g \ni G} \frac{n_a}{N} [D_a(g^{-1})]_{kj} [D_b(g)]_{lm} = \delta_{ab} \delta_{jl} \delta_{km} \quad (88)$$

In this equation, the terms $D_a(g^{-1})$ and $D_b(g)$ represent the irreducible representations of a particular symmetry group. Any non-abelian group (meaning any group in which not all of the elements commute) has a multi-dimensional irreducible matrix representation. Since Georgi also proves the theorem that any representation of a finite group like the symmetry group considered here is equivalent to a unitary representation, I shall assume that $D_a(g^{-1})$ and $D_b(g)$ are both unitary matrices. The symbol n_a in the above equation represents the dimension of this representation, and the symbol N represents the order of the group. The terms g are the elements of the group G , the symmetry group under consideration here. The above equation is derived from first principles by Georgi, so those interested in the origin of this equation are referred to his excellent book, but I will not reproduce Georgi's proof here.

Since only one representation is under consideration, the equation above simplifies into the following form:

$$\sum_{g \ni G} \frac{n}{N} [D(g^{-1})]_{kj} [D(g)]_{lm} = \delta_{jl} \delta_{km} \quad (89)$$

where D is now the unitary, irreducible matrix representation of the symmetry group. Bohr and Ulfbeck use the symbol U to denote these matrices instead to emphasize the fact that they are unitary matrices and write the term $[D(g^{-1})]_{kj}$ as $\langle k | U | j \rangle$, but aside from these notational differences, Bohr and Ulfbeck's proof follows SSC's precisely. Since SSC's notation follows Georgi's, I will follow SSC's notation in my own derivation of the density matrix.

Now, multiplying the above equation by the quantity $[D(g')]_{jk}$ and summing over j and k changes the equation to:

$$\sum_j \sum_k \sum_{g \ni G} \frac{n}{N} [D(g^{-1})]_{kj} [D(g)]_{lm} [D(g')]_{jk} = \sum_j \sum_k \delta_{jl} \delta_{km} [D(g')]_{jk} = [D(g')]_{lm} \quad (90)$$

The sums on the left-hand side of the equation can be rearranged as follows (since $[D(g)]_{lm}$ is some constant whose bra and ket operators are not summed over):

$$\begin{aligned}
\sum_{g \ni G} \frac{n}{N} [D(g)]_{lm} \sum_k \sum_j [D(g^{-1})]_{kj} [D(g')]_{jk} &= \sum_{g \ni G} \frac{n}{N} [D(g)]_{lm} \sum_k [D(g^{-1}D(g'))]_{kk} \\
&= \sum_{g \ni G} \frac{n}{N} [D(g)]_{lm} \text{Tr}[D(g^{-1})D(g')]
\end{aligned}
\tag{91}$$

Where summing over k on the right-hand side of the first equation above produces the trace of the product $D(g^{-1}D(g'))$. Now, suppose that an experiment is taking place in which the eigenvalues of the symmetry group matrix representation $D(g)$ are being measured. The results of this experiment, $\langle D(g) \rangle$, are the average outcome:

$$\langle D(g) \rangle = \sum_i \lambda_i p(\lambda_i)
\tag{93}$$

Where λ_i are the eigenvalues of $D(g)$ mentioned above and $p(\lambda_i)$ are the distribution frequencies of the measurement of the various eigenvalues. In terms of the average value of $D(g)$ the equation is:

$$\sum_{g \ni G} \frac{n}{N} \langle D(g) \rangle \text{Tr}[D(g^{-1})D(g')] = \langle D(g') \rangle
\tag{94}$$

A necessary definitional requirement of the density matrix is that:

$$\text{Tr}[\rho D(g')] = \langle D(g') \rangle
\tag{95}$$

where ρ is the density matrix. Thus, in terms of ρ , Equation 94 reduces to:

$$\sum_{g \ni G} \frac{n}{N} \langle D(g) \rangle \text{Tr}[D(g^{-1})D(g')] = \text{Tr}[\rho D(g')]
\tag{96}$$

And, thus, the density matrix's form is defined as:

$$\rho \equiv \frac{n}{N} \sum_{g \ni G} D(g^{-1}) \langle D(g) \rangle
\tag{97}$$

Thus, in the above equation, the form of the density matrix is fully derived.

Equation 97 provides a great deal of information about many of the properties of the density matrix as well. The first of these properties is the fact that the density matrix is Hermitian. A Hermitian matrix H is one that satisfies the property that $H = H^\dagger$. This property of the density matrix follows from the fact that the matrices $D(g)$ are unitary and thus $D(g^{-1}) = D^\dagger(g)$:

$$\rho^\dagger = \frac{n}{N} \sum_{g \ni G} (D(g^{-1}) \langle D(g) \rangle)^\dagger \quad (98)$$

$$= \frac{n}{N} \sum_{g \ni G} \langle D^\dagger(g^{-1}) \rangle D^\dagger(g) \quad (99)$$

$$= \frac{n}{N} \sum_{g \ni G} \langle D(g^{-1}) \rangle D(g) \quad (100)$$

$$= \frac{n}{N} \sum_{g \ni G} D(g^{-1}) \langle D(g) \rangle = \rho \quad (101)$$

The last line of this proof holds because there exists a g^{-1} for every g and since g is summed over. Thus, the density matrix is hermitian and has real eigenvalues.

Bohr and Ulfbeck carry the derivation further to prove other properties of the density matrix. First, they characterize the density matrix in terms of its eigenvalues(r) and eigenvectors($|r\rangle$) as:

$$\rho = \sum_r |r\rangle r \langle r| \quad (102)$$

Likewise, the matrix $D(g)$ can be characterized by its eigenvalues(λ_i) and eigenvectors($|\lambda_i\rangle$) as:

$$D(g) = \sum_i |\lambda_i\rangle \lambda_i \langle \lambda_i| \quad (103)$$

I will now go into more detail than Bohr and Ulfbeck to derive their density matrix results from the characterization of the density and symmetry matrices in terms of their eigenvalues and eigenvectors. Starting from Equation 95 and Equation 93:

$$\langle D(g) \rangle = Tr[\rho D(g)] \sum_i \lambda_i p(\lambda_i) = Tr[\rho D(g)] \quad (104)$$

Substituting in the equation for $D(g)$ in terms of its eigenvectors and eigenvalues gives:

$$\sum_i \lambda_i p(\lambda_i) = \sum_i \lambda_i Tr[\rho |\lambda_i\rangle \langle \lambda_i|] \quad (105)$$

The above equation can be solved for $p(\lambda_i)$ as follows:

$$p(\lambda_i) = \text{Tr}[\rho |\lambda_i\rangle \langle \lambda_i|] \quad (106)$$

$$= \sum_r \text{Tr}[|r\rangle \langle r| \rho |\lambda_i\rangle \langle \lambda_i|] \quad (107)$$

$$= \sum_r \langle r | \lambda_i \rangle \text{Tr}[|r\rangle \langle r| \rho] \quad (108)$$

$$= \sum_r \langle r | \lambda_i \rangle \sum_j \langle \lambda_j | r \rangle \langle \lambda_i | \lambda_j \rangle \quad (109)$$

$$= \sum_r \langle r | \lambda_i \rangle \sum_j \langle \lambda_j | r \rangle \delta_{ij} \quad (110)$$

$$= \sum_r \langle r | \lambda_i \rangle \langle \lambda_i | r \rangle \quad (111)$$

$$= \sum_r |\langle \lambda_i | r \rangle|^2 \quad (112)$$

This equation for $p(\lambda_i)$ agrees with the Born rule for determining probabilities since $p(\lambda_i)$ is the distribution frequency for a specific λ_i and $|\langle \lambda_i | r \rangle|^2$ thus corresponds to the probability of obtaining a certain value of λ_i given an initial state $|r\rangle$.

Finally, two other important results concerning the density matrix can be derived from this equation for $p(\lambda_i)$:

$$\langle D(g) \rangle = \sum_r \langle r | D(g) | r \rangle \quad (113)$$

$$|r\rangle = \sum_i |\lambda_i\rangle \langle \lambda_i | r \rangle \quad (114)$$

These two results, the first of which follows from the equation for $p(\lambda_i)$ and Equation 93 and the second of which merely follows from identity operator, also reflect important tools for quantum mechanical calculations that have been derived here without appealing to the laws of QM at all. Thus, this derivation has shown that many important quantum mechanical results can be derived directly from space-time symmetries alone.¹⁸

This result, like the CCR result previously derived, evinces the power of space-time symmetries. The important properties of the density matrix were all derived from a symmetry relation, some basic facts about algebra, and a simple operational definition of the density matrix. Unlike the CCR result, the density matrix derivation does not impose a strict space-time metric upon quantum mechanics; rather, it shows that, once such a metric has been imposed, the symmetry itself is sufficient for one to

¹⁸In many ways this derivation of a property of the quantum world from the algebra of the space in which QM is thought to be relevant is similar to the derivation of the Heisenberg Uncertainty Principle from the Cauchy-Schwarz Inequality. The difference, of course, is that the Bohr and Ulfbeck's result assumes an M4 space-time background while a derivation of the Heisenberg Uncertainty Principle from the Cauchy-Schwarz Inequality would require a Hilbert Space background.

derive necessary quantum mechanical constructs like the Born rule and density matrix formalism from first principles.

3.3 Bohr and Ulfbeck's Matrix Variables

From these two results (the derivation of the CCR from symmetry groups and the derivation of the essential properties of the density matrix from algebra) it is apparent that a new interpretation of quantum mechanics is in order: specifically, Bohr and Ulfbeck's "genuine fortuitousness". The heart of genuine fortuitousness lies in a reinterpretation of the quantum mechanical matrix variable, and so, in order to provide the necessary background for an understanding of genuine fortuitousness as a whole, I will first focus on Bohr and Ulfbeck's conception of a matrix variable.

Traditionally, the formalism of quantum mechanics calls for several physical quantities to be translated from classical values to quantum operators. Examples of such transformations are the position and momentum operators that appear in the CCR; in classical mechanics, position and momentum are both single-valued observables for a single particle (or system) while in quantum mechanics position and momentum take on the role of matrices that operate on the state vector. These matrices are no longer single-valued but are characterized by any number of eigenvalues. The dimensionality of these matrices is determined by the Hilbert space needed for the system. For instance, a spin- $\frac{1}{2}$ particle exists in a 2-dimensional Hilbert space because the only possible spins are spin up and spin down; thus, the operators in this space are 2x2 matrices. Likewise, a spin 1 particle exists in a 3-dimensional Hilbert space, and its operators are 3x3 matrices. This suggests that the operator for spin in the x-direction, say, can be represented by matrices of varying dimensions depending on the Hilbert space necessitated by the particle or system of particles under consideration. It is appropriate, then, to call such an operator a "matrix variable" since it can assume any number of matrix representations.

The results of Bohr and Ulfbeck's previous derivations suggest that another method for accounting for matrix variables must be in order, for in both the CCR derivation and the density matrix derivation the concept of Hilbert space is superseded by the structure of Minkowski or K4 space-time. Thus, the previous results might suggest that the cause of matrix variables lies in space-time symmetries instead of Hilbert space¹⁹. The symmetries inherent in space-time via the Poincaré group suggest the nature of matrix variables in the same way that these symmetries allowed Bohr and Ulfbeck to derive the density matrix if these space-time symmetries are to be viewed as more fundamental entities than Hilbert space.

Bohr and Ulfbeck describe matrix variables primarily in their first paper on genuine fortuitousness[16]. Their second paper on the subject[17] also spends a good deal of time discussing the nature of these entities, while the most recent paper on the subject[18], which addresses the same issues raised by matrix variables as the other two papers, does not make any direct reference to matrix variables by name. My

¹⁹Interestingly, time is not considered among these symmetries(see [16] pages 4-5, 16-17, and 30) because Bohr and Ulfbeck take the arrow of time to be fundamental to time's (a)symmetry. More will be said about the relationship between Bohr and Ulfbeck's theory and its treatment of time later in this paper.

characterization of Bohr and Ulfbeck's conception of matrix variables therefore comes mainly from Bohr and Ulfbeck's first two papers on genuine fortuitousness.

Bohr and Ulfbeck begin their discussion of matrix variables by discussing two types of symmetry manifestation which they call "primary manifestation" and "secondary manifestation". Most physicists are probably most familiar with the concept of secondary manifestations of symmetry. Such manifestations, according to Bohr and Ulfbeck, consist of transformations of physical quantities. Secondary manifestations are the appearance of symmetry in the coefficients of a symmetry transformation. Such a manifestation of symmetry requires an object to act upon since a secondary manifestation is a transformation of another object. A rotation is an instance of such a symmetry; it operates on a certain object in such a way that the changes in the x-position of the object are symmetrically or anti-symmetrically aligned with the y-position of the object, but the rotation itself is not the object being rotated. Secondary manifestations do not appear as objects in its own right; rather, they are properties of other objects (or series of objects) that force these objects to adhere to certain laws of symmetry.

The other manifestation is called a primary manifestation, and it is this manifestation of symmetry with which Bohr and Ulfbeck are most concerned. This manifestation of symmetry reveals the symmetry variables taking on an existence of their own. Instead of being tied to an object and thus being categorized as properties, these variables exist in their own right and are fundamentally treated as objects. Bohr and Ulfbeck argue that these primary manifestations of symmetry are the matrix variables previously referenced. Primary manifestations of symmetry are not just *some* objects in space-time, however; they are the *fundamental* objects of space-time. Bohr and Ulfbeck[16] refer to primary manifestations of symmetry as the "elementary substance" (3) and go so far as to define the fundamental quanta of the universe as nothing more than bundles of symmetry variables without objects. These symmetry variables, matrix variables, are responsible for all of matter, and thus secondary manifestations of symmetry depend on primary manifestations of symmetry since the primary manifestations of symmetry produce the variables that define how and provide the material upon which the secondary manifestations of symmetry act.

The fact that it is these manifestations, matrix variables, that are fundamentally "real" in space-time and the fact that they represent embodiments of the symmetries of space-time reveal Bohr and Ulfbeck's collapse of the traditional dichotomy in physics between matter and geometry. The theory of relativity, for instance, is usually interpreted as a theory of geometry while QM is a theory concerned with the behavior of matter. For this reason, the two have coexisted without either one necessitating the revision of the other despite their disagreements. However, as the derivation of the CCR clearly shows, the symmetries and geometry of space-time play a crucial role in the way that objects in that space-time act and interact. Such a dichotomy is thus seen as a flawed interpretation of space-time; the geometry of space-time and its material contents must be related in some deeper way than the Copenhagen interpretation of quantum mechanics and special relativity would suggest. Bohr and Ulfbeck make this relation explicit through their primary manifestation of symmetry; it is the geometry of space-time that forms the elementary substance of the material world. Thus, the geometry-matter dichotomy collapses; the fundamental character of matter is not

separable from the symmetries of the space-time in which that matter is said to exist.

This characterization of primary manifestations of symmetry provides a general outline of how matrix variables arise in space-time. Bohr and Ulfbeck explain how these matrix variables express symmetries in space-time in more depth when they address the indeterminacy and complementarity of matrix variables. The indeterminacy of matrix variables reflects the fact that there is no law that can determine exactly what value a measurement of some property of a quantum superposition will yield. For instance, a spin $\frac{1}{2}$ particle may exist in a superposition of the spin up and spin down states with respect to the x-direction. There is no way to determine whether a spin-x measurement of this state will yield a spin up or spin down result, thus leading to the phenomenon that has been traditionally termed “indeterminacy”. Complementarity is another property of quantum systems by which a measurement of one property changes the value of another property. For instance, in the example above, suppose that, after measuring the spin in the x-direction to be spin up, one then measures the spin in the y-direction. Such a measurement gives a definite value for the spin in the y-direction but forces the spin in the x-direction back into a superposition. This phenomenon is traditionally referred to as “complementarity” since the spin-x and spin-y values seem to be inextricably linked in such a way as both cannot have definite values simultaneously. Both indeterminacy and complementarity seem, upon traditional interpretation, to be a bit “spooky”; indeterminacy seems to mandate that “nature chooses indiscriminately” among several different outcomes with no dynamic mechanism proposed to explain this process while complementarity suggests a strange linkage between certain non-commuting operators without explaining why these are linked while commuting operators are not.

Bohr and Ulfbeck are able not only to reproduce “indeterminacy” and “complementarity” in their interpretation of matrix variable but also to liberate these phenomena from traditional interpretations that might make them seem “spooky”. Bohr and Ulfbeck first address the issue of indeterminacy, beginning with the Poincaré group. Since this group, which represents the symmetries of space-time, is non-Abelian (that is, not all of its elements commute), there must exist a multi-dimensional, irreducible representation of the elements of this group. This assertion comes from the fact that all real numbers (i.e. 1-dimensional representations of a group) are Abelian[34], and thus since a non-Abelian group cannot be completely reduced to an Abelian representation, there must be an irreducible representation of the Poincaré group in higher dimensions.

However, the observable values of the elements of the Poincaré group correspond to the eigenvalues of the irreducible representations of these elements by the eigenstate-eigenvalue link assumed in the measurement process. Thus, because there are multi-dimensional representations, there must be matrix variables that have multiple eigenvalues and thus have multiple possible results upon measurement. The possibility of multiple results from the same matrix variable is essentially the same in character as the notion of indeterminacy, and thus this phenomenon has been shown to appear in Bohr and Ulfbeck’s matrix variables as well; however, to call this phenomenon “indeterminacy” would be misleading since there is nothing in the derivation of the phenomenon to suggest the same “spookiness” inherent to the previous usage of “indeterminacy”. Rather, the phenomenon of indeterminacy falls out of the mathematics used to represent the group of symmetries whose primary manifestations appear on the

space-time scene. The indeterminacy of matrix variables only seems “spooky” to those accustomed to single-valued physical results, and thus the very nature of a matrix variable in multiple dimensions entails such “indeterminacy”. Thus, the phenomenon of “indeterminacy”, rather than posing a problematic interpretation of a physical event, reflects the mathematical symmetries and simplicities of the Poincaré group.

Likewise, Bohr and Ulfbeck deduce complementarity among their matrix variables by first appealing to the notion of a closed group algebra. For a set of elements to compose a group, one must be able to construct a multiplication table showing that each element of the group can be created by the multiplication of two other elements of the group together. Consider an element of the Poincaré group which appears in the form of a matrix variable as a primary manifestation of space-time symmetries. This matrix variable (and its eigenvalues) results from the multiplication of two other matrix variables (and their eigenvalues), and thus there is an inherent correlation between any element of the Poincaré group and the two elements that multiply together to form it. Because this correlation implicates eigenvalues too, not only the matrix variables themselves but the values linked with them are complementary, and thus the phenomenon of complementarity emerges as a consequence of the elements of the Poincaré group forming a closed algebra. This closed group algebra reveals itself as the mechanism for complementarity, meaning that one no longer needs to appeal to some innate, “spooky” connection between two complementary matrix variables to describe their relationship to one another.

This mechanism for complementarity leads to some interesting consequences. Since the matrix variables (and the symmetry elements of the Poincaré group) are not always present, they must come from somewhere and emerge in a certain form to fit the environment. For instance, the x-spin matrix cannot be always present since the symmetries it reflects are context-dependent; it must emerge as a 2-dimensional matrix given a spin $\frac{1}{2}$ particle or a 3-dimensional matrix given a spin 1 particle. Thus, a symmetry element manifests itself on the space-time scene as a matrix variable, and in doing so, it emerges from somewhere. However, if such an element of symmetry really does “emerge” into space-time by entering it from some other place (i.e. if the matrix variable appears as a result of the element of symmetry punching through the fabric of space-time to appear on the scene), then problems emerge. Any one element, as has already been discussed, can be seen as two elements multiplied together, and thus two elements emerge upon the scene any time one does. By complementarity, which links all of these matrix variables, it would appear that all symmetry elements are bound together in a unified whole, and thus no part of the Poincaré group can appear anywhere in space-time without the entire Poincaré group appearing along with it.

A paradox thus emerges. How can a matrix variable emerge in space-time (as it seems to) if doing so would cause all other symmetry elements to force other matrix variables onto the scene (as they do not seem to)? Bohr and Ulfbeck conclude that this paradox is inadmissible, and thus a matrix variable cannot directly appear on the space-time scene as a direct manifestation of an element of symmetry. Bohr and Ulfbeck[17] describe the emergence of a matrix variable as, “a matrix variable manifest[ing] itself on the space-time scene without entering the scene” (8). What they mean by this puzzling statement is simply that a mathematical object with properties that derive from a symmetry element appears on the scene but is not a physical object itself;

the true object whose reality is reflected by this mathematical object is not on the space-time scene, and thus the paradox produced by complementarity does not arise.

More should be said, however, about Bohr and Ulfbeck's answer to the most obvious question at this point: where are the elements of symmetry if not on the space-time scene? Bohr and Ulfbeck do not spend much time addressing this issue; it seems that these elements must underlie space-time in some way, but it is not clear exactly how. Bohr and Ulfbeck[17] do make a point, however, of stating that: "an expression like 'the world of matrix variables' has been avoided, since it might convey the notion that something exists beyond the world of experience" (771). Thus, Bohr and Ulfbeck cautiously dodge the issue of where the matrix variables truly reside. Because these variables do not interact directly with human experience, Bohr and Ulfbeck refuse to comment on the reality of a realm for their variables aside from assigning them a place outside of space-time. The implications of Bohr and Ulfbeck's agnosticism on this issue will be addressed, along with other philosophical implications of this theory, later on in this paper.

The emergence of the matrix variable is indeed a puzzling process, and it is treated in more thorough detail in the next section since "genuine fortuitousness" is a term originally used to describe the manifestation of a matrix variable in space-time. Before proceeding to this theory, however, several of Bohr and Ulfbeck's comments further characterizing the connection between matrix variables and space-time should be addressed. The first of these comments concerns a problem that has traditionally plagued QM: why are quantum phenomena like indeterminacy and complementarity not observed on the macroscopic scale like they are on the microscopic scale? Bohr and Ulfbeck assert that classical mechanics operates at such low resolution that the non-locality of the symmetry variables is quenched²⁰ and all of the matrix variables are reduced to classical variables. Such a low resolution reduces the matrices to 1-dimensional representations, thus losing indeterminacy. The 1-dimensional representations are the real numbers which form an Abelian group, meaning that all of the elements commute and commutators like the CCR are lost. Finally, in the 1-dimensional representation the eigenvalues of the matrices are the matrices themselves, leading to a loss of correlation among matrix variables. These claims are explained in slightly more detail by Bohr and Ulfbeck[16] on page 15 of their first paper on genuine fortuitousness.

Bohr and Ulfbeck's characterization of matrix variables also has consequences for the measurement process. Measurement is traditionally viewed in the Copenhagen interpretation as an event that collapses a wave function and forces a superposition to assume a single value with respect to a certain property. Bohr and Ulfbeck argue, however, that measuring devices, insofar as they are matter, must consist of many matrix variables, and thus the measurement device itself is a manifestation of space-time symmetries. This device produces a secondary manifestation of symmetry which then allows one to determine the space-time symmetries it embodies. An example of such a calculation for a measuring device is SSC's characterization of the beam splitter

²⁰The exact nature of this quenching is discussed in supplementary section appended to the end of reference [16]. Due to the detailed nature of the proof, a digression into the exact details of the transition between the quantum and classical regimes is beyond the scope of this paper. However, the result of this work is essential to an understanding of the merits of Bohr and Ulfbeck's matrix variable formalism, and as such this short paragraph explaining their conclusions on the matter has been included.

used the quantum liar paradox²¹. Thus, collapse does not occur because of some “spooky” property of measurement but because a small system whose interactions are characterized by primary manifestations of symmetry is interacting with a classical system whose interactions are characterized by secondary manifestations of symmetry. The result of a measurement, then, is a kind of conversion from a multi-valued matrix variable into an eigenvalue of the matrix variable used to characterize the system in the same way that the matrix variable becomes simply a numeric variable on the classical scale. Bohr and Ulfbeck[16] write “The specification of the conditions under which a symmetry variable appears with a value in a measurement is not a requirement of the recognition of the primary manifestation of symmetry” (8). The measurement problem thus loses its power; the more fundamental question to ask is not what happens in measurement but what happens in interactions due to symmetry in general. More will be said on collapse, measurement, and symmetry interactions in the next section, and it will become more apparent how a matrix variable ought to be physically interpreted.

For now, it suffices to examine what work Bohr and Ulfbeck’s conception of matrix variables does for their theory. First, the matrix variables described by Bohr and Ulfbeck provide a mechanism by which derivations of the CCR and the density matrix from space-time symmetries are made possible. If matrix variables did not have the relation to symmetry that Bohr and Ulfbeck describe, one might be tempted to dismiss these previous two derivations as mere coincidence. Thus, while these two derivations are supported by Bohr and Ulfbeck’s theory of matrix variables, they also support the theory of matrix variables as well, showing that matrix variables are entitled to all the explanatory power that Bohr and Ulfbeck bestow upon them in their theory.

Secondly, Bohr and Ulfbeck’s matrix variables explain the quantum features of indeterminacy and complementarity while removing them from the realm of “spookiness”. The matrix variables provide underlying causes for these phenomena since both are consequences of mathematics that characterizes space-time symmetries. Thus, the matrix variable theory of Bohr and Ulfbeck seems promising because it can easily explain previously unexplained and difficult to explain phenomena of the quantum world.

Finally, Bohr and Ulfbeck’s matrix variable characterization does away with the centrality of measurement to the quantum picture of the world. The question of the unexplained phenomenon of collapse itself collapses into a question concerning how different manifestations of space-time symmetries interact with each other. The even deeper virtues of Bohr and Ulfbeck’s theory of matrix variables for dealing with collapse will be discussed in the next section.

Bohr and Ulfbeck’s theory of matrix variables does leave several questions, of course. As previously discussed, the issue of where exactly the matrix variables reside is never addressed. It is also not clear what the mechanism is for the appearance of something in space-time; one could, in fact, argue that Bohr and Ulfbeck have done away with spooky concepts like indeterminacy only to posit an even spookier one in which the symmetry variable’s “ghost” appears in space-time though the entity itself does not. These potential problems for the theory, as well as others, will be addressed later on in this paper. For right now, it is enough to reflect on the nature of matrix variables, the representations of symmetry which appear in space-time though they never enter

²¹See Section 6 for more details and for this characterization of the beam splitter

it, for it is these entities that embody the space-time symmetries that make Bohr and Ulfbeck's theory of genuine fortuitousness run. It is to the appearance of these entities in space-time and the physical (and philosophical) consequences that their appearance present that are the focus of the next section.

3.3.1 Supplement: Why Matrix Variables?

Before moving on to the next section, however, it is important to address any concerns one might have that matrix variables are not the proper means of characterizing the appearance of observables in space-time. Since Bohr and Ulfbeck's argument hinges on the fact that it is *matrix* variables and not real numbers that characterize quantum observables, this proof is of the utmost importance if one is to accept Bohr and Ulfbeck's reasoning. What follows is a proof, reproduced from several places in [16] with the addition of some steps omitted from the original paper, that quantum observables are matrix variables and not numerical variables. This proof is important because it supports Bohr and Ulfbeck's matrix variable interpretation, but if the idea of treating quantum observables as matrix variables instead of numerical variables is a familiar one to you, feel free to skip this section and proceed to the next one.

To begin with, consider the symmetry of reflections (which will be henceforth denoted by S) and translations (which will henceforth be denoted by J). It is obvious that two reflections across the same axis result in no net change to a system, and thus the symmetry reflected by this principle can be represented as:

$$S^2 = I \tag{115}$$

where I is the identity matrix. Another important physical result involves the symmetry of translations. A translation of a sum is the product of translations, or:

$$J(a_1)J(a_2) = J(a_1 + a_2) \tag{116}$$

Where $J(a)$ now denotes a translation of some distance a . This kind of argument should be familiar since it follows the same logic as the previous symmetry discussion in my discussion of the Poincaré group. Now, as then, it appears that translation can be written as an exponential:

$$J(a) = e^{-iak} \tag{117}$$

Where k is the generating function for translations. This is a 1-dimensional representation of this element of the symmetry group.

There are two more important identities concerning $S(a)$ (a reflection across an axis at a position a) and $J(a)$ that will allow me to write out the 2-dimensional representation of $S(a)$ and $J(a)$. The first of these is:

$$SJ(a) = J(-a)S \tag{118}$$

Which reflects the fact that S and $J(a)$ do not commute, and:

$$S(a) \equiv J(a)SJ^{-1}(a) \tag{119}$$

Where $S = S(0)$. These facts about the relationship between $S(a)$ and $J(a)$, along with the symmetry requirements that any representation of $S(a)$ and $J(a)$ must fulfill, are satisfied by the 2-dimensional representation below:

$$J(a) = \begin{pmatrix} e^{-ik_0a} & 0 \\ 0 & e^{ik_0a} \end{pmatrix} \text{ and } S(a) = \begin{pmatrix} 0 & e^{-i2k_0a} \\ e^{i2k_0a} & 0 \end{pmatrix} \quad (120)$$

Where $k_0 = |k|$.

At this point, Bohr and Ulfbeck note that the matrix $S(a)$ can be written in the following form:

$$S(a) = S \cos 2k_0a + S(a_0) \sin 2k_0a \quad (121)$$

Where $k_0a_0 = \frac{\pi}{4}$. The following proof shows that the two terms are equivalent by substitution into the matrix form of $S(a)$:

$$S(a) = \cos 2k_0a \begin{pmatrix} 0 & e^0 \\ e^0 & 0 \end{pmatrix} + \sin 2k_0a \begin{pmatrix} 0 & e^{-i2k_0a_0} \\ e^{i2k_0a_0} & 0 \end{pmatrix} \quad (122)$$

$$= \cos 2k_0a \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \sin 2k_0a \begin{pmatrix} 0 & e^{-i\frac{\pi}{2}} \\ e^{i\frac{\pi}{2}} & 0 \end{pmatrix} \quad (123)$$

$$= \begin{pmatrix} 0 & \cos 2k_0a \\ \cos 2k_0a & 0 \end{pmatrix} + \begin{pmatrix} 0 & -i \sin 2k_0a \\ i \sin 2k_0a & 0 \end{pmatrix} \quad (124)$$

$$= \begin{pmatrix} 0 & \cos 2k_0a - i \sin 2k_0a \\ \cos 2k_0a + i \sin 2k_0a & 0 \end{pmatrix} \quad (125)$$

$$= \begin{pmatrix} 0 & e^{-i2k_0a} \\ e^{i2k_0a} & 0 \end{pmatrix} \quad (126)$$

$$= S(a) \quad (127)$$

Thus, Equation 121 has been proven, and since this relationship is true for the representation in 2 dimensions and can be generalized in terms of S , $S(a)$, and $S(a_0)$, it holds for all representations of $S(a)$.

I will now shift gears to discuss the idea of conditional probability, which is defined as $W(u;v)$ for observable values of u and v . The conditional probability $W(u;v)$ represents the probability of a value v given the value u . The conditional probability $W(u;v)$ is the same as the conditional probability $W(v;u)$ since:

$$W(u;v) = |\langle u|v \rangle|^2 = W(v;u) \quad (128)$$

The probability $W(u;v)$ also allows for one to compute average values. The average value of a matrix V , which is represented by \bar{V} , can be written in terms of a known value u and the conditional probability $W(u;v)$ as follows:

$$\bar{V} = \sum_v v W(u;v) = \langle u|V|u \rangle \quad (129)$$

The above equation holds for all matrices V and all eigenvalues of these matrices v .

There is another important probability, however, which is the joint probability of u and v , which will be represented by $p(u, v)$. This probability represents the probability of finding both a value u and a value v . It is related to the conditional probability by the relation

$$p(u, v) = p(u)W(u; v) \quad (130)$$

where $p(u)$ is the probability of getting a result u . The above relation is a theorem of probability theory. To determine this relation in more detail, I use Bohr and Ulfbeck's result for invariant symmetry matrices that:

$$p(u) = \frac{1}{d} \quad (131)$$

Where d is the dimension of the representation. The logic for this definition is that, if u is a symmetry variable and U is a matrix that leaves a system invariant in space-time, then all possible eigenvalues are equally likely. This corresponds to disavowing any knowledge of the system such that one eigenvalue would be preferred over any other.

Finally, then, it is possible to utilize the above results to determine the average value for the products of two matrices U and V . This average value is called the pair correlation, and it can be written as follows:

$$\overline{UV} = \sum_u up(u) \sum_v vp(v) \quad (132)$$

$$= \sum_{uv} uv p(u, v) \quad (133)$$

$$= \sum_{uv} uv \frac{1}{d} W(u; v) \quad (134)$$

$$= \frac{1}{d} \sum_u \langle u | uV | u \rangle \quad (135)$$

$$= \frac{1}{d} \sum_u \langle u | UV | u \rangle \text{ using the fact that } U = \sum_u u |u\rangle \langle u| \quad (136)$$

$$= \frac{1}{d} Tr(UV) \quad (137)$$

Where $Tr(UV)$ is the trace of the matrix UV . This final result is incredibly important since the pair correlation between any two matrices can be calculated simply by finding the trace of their product. No calculation of eigenvalues is needed at all in this process, nor is any knowledge of probabilities or calculation of sums necessary.

Using the previously-derived 2-dimensional representation of the reflection matrix, it is now possible to derive the pair correlation between any two reflections $S(a_1)$ and $S(a_2)$ as follows:

$$\overline{S(a_1)S(a_2)} = \frac{1}{d} \text{Tr}(S(a_1)S(a_2)) \quad (138)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{pmatrix} 0 & e^{-i2k_0 a_1} \\ e^{i2k_0 a_1} & 0 \end{pmatrix} \begin{pmatrix} 0 & e^{-i2k_0 a_2} \\ e^{i2k_0 a_2} & 0 \end{pmatrix} \right) \quad (139)$$

$$= \frac{1}{2} \text{Tr} \begin{pmatrix} e^{-2ik_0(a_1-a_2)} & 0 \\ 0 & e^{2ik_0(a_1-a_2)} \end{pmatrix} \quad (140)$$

$$= \frac{1}{2} (e^{-2ik_0(a_1-a_2)} + e^{2ik_0(a_1-a_2)}) \quad (141)$$

$$= \frac{1}{2} (2 \cos 2k_0(a_1 - a_2)) \quad (142)$$

$$= \cos 2k_0(a_1 - a_2) \quad (143)$$

This result will be extremely useful later on for calculations of pair correlations between different reflections. Notice that, once again, the result has turned out beautifully; knowing only the difference $a_1 - a_2$, one can determine the average value of the product of the two matrices. Also, note that it is now more apparent why this quantity $\overline{S(a_1)S(a_2)}$ is called a pair correlation: the only case where it can yield a value of 1 is when a_1 and a_2 are equal or differ by a factor of 2π . Likewise, the pair correlation yields a value of 0 only when a_1 and a_2 are out of phase by $\frac{\pi}{2}$. Thus, the quantity $\overline{S(a_1)S(a_2)}$ gives a great deal of information about the relationship between the two variables a_1 and a_2 , and as such it seems appropriate to call it the pair correlation.

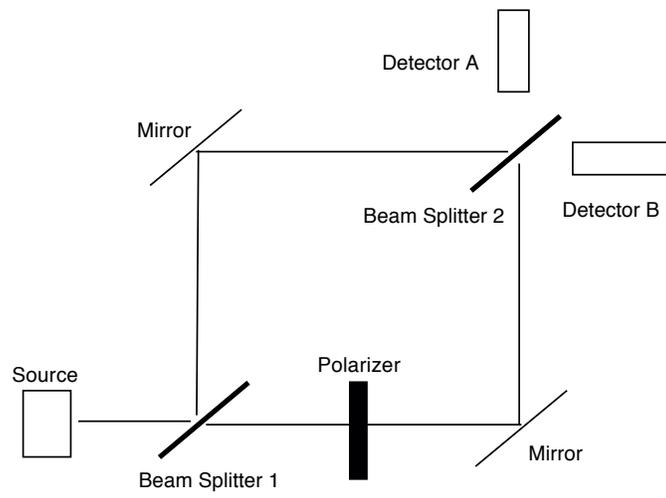


Figure 1: The Mach-Zehnder Interferometer (MZI) Experimental Setup with Polarizer

Support for the applicability of this result to real, physical situations can be shown from the experimental setup of a Mach-Zehnder Interferometer (MZI) with a polarizer along one leg of the apparatus, as shown in Figure 1. In this setup light exits the source, is split into two different paths by the beam splitter 1, and then is recombined at beam splitter 2 after traveling along the legs of equal length between the beam splitters before hitting either detector A or detector B. The mathematics of this setup can be characterized as follows:

First, define two orthogonal bases, one corresponding to detector clicks ($|A\rangle$ is a click in detector A and $|B\rangle$ is a click in detector B) and one corresponding to legs of the MZI ($|1\rangle$ is the upper leg of the apparatus and $|2\rangle$ is the lower leg of the apparatus). Since the beam splitter sends half along the upper leg and half along the lower leg equally, it is possible to write the relation between the two different eigenbases as:

$$|A\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \quad (144)$$

$$|B\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \quad (145)$$

Now, assuming that the source emits light in the $|A\rangle$ eigenstate, the light incident on beam splitter 1 can be described by:

$$|\psi\rangle = |A\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \quad (146)$$

This state results in a detector A click 100% of the time. However, the polarizer on the lower leg shifts the coefficient in front of $|2\rangle$ by a phase factor of $e^{i\theta}$. Thus, the state leaving beam splitter 2 is not the same as the state entering beam splitter 2. This new state can be written as follows:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\theta}|2\rangle) \quad (147)$$

$$= \frac{1}{2}(|A\rangle + |B\rangle + e^{i\theta}(|A\rangle - |B\rangle)) \quad (148)$$

$$= \frac{1}{2}((1 + e^{i\theta})|A\rangle + (1 - e^{i\theta})|B\rangle) \quad (149)$$

$$= \frac{1}{2}(e^{i\frac{\theta}{2}}(e^{-i\frac{\theta}{2}} + e^{i\frac{\theta}{2}})|A\rangle + e^{i\frac{\theta}{2}}(e^{-i\frac{\theta}{2}} - e^{i\frac{\theta}{2}})|B\rangle) \quad (150)$$

$$= \frac{1}{2}(2e^{i\frac{\theta}{2}}\cos\frac{\theta}{2}|A\rangle - 2ie^{i\frac{\theta}{2}}\sin\frac{\theta}{2}|B\rangle) \quad (151)$$

$$= e^{i\frac{\theta}{2}}\cos\frac{\theta}{2}|A\rangle - ie^{i\frac{\theta}{2}}\sin\frac{\theta}{2}|B\rangle \quad (152)$$

Thus, the probability distributions of detector clicks are

$$P_A = |\langle\psi|A\rangle|^2 = \cos^2\frac{\theta}{2} \quad (153)$$

$$P_B = |\langle\psi|B\rangle|^2 = \sin^2\frac{\theta}{2} \quad (154)$$

The difference between these two probabilities corresponds to the correlation between detector A clicks and detector B clicks. This correlation is then:

$$\overline{AB} = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \quad (155)$$

$$= \cos \theta \quad (156)$$

If, then, we define this angle θ such that $\theta = 2k_0(a_1 - a_2)$, it can be seen that the pair correlation result previously defined emerges. This argument show two things: first, it shows how it is possible for a physical observable to reflect symmetry relations since the polarizer embodies reflection symmetry across some distance $a_1 - a_2$, and secondly, this argument provides further support for the previously-derived pair correlation result.

Now that the proper background has been established, Bohr and Ulfbeck define the following quantity:

$$\Delta = S(a) - S \cos 2k_0a - S(a_0) \sin 2k_0a \quad (157)$$

Of course, using the definition for $S(a)$ in Equation 121, Δ is 0 in the quantum regime if matrix variables are used. However, if one were to consider numerical variables instead of matrix variables for $S(a)$, then the eigenvalues of S , which are -1 and 1, would appear instead of the $S(a)$, S , and $S(a_0)$ terms, which may represent matrices. Thus, the value for Δ in real number variable regime would be 0 only when $a = na_0$ for integer n ²². Thus, there is a discrepancy between the predictions based on the use of matrix variables and the predictions based on the use of numerical variables for the value of Δ . If $\overline{\Delta^2}$ is positive, for instance, then the matrix variable prediction must be incorrect since this would show that at least some values of Δ were not 0. On the other hand, if $\overline{\Delta^2}$ is always 0 and does not depend on a , then the matrix variable prediction defeats the numerical variable prediction.

To start with, then, I will calculate the predicted value of Δ^2 for numerical variables. The value for Δ^2 is found to be

$$\Delta^2 = (S(a) - S \cos 2k_0a - S(a_0) \sin 2k_0a)^2 \quad (158)$$

$$= S^2(a) + S^2 \cos^2 2k_0a + S^2(a_0) \sin^2 2k_0a - 2S(a)S \cos 2k_0a - \quad (159)$$

$$2S(a)S(a_0) \sin 2k_0a + 2SS(a_0) \cos 2k_0a \sin 2k_0a$$

$$= 1 + \cos^2 2k_0a + \sin^2 2k_0a - 2S(a)S \cos 2k_0a - 2S(a)S(a_0) \sin 2k_0a \quad (160)$$

$$+ 2SS(a_0) \cos 2k_0a \sin 2k_0a$$

$$= 2 - 2S(a)S \cos 2k_0a - 2S(a)S(a_0) \sin 2k_0a + 2SS(a_0) \cos 2k_0a \sin 2k_0a \quad (161)$$

$$(162)$$

Thus, the average value of this quantity is

²²The proof for this is as follows: For the numerical variable regime, the only way for combinations of 1 and -1 substituted for $S(a)$, S , and $S(a_0)$ to yield a value of 0 for Δ would be for $S(a)$, S , and $S(a_0)$ to have the same sign and for either $\sin 2k_0a$ or $\cos 2k_0a$ to be 1 so that the sum of these quantities would be 1. This only occurs if $2k_0a = \frac{n\pi}{2}$ for integer n , and thus Δ is 0 only for $a = \frac{n\pi}{4k_0} = na_0$.

$$\overline{\Delta^2} = 2 - 2\overline{S(a)S} \cos 2k_0a - 2\overline{S(a)S(a_0)} \sin 2k_0a + 2\overline{SS(a_0)} \cos 2k_0a \sin 2k_0a \quad (163)$$

and thus the pair correlation result previously derived is needed to determine each of these average values. One might question whether this result is valid given that it derives from treating the quantity $S(a)$ as a matrix already, thus suggesting that Bohr and Ulfbeck's argument is begging the question, but because of the result of the MZI experiment which confirmed the pair correlation result previously derived by Bohr and Ulfbeck, their use of the pair correlation function derives simply from an experimentally confirmed result (which just so happens to be derivable from the first principles of symmetry).

The needed correlations are as follows:

$$\overline{S(a)S} = \cos 2k_0(a - 0) = \cos 2k_0a \quad (164)$$

$$\overline{S(a)S(a_0)} = \cos 2k_0(a - a_0) = \cos(2k_0a - \frac{\pi}{2}) = \sin 2k_0a \quad (165)$$

$$\overline{SS(a_0)} = \cos 2k_0(0 - a_0) = \cos(0 - \frac{\pi}{2}) = 0 \quad (166)$$

Thus, the above equation for $\overline{\Delta^2}$ can be reduce to the following:

$$\overline{\Delta^2} = 2 - 2\cos^2 2k_0a - 2\sin^2 2k_0a + 0 \quad (167)$$

$$= 2 - 2(\sin^2 2k_0a + \cos^2 2k_0a) \quad (168)$$

$$= 0 \quad (169)$$

Thus, no matter what value a takes, the quantity $\overline{\Delta^2}$ must be equal to 0. This contradicts the fact that Δ^2 should always be positive if it is the numeric eigenvalues interacting with the system and not the matrix variables, and as such numeric variable must be ruled out in favor of matrix variables. Thus, it is not the measured eigenvalues that have real, physical meaning but rather the matrix variables themselves which directly interact via symmetry.

3.4 Genuine Fortuitousness (GF): A New Interpretation of Quantum Mechanics

Before beginning a discussion of genuine fortuitousness, it is first important to distinguish between two terms that I will be using rather frequently for the rest of the paper. Genuine fortuitousness can refer to either the phenomenon of the "genuinely fortuitous" appearance of a matrix variable on the space-time scene, which is a specific phenomenon, or it can be used to refer to Bohr and Ulfbeck's interpretation of QM as a whole which encompasses their theory of matrix variables as well as the phenomenon of "genuine fortuitousness". Thus, I will use the term "genuine fortuitousness" with lowercase letters to refer to the phenomenon posited by Bohr and Ulfbeck and the term "Genuine Fortuitousness" with capital letters and abbreviation "GF" to refer

to the theory. Hopefully this distinction will clear up any potential confusion due to terminological ambiguity.

To recapitulate some of the relevant results of the previous section, I will begin my discussion of GF with some facts about Bohr and Ulfbeck’s matrix variables. First, Bohr and Ulfbeck do away with all classical ideas of numerical variables with definite values. Classical variables like force, distance, and action are not applicable to the quantum world. Instead, Bohr and Ulfbeck posit matrix variables as the quantities of interest. These variables arise due to symmetry transformations that leave space-time invariant. To interact with the world of space-time, however, these matrix variables must assume values. Spin in the x-direction is an example of such a matrix variable, and it remains in its matrix variable form until spin in the x-direction is asked for, in which case the matrix variable appears as one of its eigenvalues.

The assumption of a value by a variable may seem like a strange thing, for such a process is both uncaused (since the matrix variable is outside of space-time and therefore cannot affect what happens in space-time) and discontinuous (the matrix variable “puts on” a value where before there was none). More will be said about the non-causality and discontinuity of the “genuinely fortuitous” appearance of matrix variables later; for now, I will consider a simple analogy to help explain how such a variable can “appear” on the scene discontinuously. Imagine a property called “shirt color”. I consider my friend Alice and attempt to determine her shirt color. However, she is currently wearing a dress, and as such she has no shirt color to speak of. When she puts a red shirt on over her dress, however, her property of “shirt color” is now changed to red, theoretically discontinuously at the moment where the red shirt can be considered “on” her. This analogy using “shirt color” is an apt one for several other properties of the process by which a matrix variable assumes a value, so I will refer to it again later, but I should stress first that this analogy is not a perfect representation of the process because it still suggests causality in the process (putting on the red shirt *causes* the property “shirt color” to assume a value) and assumes the existence of Alice, the analogue of the particles that Bohr and Ulfbeck will attempt to do away with.

Returning to the characterization of matrix variables, the final (and perhaps most important) aspect of Bohr and Ulfbeck’s matrix variables is that these variables never appear on the space-time scene. Even though these matrix variables are physical and their eigenvalues appear in and interact with space-time and its objects, the matrix variables themselves never enter space-time. Such a process is always referred to as an “appearance” or “manifestation” instead of “emergence” to emphasize the fact that while the matrix variable’s effects can be seen in space-time, the matrix variable itself remains resolutely outside of space-time.

Despite their previous emphasis on matrix variables, Bohr and Ulfbeck do not spend much time in their later papers[17][18] addressing these entities; in fact, the word “matrix variable” does not even appear in [18]. These, two later papers spend more time addressing the appearance of the eigenvalue of a matrix variable on the space-time scene than on the character of the matrix variables themselves. The first of these two papers, [17] attempts to specifically characterize the process of the matrix variable’s appearance while [18] spends more time addressing some of the major implications of the theory of GF developed in these two papers.

Bohr and Ulfbeck call the event of a matrix variable assuming one of its eigenvalues in space-time a “click” because it corresponds to the clicking of a detector in an experimental setup. An event like the click of a detector is a good way to describe the process of a matrix variable assuming a value because such clicks are discontinuous points in 4-dimensional space-time. Such clicks are also traditionally associated with the measurement of properties, and since it is the assumption of an eigenvalue by a matrix variable that allows the matrix variable to be quantified and measured on the space-time scene, the term “click” suffices to adequately describe the process.

So what is it that can be said about a click? As previously discussed, there are two important characteristics that Bohr and Ulfbeck attribute to this event: it is not caused, and it is discontinuous. The click cannot be caused since such a causal relationship would transcend the bounds that separate space-time from the matrix variable, and since nothing (not even the matrix variable itself) is able to reach across such a boundary, there can be no causal mechanism that would be able to reach across the boundary either. Likewise, the click must be discontinuous since there was nothing before it (no cause) in space time, and thus it arises from no cause at all. Thus the event must have an onset, a discontinuous, non-causal arising from itself, appearing instantaneously and thus discontinuously in space-time. Because the click has such an onset, then, there can be no physical laws that constrain the appearance of the click itself; thus, the click is said to be “purely lawless”, or, in Bohr and Ulfbeck’s words, “genuinely fortuitous”. The onset of a click marks its advent in space-time, and with the onset a causal chain can begin; however, the onset itself can only begin a causal chain, it can never be a link in it.

The nature of the click has several characteristics and important consequences, which will be discussed throughout the rest of this section. The first consequence of the click is that it naturally introduces indeterminacy in the quantum process. As was previously discussed with matrix variables, any number of eigenvalues may be assumed by the matrix variable when it appears in space-time, and since a click is genuinely fortuitous, there is no way to determine which eigenvalue will appear. This kind of indeterminacy, quantum indeterminacy, is, by this very characterization, distinguished from classical indeterminacy. In a classical system, indeterminacy arises if one knows that a particle takes a path but not which path it takes, for instance. One might imagine an interpretation of the double slit experiment in terms of classical indeterminacy. In such a picture, the experimenter would say to herself, “I do not know which slit the particle went through”. In terms of quantum indeterminacy, however, the description of the event is very different. Since the idea of “taking a path” is directly related to the classical notion of action, and since it is not such numerical variables but matrix variables that hold sway over the quantum world, the quantum experimenter would say, “It is impossible for us to ask a question about a particle taking one path or another unless we directly measure the particle at the slits. Otherwise, the particle need not take *any* ‘path’”.

The difference between classical and quantum indeterminacy can also be made more obvious by a return to the “shirt color” example. In this example, the classically indeterminate situation would be one in which you are, perhaps, colorblind and looking at Alice’s shirt. You know that her shirt has a color, but you cannot determine what color it is. The quantum indeterminate situation would arise when Alice is wearing a

dress. It is not simply that you cannot *know* what color shirt she is wearing, but to ask such a question would be silly since she has no “shirt color” until she puts on a shirt.

The difference between classical and quantum indeterminacy may be clear, but the reason for why quantum indeterminacy differs from classical indeterminacy is not. Bohr and Ulfbeck’s theory is able to account for the difference between the two forms of indeterminacy since it is the nature of the matrix variable that produces the measured results of the experiment, and thus, since “paths” only have reality once clicks have determined the necessary variables to characterize this experiment, quantum indeterminacy is established over classical indeterminacy.

Another important characteristic of the click is that it is now the fundamental unit of quantum theory. Clicks, unlike particles (the traditional units with which quantum theory concerns itself), are events in space-time, meaning that they occur at a single time in a single position instead of over a series of times and series of positions. It is to the behavior of these clicks, not the behavior of particles, that higher-order phenomena are reduced. Clicks are also unique, further supporting their fundamental nature, because they are irreversible events that arise macroscopically. Thus, any given click has almost infinite degrees of freedom because there are a very large number of microscopic atoms interacting to cause the click. With so many degrees of freedom, it is nearly impossible to replicate any particular click, and thus each click is taken to be a unique, fortuitous event.

A final point to be made about the click itself may seem trivial: it is in the counter that the click occurs, not the atom. When one discusses the nature of clicks, as I have done here, one is talking only about clicks made in the detector, nothing more. Thus, though one might be tempted to argue that the impingement of atoms on the detector is likewise a non-causal, discontinuous event, this is not a conclusion that Bohr and Ulfbeck draw. The click is a phenomenon that is intimately connected with the nature of the detector alone, and so to add anything to the theory suggesting some relationship between the click and the particle would be superfluous at best and specious at worst.

By this point in the discussion of clicks and fortuitousness, one might wonder how Bohr and Ulfbeck could possibly believe that clicks are truly fortuitous given that quantum mechanics has produced excellent laws that allow experimentalists to perfectly predict distributions of these clicks given a large enough sample. Bohr and Ulfbeck might very well claim that the result of any *one* click surely can be fortuitous (and QM in the Copenhagen interpretation relies on the fact that it is), but the fact remains that *many* clicks follow a certain pattern, and such a pattern would constitute a law that would defy the centrality of genuine fortuitousness in any physical quantum theory.

Bohr and Ulfbeck’s response, however, is that a distinction must be made between single clicks and distributions of clicks. As previously stated, Bohr and Ulfbeck argue that individual clicks are themselves completely fortuitous and thus absolutely unpredictable. Since each click is unique it does not have to be part of a probability distribution. These clicks represent a primary manifestation of symmetry as a matrix variable manifests itself on the space-time scene.

However, at low enough resolution of the quantum world²³, distributions of clicks

²³A quick note should be inserted here on coarse-graining since it is of the utmost importance to un-

emerge and patterns start to form. This low resolution, Bohr and Ulfbeck argue, is not powerful enough to see the primary manifestations of symmetry which appear on their own, and thus one observes not total randomness but distributions and trends which suggest the symmetry of the entire system. This is the secondary manifestation of symmetry previously referenced since laws do not appear as themselves but rather are attached to objects, specifically to the primary manifestations of symmetry that are the clicks.

Thus, quantum experiments yield reproducible results in the form of probabilities which express low-resolution relations among clicks. These distributions constitute laws, as previously suggested, but not particularly strict ones. The idea, then, is that clicks themselves are genuinely fortuitous events, but because of coarse graining (the fact that our resolution of the fine-grained picture of the microscopic world does not allow us to determine the lower-level structure of the microscopic world), patterns emerge in the click distribution that allow for the establishment of laws.

There should, therefore, be some kind of continuum where, as the resolution becomes lower, clicks seem more and more fortuitous. At extremely high resolution, probabilities lose all of their meaning and all actions would be perfectly fortuitous. Also, this high resolution represents a break in symmetry as perceived in the secondary manifestation. Since the laws reflected by probability distributions derive from the secondary manifestations of symmetry for a system, a totally lawless regime represents a breakage of this symmetry. The only type of symmetry that remains at this level is the primary manifestation of symmetry, the opposite result from the classical world where only the secondary manifestation of symmetry in which symmetry is inherently attached to other objects is the only kind of observed symmetry. The quantum world, then, produces strange results that given examples of both manifestations of symmetry and their roles in space-time.

With a reinterpretation of quantum probabilities and the probability distribution, a new interpretation is needed for the wave function, the metaphysical entity whose interpretation has confounded physicists and philosophers alike for so long. The wave function, like probabilities, must be a result of the coarse-graining of the quantum world, and as such it must not have any sort of fundamental reality. Bohr and Ulfbeck disregard not only the fundamental role of particles in space-time but the fundamental role of the wave function in space-time as well, rebelling against many canonical interpretations of the nature of particles and the wave function as they do so.

However, the fact that the symmetry of the system (in its secondary manifestation)

derstanding Bohr and Ulfbeck's theory although they do not address it themselves. The basic idea of coarse-graining in this context is that the quantum world exists with such fine structure that the implements used to measure the structure do not pick out these particularities. For example, consider trying to model a bell curve with a certain number of sticks. Modeling the bell curve with two sticks would result in a loss of many of the features that make the bell curve distinct, such as the tails that trail off and the rounded top of the curve. Such a bell curve, at this resolution, would seem identical to a triangular wave. Bohr and Ulfbeck's argument is, then, that all of the detailed information that makes each click unique is eventually flattened into either a click or a no-click on the classical scale, thus forcing fortuitous events to seem different as their randomness is hidden by the low-resolution. Another important thing to note here is that Bohr and Ulfbeck are implicitly assuming that classical mechanics is the result of coarse-graining QM. This view is not yet canonical and should be noted as an unstated assumption of their theory

decides the probability distribution for a system means that the wave function itself is a representation of the secondary manifestations of symmetry of a system. Unlike the click, which is a primary manifestation of symmetry, the wave function has no reality in space-time but is a helpful tool that contains all of the information concerning the symmetries of a macroscopic system. Thus, as resolution increases and secondary manifestations of symmetry splinter, the wave function also loses its meaning, but at the macroscopic scale, it is an extremely useful tool for calculations.

The exact nature of the wave function is now viewed in terms of clicks, not corresponding particles, and thus the symmetry reflected by the wave function is the symmetry of detectors, sources, and other important instruments in a given apparatus. The click distribution is encoded in the wave function, as is the very geometry of space-time. There is no longer any association of the wave function with particles, as was previously the case, for the wave function is now revealed to be merely a statistical tool that describes the large-scale behavior of small-scale systems.

Bohr and Ulfbeck's final interesting note on the wave function is that it evolves in time, thus linking spacial symmetries embodied by the wave function to time. Bohr and Ulfbeck do not take time to be one of the fundamental space-time invariant symmetries that is reflected by the wave function as a secondary manifestation of symmetry, mainly because they take the asymmetry of time²⁴ to be evidence that transformations with respect to time are not invariant, thus breaking the symmetry. Therefore, since time cannot be a symmetry reflected in the very nature of secondary manifestations of symmetry via the wave function, there must be some way of incorporating time symmetries into this theory, and this role is provided by the time evolution of wave function. Since the wave function, which embodies spatial relations, is constrained to rules about its evolution in time, there must be some fundamental link between space and time, according to Bohr and Ulfbeck. Thus, this interpretation of the wave function itself suggests a "footprint of relativity" like the results of the CCR derivation.

If the wave function is reinterpreted by GF, it seems to follow that a new interpretation of the nature of particles and matter is in order too. As previously discussed, particles are removed from the center of the natural world in GF in favor of clicks. With clicks separated from the particles that supposedly cause them and QM derived without a single appeal to particles, Bohr and Ulfbeck conclude that the notion of a particle is superfluous. No particles are needed to model the physical world. In an experimental apparatus consisting of a photon source and a photon detector, one need not appeal to any entity called the photon to describe the events that follow. Bohr and Ulfbeck compare discarding of the electron and photon to dismissing of the theory of the ether in the 20th century; in both cases, they argue, a misleading notion obscured true facts about the physical world, and it was not until the offending notion was rejected that progress in physics could be made.

By discarding the particle interpretation, several other troubling problems in QM go by the wayside as well. Perhaps the most troubling of these is the result of the double slit experiment. As previously discussed, one cannot talk about the motion of a particle along either of the paths in the double slit experiment unless a device is

²⁴That is, the arrow of time which forces us to live life always in a conceived present moving toward the future and away from the past

configured to measure particles before they enter one of the slits. The problem with this is that one is forced to resort to strange interpretations about the nature of matter. For instance, one could claim that, if the measuring device is not present, the particle is not really a particle but is acting as a wave²⁵. One could also argue that the particle is somehow interfering with itself or perhaps that the particle changes back and forth from wave to particle on a whim. All of these interpretations introduce some sort of new, spooky character to what was previously well-defined particle nature.

Bohr and Ulfbeck, however, have no trouble explaining the results of the double slit experiment. First, they do away with the notion of a particle and concern themselves with “click distributions” (here interpreted as the pattern on the screen) only. Then, they note that the addition of the measuring device before the two slits changes the symmetry of the entire experimental setup. This change in symmetry must, of course, result in a change in the distribution of clicks in agreement with the wave function amplitude changes predicted by QM, and so it is natural that, by adding another instrument to the experimental apparatus, the distribution of clicks should change accordingly. GF is equally able to interpret the situation when the intensity of the source is reduced so that only one photon is emitted at a time, for such a change only modifies the rate at which the click distribution emerges, not the pattern of the click distribution itself. Thus, GF’s dismissal of the reality of particles, while it may originally seem off-putting, actually resolves some of the deep problems that have plagued interpretations of QM for decades.

With particles marginalized, one might wonder about the phenomenon of entanglement. One might also raise one of the more serious questions about GF, which is how can two clicks, each of which is unique and “genuinely fortuitous”, give rise to a click distribution that is not fortuitous. The answer, once again, involves the distinction between microscopic and macroscopic phenomena. Bohr and Ulfbeck argue that it is not particles that are entangled (for these do not exist) but the instruments that reflect secondary manifestations of symmetry themselves. Thus, in the previous example of an experimental setup consisting of a photon source and a photon detector, the source and detector are themselves entangled. This means that coarse-graining in one detector is linked to coarse-graining in the other detector, thus leading to entangled clicks and explaining how click distributions that are not fortuitous can arise from fortuitous clicks.

Entanglement represents an innate connection between the source and the detector in this experiment. This connection has a distinct character because entanglement is a non-local phenomenon; that is, entanglement allows two events to be connected despite the fact that they are space-like separated. Thus, entangled clicks represent essentially the “same” event in that the “cause” of both clicks, which is the space-like relation between the source and the detector, is the same emergent phenomenon. Since the pattern of clicks arises as a result of space-time symmetries, and since the non-local connection between the source and the detector constitutes such a symmetry, the two clicks are, in fact, manifestations of the same symmetry and thus are, essentially, the same in character.

²⁵This claim runs into trouble in the experiment where the intensity where the source is lowered to the point where only one photon is emitted from the source. In this case, an interference pattern emerges on the screen behind the double slits, but it appears one dot at a time.

There are several other important consequences for quantum mechanics (and, in fact, for all of space-time) that derive from GF aside from the non-reality of particles and the reinterpretations of the wave function and entanglement. The first of these that I shall address is measurement. In the Copenhagen interpretation of quantum mechanics, measurement is a tricky thing to define, and the results of measurement are even trickier. The traditional interpretation of measurement is that measuring a wave function “collapses” a superposition into a certain eigenfunction. There is no mechanism proposed for this collapse; rather, collapse is a discontinuous process that randomly changes the form of the wave function by changing the amplitudes associated with certain eigenstates within the wave function.

In GF, however, the measurement problem (that is, the problem of how measurements collapse the wave function) is a non-problem, and the mechanism of collapse is not needed. The act of introducing a new measuring device into a system changes the wave function because a new symmetry that can be used to describe the entire system has been added. Non-local connections between the new measurement device and the other devices become reflected in the symmetries of the wave function, changing its very nature to reflect the new configuration. Thus, collapse ceases to be a problem for Bohr and Ulfbeck; if anything causes “collapse”, it is the experimenter herself, introducing the new measuring device to the system!

What is more, “measuring” devices are no longer capable of “measuring” any “thing” in the traditional sense. The clicks produced by a device do not correspond to the existence or state of some quantum particle; instead, they are simply clicks, manifestations of matrix variables that exist as events in space-time. They are uncaused and discontinuous, like collapse, but they are not problematic like collapse because they do not suggest an instant and spooky change in a physical system due to some unseen force; rather, they express the emergence of a random phenomenon, transferred from the microscopic world to the macroscopic world via a coarse-grained measuring device. In this way, measurement and all of its difficulties are removed from their central place in quantum mechanics by the fortuitous nature of clicks in space-time.

Another important consequence of GF concerns causality. The non-causal nature of the click suggests that there is nothing fundamental about the notion of causality on the quantum scale. Dynamical laws, which are causal, emerge as the resolution becomes more coarse-grained, and as such causality is not required at all scales but is a consequence that follows from the onset of the click at low resolution. This coarse-graining interpretation is further supported by the interpretation of entanglement which characterizes the source and detector of a system as entangled instead of the “particles” that we mistakenly think travel between them. Since the detector and source are non-locally connected, there can be no causal relation between them such that a click in the source “causes” a click in the detector. Instead, the result of a click in the detector derives from the symmetry represented by the entangled state vector. The secondary manifestation of symmetry determines how the detector will click, but the determination of the clicks themselves is fortuitous and non-causal, though it must obey the secondary manifestation of symmetry that is the probability distribution dictated by the wave function.

The issue of causality might seem a little bit muddled, however, because there are certain entities, namely matrix variables, whose properties do determine the possible

outcomes of any experiment and any distribution of clicks. One might even want to go so far as to say that the nature of matrix variables “causes” clicks to occur (except, of course, for the fact that clicks are inside space-time while the matrix variables that “cause” them are not). However, before proceeding to discuss causality in too much detail, I wish to propose the following two necessary conditions for a causal relation:

If A causes B, then:

1. The conditional probability $P(B|A)$ (that is, the probability of event B given event A) must be larger than $P(B)$ alone. This means that B is more likely if A has already occurred.
2. There exists some mechanism by which A can directly interact with B.

From this propositional definition of causality, one can see that while condition 1 may be met by the matrix variables (since a three-dimensional matrix with equally probable eigenvalues would increase the probability of measuring any of those eigenvalues over some infinite-dimensional matrix, for example), condition 2 is *never* met by matrix variables since they never actually emerge on the space-time scene. Thus, there is no mechanism by which a matrix variable “becomes” its eigenvalue, and thus there is a non-causal, perfectly fortuitous relation between the matrix variables and space-time. Causality thus falls by the wayside as a effective tool for describing the quantum world.

A final consequence of GF concerns the nature of mass and of Planck’s constant. Both of these quantities, which are linked because Planck’s constant provides the units of mass, are conspicuously absent from GF. Like particles themselves, then, mass and Planck’s constant, which provide a means of analyzing such particles, are seen as “phantasms” as well, emerging from the low resolution of macroscopic systems. To determine why Bohr and Ulfbeck treat mass and Planck’s constant so, I return to their discussion of the very first CCR proof.

In the CCR proof, the quantities of mass and Planck’s constant are only introduced to make the generators of space-time symmetry (specifically the Lorentz boost generator and the spatial translation generator) analogous to the physically observable quantities of the macroscopic world. Planck’s constant is used as a scaling factor for converting the units of the generators into the more familiar macroscopic units of position and momentum while mass was introduced first as an approximation of the energy in the relativistic limit and then cancelled out by a mass term that derived from transforming the generator of Lorentz boosts into the position operator. Thus, terms involving mass only arise when dealing with classical analogues, not when dealing with the actual space-time variables themselves.

The CCR proof, then, reveals the non-essential nature of mass. Like causality, mass is treated as a macroscopic phenomenon that emerges from our coarse-grained perception of the fine-grained quantum world. Mass has no fundamental physical reality and should be dismissed as a physically significant quantity on the quantum scale. This stance towards the reality of mass makes sense given that only particles have mass, and since Bohr and Ulfbeck dismiss the particle description of the quantum world, they should likewise dismiss the importance of properties that particles alone

possess. Thus, Planck's constant is no longer viewed as a fundamental entity whose existence reveals a deep fact about operations of the quantum world but as a scaling factor that determines how coarse-grained the macroscopic perception of the quantum world truly is.

With these considerations taken into effect, GF reveals itself as a complete theory of QM. There are no phenomena that Bohr and Ulfbeck are aware of that are unexplained by their description of the quantum world in terms of fortuitousness and matrix variables. There are some processes, such as how insight into the physical world changes as resolution increases, for instance, that need to be described in more detail, but in general Bohr and Ulfbeck have created a complete description and interpretation of the quantum world. The following section will examine other interpretations of how symmetry plays a role in QM and what consequences these interpretations entail for space-time and the quantum world.

4 Other Relevant Interpretations of Quantum Mechanics

Aside from the Copenhagen interpretation discussed in Section 2, there are several other important interpretations of QM that are relevant to the discussion of GF and, eventually, SSC’s relational blockworld (RBW). The following sections describe two of these interpretations of QM: the Ithaca interpretation from the writings of David Mermin and Jeeva Anandan’s interpretation of QM. These interpretations of QM are similar to Bohr and Ulfbeck’s in several ways, yet both diverge from Bohr and Ulfbeck in interesting and important ways. Both interpretations also provide background for the work of SSC, and so a discussion of these two interpretations serves as a bridge between Bohr and Ulfbeck’s older GF and SSC’s newer RBW.

4.1 The Ithaca Interpretation of Quantum Mechanics

4.1.1 Everett’s “Relative State” Interpretation

The first interpretation to be examined in this section is David Mermin’s “Ithaca Interpretation of Quantum Mechanics” (IIQM), which is presented in two papers in the mid-to-late 1990’s[38][39]. However, the roots of IIQM extend further back to the doctoral thesis of H. Everett[27] in the late 1950’s. This paper of Everett’s was written before his infamous “many worlds” interpretation of QM was fully developed, so while his paper does not yet contain the many references to other worlds that many physicists today find repugnant, one can certainly see the seed of these ideas in Everett’s “Relative State” formalism. Even though Mermin’s interpretation does not follow Everett’s precisely, the former follows the latter so closely that it is worth discussing Everett’s paper in detail.

In his paper Everett stresses repeatedly that his “Relative State” formalism is not a “theory” of quantum mechanics but rather a “metatheory”. Everett agrees with the experimental results and predictions of Copenhagen, and what is more, Everett believes that the Copenhagen interpretation is complete; that is, that there are no “hidden variables” or other unaccounted-for factors whose influences come through in the counterintuitive facets of quantum theory. Thus, Everett’s purpose is not to present some alternative theory or interpretation that will change quantum mechanics but to essentially present a different perspective on quantum mechanics that could make it easier to use.²⁶

The new interpretation that Everett presents rejects of notions like “particles” and “waves” from QM. Instead of making QM a theory concerned with objects, Everett suggests that it is the correlations, not the correlata, that are the physical entities treated by QM. What is important, then, is not the given value for a particular property but rather the correlation between this value and other values. The states of a system are not absolute but relative, and thus quantum mechanics is a theory concerned with relations. In the face of this interpretation, absolute states lose their meaning and the

²⁶Specifically, Everett is interested in providing a “metatheory” that makes it easier for physicists to utilize QM to quantize general relativity

most that can be said about a property is its relation to other properties, not its value in and of itself.

Everett's emphasis on relational states is especially effective at dealing with the measurement problem in QM. Collapse ceases to be a discontinuous process when viewed in terms of correlations. Consider the following situation: a particle, originally prepared in a superposition of two states (call them $|A\rangle$ and $|B\rangle$) is measured and collapses into state $|A\rangle$. The usual interpretation of this process is that there has been a change here since a superposition represents indeterminacy while the post-collapse particle certainly has a determinate value.

According to Everett's interpretation, however, both states $|A\rangle$ and $|B\rangle$ already exist *prior* to collapse since these quantities are relations and have ontological priority. The process of collapse is not one that introduces a new value of determinate nature to the world but one that forces the measuring device to filter out all of the possible measurable relations except for one. Thus, the measurement process becomes no more mysterious than the workings of a gumball machine!

Everett allows for what he calls the "branching" of states because these states are relations, not tactile, spatial entities like particles. There is no reason why a multitude of relations cannot exist simultaneously on the quantum scale where none of these relations are observed in the traditional sense of the word. What is more, due to the nature of the relation between a macroscopic object like a measuring device and a microscopic object like a quantum system, Everett can posit that no more than one relation can be observed macroscopically, no matter how many relations may have existed prior to measurement. This characterizes and explains the phenomenon of measurement, and thus Everett's relational interpretation of QM succeeds in "collapsing" the measurement problem.

4.1.2 Rovelli and Information Theory

Carlo Rovelli[47] provides a similar interpretation to Everett's, though he attempts to pin down the nature of Everett's relations a bit more. Rovelli agrees with Everett that the Copenhagen interpretation is complete and makes it his task to derive the results of the Copenhagen interpretation of QM through arguments involving information theory. By invoking information theory in his analysis, Rovelli suggests that Everett's relations that form the physical basis for quantum theory are information and that the information exchange between systems constitutes the interactions and relations on the quantum scale.

Rovelli suggests the validity of his information theoretic approach by discussing the two-observer²⁷ problem. He discusses a situation in which an observer (call him observer 1) makes a measurement on a superposition state (again, assume that the superposition is between states $|A\rangle$ and $|B\rangle$). Observer 1 knows that the state is initially a superposition, so he knows that his measurement changes the system in the following manner:

²⁷It should be noted here that observer, for both Rovelli and Mermin, does not necessarily imply a human observer. Both theorists posit that observers can be the measurement devices themselves instead of the humans that interpret them. However, in my formulation of Rovelli's example I will assume that the observers discussed are both human, as Rovelli does in his original paper

$$\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \rightarrow |A\rangle \quad (170)$$

The same situation is viewed by another observer (call her observer 2) who makes no measurement but knows the initial conditions not just of the state vector but of observer 1 as well. If the state $|1\rangle$ corresponds to a report by observer 1 of an A value, and if the state $|2\rangle$ corresponds to a report by observer 2 of a B value, and if observer 2 does not receive any information from observer 1, she is able to report the transition in this situation as follows (assuming that the state $|initial\rangle$ corresponds to the initial state of observer 1):

$$\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \otimes |initial\rangle \rightarrow \frac{1}{\sqrt{2}}(|A\rangle \otimes |1\rangle + |B\rangle \otimes |2\rangle) \quad (171)$$

The same events, then, can be interpreted in two different (and entirely correct) ways depending on which observer's perspective one assumes. Rovelli argues that this situation reveals that different observers can tell different, correct stories about the exact same quantum phenomena. In the example above, this difference arises from the fact that observer 1 was observing some quantum mechanical state vector while observer 2 was observing the system composed of observer 1 and the state vector. Thus, the two different observers obtained different data, leading to different stories about the same physical events.

The example above and the conclusions that Rovelli draws from it suggest several important consequences. The first of these consequences is that there is no objective "state of the system". Rather, the "state of the system" can be determined only from the perspective of an observer, and one observer's perspective precludes the assumption of other perspectives. Since all observers' perspectives are equally valid ways of describing the physical system (i.e. there is no "preferred quantum observer" frame), one can only conclude that each of these system states is equally valid and thus that there is no one, single state that both describes the system and contains all of the information obtained by different observers.

One might object to Rovelli's example on the grounds that two different events are observed (the state vector for observer 1 and the system of the state vector and observer 1 for observer 2), but Rovelli states that no self-observations are possible, and thus observer 1, not being able to measure himself, obtains as close a picture to the "observer 1"-state vector system as he can get by simply observing the state vector. Rovelli speaks about this attitude towards self-measurement later on his paper when he compares his interpretation to others'.

Another consequence that Rovelli draws from the above example is that an observer in observer 2's position may determine the property that observer 1 will measure²⁸ but not what value unless the final conditions are specified. Rovelli concludes that an observer can, at most, know *that* another observer knows something; however, unless the former actually makes a measurement on the final state of the latter, the former cannot know *what* the other observer knows.

²⁸Again, this fact comes from knowing the initial conditions of the experimental setup

This leads to Rovelli’s information theoretic interpretation of QM. The difference between the interpretations given by observer 1 and observer 2 comes down to information about the situation. Neither observer gets the same data, and thus they interpret the same physical situation differently. These data, Rovelli suggests, are converted from ontic to epistemic entities via Planck’s constant, thus taking physical units and transforming them into bits of information. One might wonder at this point whether Rovelli would be so bold as to address the issue of consciousness in his theory considering its involvement in other epistemic phenomena, but Rovelli prefers to end his article with computational information theory without involving the messy issues of the mind.

4.1.3 Mermin’s Ithaca Interpretation

With Rovelli and Everett having been discussed and providing the appropriate background, I now turn to Mermin and his Ithaca interpretation of QM. As has already been suggested, Mermin agrees with Everett and Rovelli to a large extent. First off, Mermin states explicitly that his “interpretation” of QM ought not be called an “interpretation” but rather an “attitude” towards QM since it does not suggest any different results from the canonical Copenhagen interpretation. Mermin’s goal in proposing the Ithaca interpretation of quantum mechanics (IIQM) is to suggest a new way of thinking about QM, not to overturn the pre-established quantum theory.

Mermin presents two theorems from which the IIQM follows in his first paper proposing the IIQM [38]. The first of these theorems is that all meaningful quantum mechanical information of a system can be derived from the density matrix of that system, and the second theorem is that the density matrix of a system can be constructed from the local correlations inside that system. Both of these theorems are supported by proofs whose reproduction is beyond the scope of this paper. The conclusion from these two theorems is that the local correlations inside a system are sufficient for a complete characterization of the system itself. All that needs to be known about a system, then, is the character of its internal correlations, according to the IIQM.

Thus, for Mermin, correlations serve as the fundamental objects for QM. These correlations are then between the subsystems of a system and are thus the only proper subjects of any physical theory about the world. Unlike Everett and like Rovelli, however, Mermin does not stop with an assertion of the primacy of these correlations but rather attempts to determine more specific information about the character of these correlations. In [39], Mermin defines small-scale correlations as, “the mean values, at any given time, of all system observables(Hermitian operators) that consist of products over subsystems of individual subsystem observables” (4). The quantities Mermin describes as correlations can be properly described as correlations because relationships among the subsystems are established by the product of the subsystems’ observables while the information regarding the absolute value of each individual subsystem’s observable is lost.

Mermin also describes large-scale correlations, which are different in that they involve two systems instead of two subsystems. The difference in terminology here reflects the fact that two systems are not both parts of a larger system while two subsystems can be. Correlations between systems are the cause of the collapse phenomenon, ac-

ording to Mermin, because such correlations consist of a macroscopic observer directly interacting with a microscopic system. Mermin’s system correlations, then, are different in character from the correlations between two subsystems of a given system, and thus one would rightly expect such system-system correlations to behave differently from subsystem-subsystem correlations. It is the nature of this interaction, then, that leads to the phenomenon of collapse.

Mermin is not content to leave his description of collapse here, however. He continues his analysis by agreeing with Rovelli that all correlations, which correspond to wave functions, are equally real and present in an unobserved system. Thus, the phenomenon of collapse is not a discontinuous process that changes the basic character of the quantities observed but rather a kind filtering of possibilities, allowing only one correlation at a time to manifest itself.

One might ask the question of why collapse might have seemed like a problem to others who followed the Copenhagen interpretation. Mermin answers this question by appealing to the Copenhagen interpretation’s emphasis on defining the correlata of Mermin’s correlations. Unlike Copenhagen, IIQM does not attempt to define what individual observable values, the correlata of the correlations described above, are. Such correlata have no meaning in the IIQM. Primary ontological status is reserved for correlations alone.

The final facet of Mermin’s IIQM that bears mentioning is its attitude towards probability. Mermin points out that many interpretations of QM seem problematic or confusing because probability is not understood properly. Usually, people think of probabilities as states of knowledge that reflect uncertainty. Under this view, probabilities are essentially epistemic or knowledge-based. However, Mermin argues that, once the IIQM is fully understood, probabilities reveal themselves to be essentially *ontic*, existing as objective correlations between subsystems. Probabilities are mathematical objects that do not reflect a lack of knowledge about a system but a predictive power about the internal functioning of a given system, and as such they are objective facts about the world. According to Mermin, it is because of epistemic probability interpretations that spooky entities like consciousness enter discussions of QM, and once the IIQM is fully accepted, the role of such ghosts in the physical theory will vanish.

4.1.4 Comparison of Mermin with Everett and Rovelli

Mermin’s IIQM certainly differs from Rovelli’s and Everett’s interpretations in various important ways. First, however, the similarities that Mermin, Rovelli, and Everett share should be emphasized and discussed. One characteristic shared by all three theories is that they hold the Copenhagen interpretation of QM to be complete. There is no “hidden variable” that Copenhagen has missed, and all of the strangeness of QM arises as a reflection of the strangeness of the physical world and not as a suggestion that QM lacks something as a physical theory. Thus, Mermin, Rovelli, and Everett are not so much looking to change the Copenhagen interpretation as to redefine it in such a way that traditionally troublesome facts and processes, especially collapse, lose their “spookiness”.

All three theorists also agree on the fact that it is correlations, not their correlata, that are fundamentally real. Thus, QM is not a theory of “things” but a theory

of relations among “things”. While it is Mermin who most ardently argues against the existence of any sort of correlata at all, Everett and Rovelli also agree that it is correlations and not correlata that have ontological significance.

The final major point of agreement among Mermin, Rovelli, and Everett is that the measurement problem can be dismissed if one views measurement as just another kind of correlation. Relations on the quantum scale are between microscopic entities while measurement establishes a relationship between a macroscopic observer and a microscopic system. Thus, the character of the correlation established by the measurement process must be fundamentally different from the character of microscopic correlations. What is more, not only is the collapse process not mysterious, it is also continuous. Instead of the entire system evolving from some superposition of indeterminate reality to a eigenfunction which is defined, the system evolves from a collection of many equally real states representing many equally real correlations to a single defined state. The indeterminacy of the first relation, Mermin reveals, is due to an interpretation of QM that assumes the reality of properties instead of relations among properties, and thus Mermin, Rovelli, and Everett do away with the troubling aspects of measurement when they shift their focus from correlata to correlations.

However, Mermin provides a different interpretation of quantum mechanics from Rovelli’s and Everett’s in several important ways. First, Mermin elucidates the nature of the correlations described by the IIQM much better than either Rovelli or Everett do in their papers. Mermin’s interpretation involves well-defined terms involving systems and subsystems instead of the vaguely defined relations of Everett . One might contend that Rovelli’s conception of information is equally elucidating, but while Rovelli describes information transfer in great detail, he never explains exactly how it is that Planck’s constant is supposed to transform an ontic entity into an epistemic one.

This leads to a major difference between Mermin and Rovelli which lies in Mermin’s emphasis on the objective. On the issue of probability, Mermin resolutely denounces the epistemic interpretation of probability in favor of objective probability, emphasizing the fact that all observers will agree on the same probabilities and the same relations among wave functions. Rovelli, on the other hand, prefers to emphasize the relativity of the quantum world, suggesting that there is no objective state of affairs and not only that two different observers will fundamentally disagree as to the state of a given system but that these two observers will also receive different data from the same setup. Each of these interpretations is equally valid, and thus subjectivity becomes the law for Rovelli’s QM where objectivity rules for Mermin.

A final important difference that sets Mermin apart from Rovelli and Everett is the fact that Mermin goes through proofs of each of his two theorems that lead to his IIQM in intricate detail. Though these proofs have not been reproduced here, suffice it to say that they are better laid-out and rely more on mathematics and less on plausible interpretations than any of Everett’s arguments or Rovelli’s initial argument for relativity and information theory in QM. For this reason, along with those previous stated, I have decided to use Mermin’s IIQM as the best representative of the relational-state attitudes towards the Copenhagen interpretation of QM, and it will be Mermin’s perspective with which I contrast GF.

4.1.5 Comparison of IIQM with GF

How does Mermin's vision of the IIQM compare with Bohr and Ulfbeck's formulation of GF? To a certain extent, both theories share a lot in common. The first similarity they share is that both IIQM and GF turn many of the classical conceptions of realism that continue in the Copenhagen interpretation of QM on their heads. GF, for instance, claims that it is the symmetries of space-time and not particles or waves that have ontological priority while IIQM claims that correlations between properties of a system have ontological priority over the properties themselves and their values. The classical picture holds that both particles and their properties are fundamentally real, and thus both GF and IIQM do away with this picture by rejecting some real entity in the Copenhagen interpretation.

Another similarity between the two theories is that probabilities are taken to be objective. In IIQM probabilities represent objective relations among subsystems while in GF probabilities are the manifestations of space-time symmetries. This ties into another similarity between the two theories: it is relations in some form (symmetries for GF, correlations for IIQM) that are real while what they relate (matter, physical observables) is not. Both theories find reality in the probabilities and relations among things they take to be, in and of themselves, unreal. Just as the idea of a single, uncorrelated value has no meaning in IIQM, matter without symmetry manifestations has no meaning in GF.

Another issue on which IIQM and GF agree regards measurement. Unlike some theories that would posit that the process of collapse happens in the mind[4] or that consciousness is somehow involved in measurement, both GF and IIQM hold that the phenomenon commonly referred to as collapse takes place physically in the world as an objective fact. What is more, the two theories agree that the measurement phenomenon results from the fact that a macroscopic object must interact with a microscopic system for "collapse" to take place. The strange nature of this interaction necessitates the phenomenon of collapse in both pictures of the quantum world.

However, despite general similarities, GF and IIQM are certainly different theories of the world. The first difference between GF and IIQM is that GF posits the non-existence of matter. This takes the non-reality of properties and their values in IIQM to a deeper level since GF rejects not only the reality of certain physical properties but the reality of entities (particles, waves, etc.) that would hold these properties as well. In a way, then, GF can be seen as an extension of IIQM, attacking common interpretations of QM at a deeper level than IIQM ever reached.

Another way in which GF can be seen as a kind of extension of IIQM is that it also makes clear the idea of what "correlations" are by tying in the idea of space-time symmetries. By appealing to symmetries in the Poincaré group, GF finds a basis for the idea of correlations inherent to IIQM without having to dig up an explanation for the cause of these correlations. GF's conception of correlations is deeper than IIQM's because GF draws its correlations from relativity and thus can give both a cause and a characterization of the correlations it describes instead of merely giving a characterization as IIQM does. Thus, to a certain extent, GF can be thought of as a deeper extension of Mermin's IIQM, and it is for this reason that GF is capable of explaining the relationship between SR and QM aside from explaining how measurement and

other quantum phenomena work while IIQM can only do the latter.

However, there is at least one issue on which GF and IIQM strongly disagree: the problem of measurement. For IIQM collapse seems a misleading phenomenon; instead of being a discontinuous shift from indeterminacy to determinacy, collapse is viewed as a gradual evolution from multiple existing relations to a single relation. For GF, however, collapse is a fundamental phenomenon, for a click in a detector (the closest thing to the “collapse” of a “particle” in the world of GF) is a genuinely fortuitous event, completely uncaused and arising from the nowhere background of space-time symmetries. Not only is the appearance of a matrix variable a discontinuous event for GF, it is a *fundamentally* discontinuous event, for it is from such fortuitous, discontinuous events that the entirety of QM is built.

Such discontinuity in GF can be seen to arise from the fact that the space-time symmetries are not present on the space-time scene before the fortuitous click. In IIQM the relations already exist before the collapse process takes place, but for GF one matrix variable cannot be on the scene without implicating others, and thus no matrix variables are on the space-time scene before the click. It is this difference in reinterpreting indeterminacy as already-present states for IIQM and not-present states for GF that leads to their great disagreement about the discontinuous (and thus fortuitous) nature of measurement and thus to different interpretations of the measurement problem.

GF entails the abandonment of the Copenhagen interpretation, but it still shares several qualities in common with IIQM. These similarities can be interpreted to suggest the following two claims about the relationship between the Copenhagen interpretation and GF: either IIQM represents a realization of several fundamental flaws in the traditional thinking of the Copenhagen interpretation and that these flaws are eventually rectified in GF, the fulfillment of IIQM, or else GF is a radicalization of IIQM, and, in the event that GF is eventually abandoned, IIQM serves as a less violent theory that keeps some of GF’s character. Whether it is GF that supports IIQM or IIQM that leads to GF will be an issue that will only be decided in the face of confirming or disconfirming experimental evidence concerning GF.

4.2 Anandan’s Relatationality

Like Mermin’s IIQM, Jeeva Anandan’s interpretation of QM differs from GF on several important counts; however, Anandan’s thinking is much more along the lines of Bohr and Ulfbeck on the issue of causation, an issue that Mermin does not explicitly address in his papers. Thus, I will now proceed to investigate Anandan’s interpretation, starting with a detailed derivation of the Born rule from a vision of QM without appeals to particles, waves, or time, and then proceeding to a discussion of the general points of Anandan’s theory. Finally, I will conclude this section with a brief comparison of Anandan’s work with Bohr and Ulfbeck’s as well as a suggestion of how SSC utilize Anandan’s work to their ends.

4.2.1 The Born Interpretation

Anandan's reinterpretation of QM depends on a mathematical derivation of the Born probability interpretation from non-dynamical quantities. To begin this derivation, Anandan first proves that the amplitude of the quantum mechanical wave function must be a complex number. The beginning of this proof uses the result, derived from Hurwitz[33], that the probability amplitude must be a real number, a complex number, a quaternion, or an octonion. Albert[3] generalizes this conclusion to non-quadratic forms, but due to the simpler nature of Hurwitz's proof, his will be the proof I reproduce here. Neither of these proofs, it should be noted, appear in Anandan's work.

The goal of Hurwitz's proof is to determine the dimensionality of matrix representations of quadratic forms. Quadratic forms can be written as

$$Q(x) = x^T Ax \tag{172}$$

where x is the vector representing the variables of the system and A is the matrix by which the quadratic form may be represented. This can be written as an inner product

$$Q(x) = \langle x | A | x \rangle \tag{173}$$

Since the form of a quantum mechanical wave function amplitude manifests itself in probabilities as $|\langle \psi_f | \psi_i \rangle|^2$, it is clear that the wave function amplitude may be represented as a quadratic form.

The question, then, is what kind of matrix is A . Since A is an $n \times n$ matrix, one can reasonably ask what value n should assume. For the real numbers, for instance, n is 1 and the matrix is a constant. For complex numbers of the form $a + bi$ there must be 2 different necessary values, and as such n is 2. Likewise, if n is 4, the matrix corresponds to a quaternion, and if n is 8, the matrix corresponds to an octonion. Therefore, to prove that the probability amplitude must be a real number, a complex number, a quaternion, or an octonion, one can prove that a quadratic form that satisfies the same properties as the probability amplitude must have the general forms of a constant, a 2×2 matrix, a 4×4 matrix, or an 8×8 matrix.

Hurwitz begins with some assumptions that allow the quadratic forms in his proof to reflect the constraints on probability amplitudes²⁹. He assumes the three quadratic forms denoted by ϕ , ψ , and χ which are quadratic forms of n variables with non-zero determinants. These forms satisfy the relation

$$\phi\psi = \chi \tag{174}$$

because the product of operators should also be an operator. At this point, one trivial solution for the dimension of the matrices above is that these matrices are of dimension 1. So, simply by stating the problem, one can infer that the probability amplitude can consist of real numbers. However, it is not proven that real numbers are the *only* form that the probability amplitude takes, and as such I must continue Hurwitz's proof for the other potential forms of the probability amplitude. By performing a change of

²⁹It should be noted here that Hurwitz was not, of course, attempting to prove anything about the quantum mechanical probability amplitude when he wrote this paper. The paper containing the proof outlined here was written in 1898, at least 30 years before the advent of QM.

variables such that $\phi = \sum x_i^2$, $\psi = \sum y_i^2$, and $\chi = \sum z_i^2$ where all of the sums are over the index i , this equation can be rewritten as

$$\left(\sum x_i^2\right)\left(\sum y_i^2\right) = \sum z_i^2 \quad (175)$$

The quantities x_i and y_i are all independent, and the z_i terms are bilinear functions of x_i and y_i .

Hurwitz now defines a given $n \times n$ matrix A in the following manner:

$$AA^T = \sum x_i^2 \quad (176)$$

$$A = \sum x_i A_i \quad (177)$$

$$A_i A_i^T = I \quad (178)$$

where I is the identity matrix. There are n matrices of form A_i which are related to the matrix A as described by the above equation. A final matrix B_i of the same dimensionality as A and A_i is defined as follows:

$$B_i = A_i A_n^T \quad (179)$$

for $1 \leq i \leq n-1$. Using the above definitions and properties of A , A_i , and B_i , one easily prove that the following is true:

$$\sum x_i^2 = AA^T \quad (180)$$

$$= \left(\sum x_i A_i\right)\left(\sum x_i A_i^T\right) \quad (181)$$

$$= \left(\sum x_i A_i\right)I\left(\sum x_i A_i^T\right) \quad (182)$$

$$= \left(\sum x_i A_i\right)A_n^T A_n \left(\sum x_i A_i^T\right) \quad (183)$$

$$= \left(\sum x_i A_i A_n^T\right)\left(\sum x_i A_n A_i^T\right) \quad (184)$$

$$= \left(\sum x_i B_i + x_n\right)\left(\sum x_i B_i^T + x_n\right) \quad (185)$$

This final relation implies that the following properties of B_i are true:

$$B_i B_i^T = I \quad (186)$$

$$B_i^2 = -I \quad (187)$$

$$B_i = -B_i^T \quad (188)$$

$$B_i B_k^T = -B_k B_i^T \quad (189)$$

These properties of B_i will be necessary for later calculations involving B_i .

For the moment I should pause to note two important facts about B_i . Since $B_i = -B_i^T$, the matrix B_i is antisymmetric, and because $B_i^2 = -I$, B_i is invertible. One of the theorems Hurwitz utilizes at this point is the fact that any $n \times n$ matrix that is

both invertible and anti-symmetric must have an even n ³⁰. Clearly, then, imaginary numbers, quaternions, and octonions are still candidates for expressing the probability amplitude. However, the pool of candidates has not been limited to only these three options yet, and thus the proof continues.

Hurwitz now considers transformations of the form $B_{i_1}B_{i_2}\dots B_{i_r}$ where i and r denote the same indices used previously. It is obvious that $r < n$ since there are $n - 1$ different B_i matrices. I wish to consider symmetric S_{ir} terms to determine which transformations are symmetric and thus, having ruled out the symmetric matrices, which matrices are antisymmetric as well. I will start with an example, $r = 3$. In this situation, the transpose of the transformation $B_{i_1}B_{i_2}B_{i_3}$ is needed. If the transpose of $B_{i_1}B_{i_2}B_{i_3}$ is the same as $B_{i_1}B_{i_2}B_{i_3}$, then the transformation $B_{i_1}B_{i_2}B_{i_3}$ is symmetric. To determine symmetry, then, I perform the following calculation using the previously-stated properties of B_i :

$$(B_{i_1}B_{i_2}B_{i_3})^T = B_{i_3}^T B_{i_2}^T B_{i_1}^T \quad (190)$$

$$= (-1)^3 B_{i_3} B_{i_2} B_{i_1} \quad (191)$$

$$= (-1)^4 B_{i_3} B_{i_1} B_{i_2} \quad (192)$$

$$= (-1)^5 B_{i_1} B_{i_3} B_{i_2} \quad (193)$$

$$= (-1)^6 B_{i_1} B_{i_2} B_{i_3} \quad (194)$$

$$= B_{i_1} B_{i_2} B_{i_3} \quad (195)$$

Thus, S_{i_3} is a symmetric matrix. However, S_{i_2} is not, as shown below:

$$(B_{i_1}B_{i_2})^T = B_{i_2}^T B_{i_1}^T \quad (196)$$

$$= (-1)^2 B_{i_2} B_{i_1} \quad (197)$$

$$= (-1)^3 B_{i_1} B_{i_2} \quad (198)$$

$$= -B_{i_1} B_{i_2} \quad (199)$$

Thus, S_{i_2} is antisymmetric. The pattern from these two cases can be generalized: for any S_{ir} , S_{ir} will be antisymmetric if the quantity $r + (r - 1) + (r - 2) + \dots + (r - (r - 1))$ is odd and symmetric if it is even. Using the identity that

$$r + (r - 1) + (r - 2) + \dots + (r - (r - 1)) = \frac{r(r + 1)}{2} \quad (200)$$

it becomes obvious that the only matrices S_{ir} that are symmetric are those with either r as a multiple of 4 or $r + 1$ as a multiple of 4. Thus, the possible values for r that yield symmetric S_{ir} are 0 mod 4 and 3 mod 4, and the possible values for r that yield antisymmetric S_{ir} are 1 mod 4 and 2 mod 4.

³⁰Though Hurwitz utilizes this theorem, he does not, in fact, give a proof of it. The following is a quick proof of this theorem: the transpose of a matrix must have the same determinant as the original matrix. However, an antisymmetric matrix has a transpose whose determinant is equal to -1^n times the original matrix's determinant. The only condition under which the two determinants are equal, then is when n is even.

I will now shift gears somewhat and address the issue of reducible and irreducible linear dependences. I will return to the previous results concerning symmetric and antisymmetric matrices later. A linear dependence R in this context refers to the sums of various transformations in a space multiplied by constants such that the sum is zero. A linear dependence is said to be reducible if it can be written in the form $R = R_1 + R_2$ where R , R_1 , and R_2 are equal to zero. The transformations S_{ir} that will be treated here are assumed to have irreducible linear dependences.

This simplifying assumption allows Hurwitz to write a given transformation S_{ir} as a linear combination of other transformations as follows:

$$S_{ir} = \sum c_{i_1 i_2 i_3} B_{i_1} B_{i_2} B_{i_3} + \sum c_{i_1 i_2 i_3 i_4} B_{i_1} B_{i_2} B_{i_3} B_{i_4} + \dots \quad (201)$$

I have started with the three-term sum here because of the fact that all symmetric S_{ir} terms are composed of 0 mod 4 or 3 mod 4 terms in anticipation of the future symmetry requirements in the proof. Hurwitz deals with symmetric transformations here because symmetric transformations yield symmetric transformations when operated on each other and added together if only irreducible linear dependences are considered.

Since the matrices are invertible and since $B_i B_i^T = 1$, the above result can be seen to simplify a great deal to give the identity matrix, a symmetric matrix, as a linear combination of transformations:

$$I = \sum c_{i_1 i_2 i_3} B_{i_1} B_{i_2} B_{i_3} + \sum c_{i_1 i_2 i_3 i_4} B_{i_1} B_{i_2} B_{i_3} B_{i_4} + \dots \quad (202)$$

Now, choosing an arbitrary B_i and multiplying it by both sides yields:

$$B_i = \sum c_{i_1 i_2 i_3} B_{i_1} B_{i_2} B_{i_3} B_i + \sum c_{i_1 i_2 i_3 i_4} B_{i_1} B_{i_2} B_{i_3} B_{i_4} B_i + \dots \quad (203)$$

This equality suggests that the following symmetry considerations be taken into effect: first, since B_i is antisymmetric, each term in the sum must also be antisymmetric. Thus, each transformation in the sum with $r = 3$ must either have a coefficient of zero or one of its elements equal to the arbitrary B_i . If $i = i_1, i_2$, or i_3 , then the fact that $B_i^2 = -1$ will lower the $r = 3$ transformations, which are symmetric, to $r = 2$ transformations, which are antisymmetric. It must be the case, then, that the arbitrary B_i term introduced is already present in the sums on the right hand side of the above equation. Since the B_i is chosen arbitrarily, the constant c terms must not depend on it, and thus c must be equal to zero unless the term multiplied by B_i includes all possible terms.

Thus, the only way to be sure that the sum on the left is equal to B_i is for there to be a single transformation which contains all n possible B_j matrices. This allows me to write:

$$B_i = B_i \prod_j^{n-1} B_j \text{ or } I = \prod_j^{n-1} B_j \quad (204)$$

This last equation allows for a differentiation between the two types of even values n can assume; values of n can be equal to 0 mod 4 or 2 mod 4, but only the values of n equal to 0 mod 4 satisfy this equation. Values of n equal to 2 mod 4 lead to

$I^T = -I$, which is false, and this contradiction means that the initial assumption of linear dependence is clearly false. Thus, two different situations arise: if n is $0 \pmod 4$, then it can consist of linearly dependent B_i matrices, while n values that are $2 \pmod 4$ lead to linearly independent B_i matrices.

At this point Hurwitz invokes the fact that the number of linearly independent transformations for a given n must be less than or equal to the total number of basis matrices. The total number of basis matrices in all cases is n^2 for a given dimensionality n . The cases of $0 \pmod 4$ and $2 \pmod 4$ lead to different numbers of linearly independent transformations. Since the $2 \pmod 4$ case for n consists of linearly independent transformations, the total number P_2 of linearly independent transformations is given as:

$$P_2 = \sum_{r=0}^{n-1} \binom{n-1}{r} = 2^{n-1} \quad (205)$$

However, for the case where n is of the form $0 \pmod 4$, the total number of linearly independent transformations is smaller since the relation in Equation 204 allows for any transformation to be unaffected when multiplied by the product of all the B_i terms in the n -space. This means that a transformation of p terms is now equivalent to a transformation of $n - p$ terms since $B_i^2 = -I$. For example, a transformation S_{i_3} in an $n = 4$ space of the form $B_{i_1}B_{i_2}B_{i_3}$ could be multiplied by the term $B_{i_1}B_{i_2}B_{i_3}B_{i_4}$, which is equivalent to the identity matrix, to yield $-B_{i_4}$, to which the expression $B_{i_1}B_{i_2}B_{i_3}$ is now seen to be equivalent. Thus, the total number of linearly independent transformations P_0 is cut in half, and thus:

$$P_0 = \frac{1}{2} \sum_{r=0}^{n-1} \binom{n-1}{r} = 2^{n-2} \quad (206)$$

Thus, the inequality above is reduced to $2^{n-2} \leq n^2$ for values of n equal to $0 \pmod 4$ and $2^{n-1} \leq n^2$ for values of n equal to $2 \pmod 4$. The only values for n that satisfy these inequalities are 2, 4, 6, and 8.

The final step in Hurwitz's proof is to show that $n = 6$ cannot be the case. To do this, it is first necessary to posit that, in an $n \times n$ space, the maximum number of anti-symmetric matrices must be $\frac{n^2-n}{2}$. This follows from the fact that, if one were to represent this space as a matrix, the n diagonal elements would be zero and half of the remaining elements would be repeats due to the nature of antisymmetry. Thus, there is a maximum number of $\frac{n(n-1)}{2}$ antisymmetric matrices for any $n \times n$ space.

However, as previously derived, the number of antisymmetric matrices comes from selecting out the number of antisymmetric matrices from the total number of independent matrices in the space, which entails looking at the $r = 1 \pmod 4$ and $r = 2 \pmod 4$ terms of the expression

$$\sum_{r=0}^{n-1} \binom{n-1}{r} \quad (207)$$

for values of $n = 2 \pmod 4$ (thus, 2 and 6) and the $1 \pmod 4$ and $2 \pmod 4$ terms of the expression

$$\sum_{r=0}^{n-2} \binom{n-2}{r} \tag{208}$$

for values of $n = 0 \pmod 4$ (thus, 4 and 8). If the number of antisymmetric matrices calculated in this way is less than or equal to the maximum number allowed ($\frac{n(n-1)}{2}$), then the value for n is allowed, but otherwise, a contradiction arises, forcing the rejection of a value for n .

The following table presents the values for the number of antisymmetric matrices allowed for a given $n \times n$ space and the number needed:

n	Number of Antisymm.	Antisymm. Allowed
2	1	1
4	3	6
6	16	15
8	28	28

Based on the results of the above table, it is clear that $n = 6$ yields a contradiction and thus 6 cannot be an admissible value for n to assume. Thus, the matrices of the form $n \times n$ must be 2×2 , 4×4 , or 8×8 matrices. The previous interpretation of these results and the inclusion of the 1×1 case that was trivially mentioned in the beginning of this proof support Anandan's assertion that the probability amplitude must be a real number (1×1), a complex number (2×2), a quaternion (4×4), or an octonion (8×8).

At this point, then, I leave Hurwitz and return to Anandan and his interpretation of the probability amplitude. More must be said about Anandan's interpretation of this quantity before continuing. To this end, I will examine Anandan's path interpretation of QM. One of Anandan's fundamental assumptions when examining the "dynamical" system of a particle traveling along a path from point A to point B is that the motion between these two points is completely random and uncaused. This line of reasoning follows the same major path as Bohr and Ulfbeck in GF, and more will be said about Anandan's general approach later in this section. The important thing to glean from Anandan's interpretation of the particle's travel is that all paths are equally probable for the particle to take, and thus the probability of a particle taking any one of an infinite number of paths is the same³¹. This criterion will be important when Anandan seeks to rule out possible numerical forms for the probability amplitude.

There are also other criteria that Anandan invokes to constrain the nature of the probability amplitude, including the fact that the probability amplitude must follow an algebra. Probabilities of sums should be calculated by adding probability amplitudes and conjoined probabilities should be calculated by multiplying probability amplitudes since a single path may be constructed from several other, smaller paths and since the path taken is said to be indeterminate in accordance with the laws of QM. This

³¹It is already clear, at this point, that if the path probabilities are assumed to be real numbers, all probabilities will have to be zero and the probability of taking any path will have to be zero. Thus, it is already clear that the probability amplitudes cannot be real numbers.

algebra criterion will also be used by Anandan to rule out possible numerical forms of the probability amplitude.

Now, citing Hurwitz's result, Anandan seeks to eliminate potential candidates for the form of the probability amplitude so that he may determine what its form must necessarily be. First of all, Anandan rules out octonions as candidates for probability amplitudes because they are not associative under multiplication in a situation where addition is involved. The algebra requirement of the probability amplitudes that I have previously stated is invoked by Anandan to rule out these quantities as potential probability amplitudes.

Likewise, Anandan rejects the real numbers as candidates for the probability amplitude because, since there are an infinite number of paths with the same probability for each, any non-zero probability for each path will yield an infinite total probability under the reals. Thus, if the probability amplitudes are real numbers, the results would either not agree with QM or would suggest that motion of any kind is impossible. For this reason, the real numbers are also rejected as candidates for the probability amplitude, as I suggested in my previous footnote.

Thus, the probability amplitude must be either a complex number or a quaternion. Anandan cites Adler[1] who, in a proof beyond the scope of this paper, attempts to determine the quaternionic analogue to the path integral formulation of QM. Adler determines an equation (p.110) for the path integral formulation which, unlike the traditional path-integral, does not allow for the combination of exponential factors and thus does not lead to a computable action for the quantum system. Adler concludes that the fundamental quantity in quaternionic QM must be the Hamiltonian rather than the action since no quaternionic formulation of the action exists. Anandan cites this result as evidence that quaternions cannot lead to a meaningful mathematical interpretation of the quantum mechanical path, and as such, probability amplitudes cannot be quaternions. Thus, by process of elimination, Anandan concludes that probability amplitudes must be complex numbers.

Complex numbers can be sufficiently defined by two quantities: the norm, or length, of the number and the argument, or phase, of the number. Anandan considers two paths ψ_1 and ψ_2 with unknown probability amplitude phases³². Because the phase between the two paths' probability amplitudes is uncertain, Anandan averages over this uncertain phase to determine the following relation between the probability of the sum of the paths and the sum of the probabilities of the individual paths:

$$\frac{1}{2\pi} \int_0^{2\pi} d\theta_1 P(\psi_1 + \psi_2) = P(\psi_1) + P(\psi_2) \quad (209)$$

where $\theta_1 = \arg\psi_1$, $\theta_2 = \arg\psi_2$, and $P(\psi_1)$, $P(\psi_2)$, and $P(\psi_1 + \psi_2)$ are the probabilities of the paths ψ_1 , ψ_2 , and $\psi_1 + \psi_2$ respectively. By integrating over θ_1 , the probability is averaged over all possible phases of θ_1 from 0 to 2π . Because the variable of integration is θ_1 in the above equation, the probability $P(\psi_1)$ cannot be a function of any component of path 1 including $\arg\psi_1$. Thus, the probability $P(\psi_1)$ depends only on the norm of the complex number represented by $|\psi_1|$. Since probabilities are required

³²This section of Anandan's proof is reproduced in greater detail by Stuckey et. al.[54]. I make use of many of their steps in my reproduction of Anandan's proof

to be positive, it seems a good assumption that the probability should be of the form $P(\psi_1) = |\psi_1|^n$. Since ψ_1 is the probability of an arbitrary path and since all paths have the same form of probability amplitude, one can generalize this result to say that all probabilities of particular paths must have the form of $|\psi|^n$ for any ψ .

A simplification to the above equation can be made by noting that all probabilities of all paths must be the same. Thus, the norms of these paths should be the same as well, meaning that, setting $|\psi_1| = b$, the right side of the above equation reduces to $2b^n$. The probability $P(\psi_1 + \psi_2)$ reduces as well. Taking the norm of this quantity, one obtains $|\psi_1 + \psi_2| = |[2b(1 + \cos(\theta_1 - \theta_2))]^{1/2}| = |2b \cos(\theta/2)|$ where $\theta = \theta_1 - \theta_2$. Making the appropriate substitutions for the probabilities in the above equation and changing the variable of integration from θ_1 to θ , Anandan obtains:

$$\frac{1}{2\pi} \int_0^{2\pi} d\theta 2^n b^n |\cos^n(\theta/2)| = 2b^n \quad (210)$$

Setting a new quantity $\alpha = \theta/2$ and changing the limits of integration yields:

$$\frac{2^n b^n}{2\pi} 2 \int_0^\pi d\alpha |\cos^n(\alpha)| = 2b^n \quad (211)$$

This equation reduces to:

$$\frac{2^n}{2\pi} \int_0^\pi d\alpha |\cos^n(\alpha)| = 1 \quad (212)$$

At this point, SSC[54] make use of the following identity:

$$\int_0^\pi d\alpha |\cos^{2m+1} \alpha| = 2 \int_0^{\pi/2} d\alpha \cos^{2m+1} \alpha \quad (213)$$

for any non-negative integer m . Since Anandan assumes that n is positive (allowing for probability amplitudes of zero), setting $n = 2m + 1$ will be useful in this situation. Before this substitution, however, SSC note that the right side of the equation can be evaluated as follows:

$$\int_0^\pi d\alpha |\cos^{2m+1} \alpha| = \frac{2^{2m+1} (m!)^2}{(2m+1)!} \quad (214)$$

Thus, substituting $n = 2m + 1$ in the equation above and substituting this identity into the probability amplitude equation above yields:

$$\frac{2^n}{2\pi} \frac{2^n}{n!} \left(\frac{n-1}{2}\right)!^2 = 1 \quad (215)$$

To get proper cancellation in this equation, a factor of π must be introduced somewhere in the numerator on the left side of the equation. This factor can only arise from the $(\frac{n-1}{2})!$ term, and such a π factor requires that $n - 1$ be odd, so one can set $n = 2k$ for some non-negative integer k and substitute back in:

$$\frac{2^{2k}}{2\pi} \frac{2^{2k}}{2k!} \left(k - \frac{1}{2}\right)!^2 = 1 \quad (216)$$

Invoking another identity:

$$\left(k - \frac{1}{2}\right)! = \frac{\sqrt{\pi} (2k)!}{2^k 2^k k!} \quad (217)$$

The equation reduces to:

$$1 = \frac{2^{2k} 2^{2k}}{2\pi 2k!} \left(\frac{\sqrt{\pi} (2k)!}{2^k 2^k k!}\right)^2 = \frac{(2k)!}{2(k!)^2} \quad (218)$$

The above equation holds for $k = 1$ but not for $k > 1$ or $k = 0$. Thus, $k = 1$, and accordingly $n = 2$. The Born Rule has therefore been proven; the probability $P(\psi)$ of a given path can be determined simply by taking the square of the norm of the probability amplitude ψ .

4.2.2 Realism for Anandan

Having shown how Anandan derives the Born Rule from non-dynamical symmetries, it is now my task to provide the adequate background on Anandan’s interpretation of QM and then explain how Anandan’s derivation of the Born Rule leads to his conception of “relational reality”. The papers in which Anandan explains his idea of relational reality best are [5][7][8], and as such most of my discussion of Anandan’s relational interpretation of QM will draw primarily from these three resources.

Anandan’s interpretation of QM begins with his discussion of laws, primarily in [5]. He claims that two of the most troubling problems in modern physics, quantum gravity and measurement, arise from what he calls the “paradigm of law”. The paradigm of law is the idea that a given law should be used to describe a physical system in the world until it is shown to be invalid, at which point another law takes its place. Anandan defines these laws in [5] as, “The ability to describe the initial state of a physical system from which the final state can be predicted, deterministically or probabilistically, using the nature of the system and its interactions with its environment” (1648). There are two types of laws that Anandan distinguishes in this definition: probabilistic laws and deterministic laws. However, both types of laws allow one to determine (or at least guess) the final state of a system given knowledge of the system’s initial state.

Traditionally, laws have been taken as fundamental facts about an absolute, external reality. The proper way to characterize the phenomenon of a cart rolling down a ramp, for instance, is to utilize Newton’s laws of motion. However, the fundamental nature of such laws as Newton’s is called into question when one considers QM and its predictions. Quantum mechanical laws are probabilistic, not deterministic, meaning that there is no way to determine, for a single trial, the final state of a system even if one has perfect knowledge of the initial state of the system. In Anandan’s words, nature is capricious, and because of such capriciousness, deterministic laws cannot truly be fundamental because such laws do not underwrite QM.

Anandan can reach his goal, the dismissal of dynamical laws as the fundamental entities of the universe, if he only rejects the fundamental nature of probabilistic laws. To this end, Anandan posits another entity that he views as more fundamental than dynamical laws. Like Bohr and Ulfbeck before him, then, Anandan posits symmetry as fundamental to dynamical laws, and he supports his position with several reasons.

First, Anandan notes that laws themselves are required to obey symmetries. Any verifiable experiment will produce the same results under the same conditions. This experimental requirement suggests that a similarity in background is what brings about a similarity in law between a first and second experiment, and thus it seems plausible that space-time symmetries could underwrite dynamical laws.

Anandan's suggestion is also supported by a CCR derivation in [8] that follows along the same lines as Kaiser's derivation and Bohr and Ulfbeck's derivation. What is more, Anandan adds gauge symmetries from the standard model to the list of symmetries from whom dynamical laws can be derived. In his other papers, Anandan also succeeds in obtaining other physically important quantities and laws from symmetry, most notably the Lagrangian[5], whose different formalisms on the classical and quantum scales are explained by differences between relevant symmetries on each of these scales. Anandan also writes at length about the specific geometry that causes these symmetries[6] which further aid in his quest to displace dynamical laws from their central place in contemporary physics.

However, Anandan has yet to show directly that probabilistic laws in particular result from symmetry. Such a discussion of how probabilistic laws emerge from symmetry would seal the fate of dynamic laws as subordinate to symmetry considerations once and for all. To this end, Anandan first attempts to relate probabilistic laws to symmetry by showing that two symmetric assumptions lead directly to probability interpretation[5]. Anandan also states that probabilities are required to be invariant under symmetry transformations, thus revealing that even probabilistic laws have requirements foisted upon them by symmetry considerations.

However, Anandan's most convincing proof comes from his derivation of the Born Rule, a rule of probabilistic interpretation in quantum mechanics, from the non-dynamical quantum mechanical action, which he interprets as an embodiment of all of the symmetry invariance of a given system. This derivation, described in detail in the previous section, shows how a probabilistic quantum mechanical law can be mathematically derived simply from symmetry considerations, thus proving that it is the symmetries reflected in physical situations and not the laws that describe them that are the fundamental quantities of physics.

It is relations among symmetries, then, and not dynamical laws that are fundamentally real in Anandan's interpretation of QM. These relations among symmetries are also reflected in Anandan's assertion that interactions between particles assume that the two particles in question share a symmetry group element. Thus, the fundamental nature of interactions, like the nature of the dynamical laws that cause interactions on the classical scale, comes from symmetries as well. The ideas of relations and interactions as the embodiments of symmetry (at least, the most basic embodiments with which macroscopic entities like human beings can interact with) leads Anandan to the heart of his interpretation of QM: relational realism.

By Anandan's relational realism, the only things that are real are those that interact with other things. Nothing exists within a void, then: existence only emerges from the interactions of individual objects. Before an isolated object interacts with something else, this object is not posited to be real; rather, it has potential which is only actualized when it interacts with something else. To support this interpretation, Anandan appeals to a simple thought experiment in which he claims that a universe with a single object

in it is indistinguishable from a universe with no objects in it at all³³. What makes things distinguishable and real has to be interactions between entities, and thus it is relations, specifically in the form of interactions, that make particles real.

Anandan's relational reality is contrasted with the notion of absolute reality that pervaded classical physics centuries ago. Absolute reality is the idea that there is single uniform, objective reality that is always present and that all those with an unhindered perspective will view in the same way. This idea differs from Anandan's relational reality greatly since different interactions among different observers could lead to many different (and equally legitimate) views of reality. For a believer in absolute reality, then, Anandan's assertion that a non-entangling interaction between two systems where the action/reaction principle is satisfied defines reality would be preposterous, for it would allow things to come into being and leave being constantly, for things would enter reality by interacting with others and leave reality when isolated. In Anandan's interpretation, there is no objective existence or identity.

The relational reality interpretation, as odious as it might seem to believers in absolute reality, is able to explain away several conceptual difficulties with quantum mechanics. For instance, the measurement problem posits two different types of evolution for the state vector which can change discontinuously. This seems strange to a believer in absolute reality who would see no reason or cause for such an abrupt change, but for the believer in relational reality the change is simply described as a new interaction entering the scene in the form of measurement. What is more, the indeterminacy of quantum events is no longer mysterious either since different interactions among different observers and the quantum state may bring different events into reality. The idea, however, is that before measurement, the state vector does not technically exist or have reality. The act of measurement, then, is an act of interaction, and such interaction forces the superposition to make real that which had previously only existed in potential. In such an act, the probability amplitude becomes a probability in line with Born's rule and collapse occurs as a reflection of a particle or wave's new state of being.

One might worry that such a relational realism would lead to strange conclusions in the classical world. For instance, one might think that if he were to close his eyes and shut out all interactions with the world than he would cease to exist. However, to draw such a conclusion from Anandan's work would be foolish. Anandan points out that countless interactions such as gravitational attraction and electromagnetic forces

³³Though the focus of this section and this paper in general is not to address and investigate Anandan's work to the same extent as Bohr and Ulfbeck's and Stuckey et. al.'s, I feel that I should address here what I believe to be a particularly weak and poorly-defined claim on Anandan's part. What does Anandan mean when he writes that a universe with only one object in it would be indistinguishable from a universe with no objects in it? General relativity suggests that the geometry of space-time would be warped by the presence of an electron in a toy universe while a universe with no electrons would remain pristine and unadulterated. Perhaps what Anandan means here is that the elements of space-time geometry would themselves count as objects, rendering my objection null and void, or that such an electron would have no one to interact with it and thus determine whether or not the electron was there, though this last explication would lead to question-begging in Anandan's argument. Either way, Anandan should be much clearer in his example, especially given that it is one of the main, "obvious" pieces of evidence he cites to support his relational realist interpretation.

between bodies in the universe occur constantly, meaning that worries of existence or non-existence for human beings on a classically-scaled universe are groundless given a relational view of reality.

Having established Anandan's view of relational reality, I will now proceed to address two issues concerning relational reality: first, I will foreshadow the work of Stuckey, Silberstein, and Cifone (SSC) by discussing the implications of relational reality for the nature of time, and secondly, I will use Anandan's own characterization of his relational reality to compare and contrast his views with the Copenhagen interpretation as well as the interpretations presented by Everett, Rovelli, and Mermin as previously discussed.

Anandan's view on time seems to shift over the course of his work on his interpretation of relational reality. In one of his earlier papers[5], he draws the conclusion that time is fundamentally asymmetric. He states that the asymmetry in time derives from the fact that the probability of reaching a specific final state given knowledge of the initial state of the system is not the same as the probability of obtaining a known initial state from a particular final state of the system. However, by the time Anandan writes [7], he has abandoned his previous view and now seems convinced that time is not asymmetrical since many physical theories are symmetric under time reversal³⁴.

The time-symmetry of physical laws and quantum indeterminacy imply that causality is not a fundamentally real and essential characteristic of the natural world. As will be discussed later in this paper, one of the main features of the physical world that many philosophers of time and physicists believe make time directional is causality, and so the abandonment of causality opens the possibility not only that time does not have to flow in the direction it flows (from past to future) but that such an "arrow of time" may not be a feature of the physical world at all! This latter possibility is one that SSC embrace, and more will be said about the arrow of time and the implications of causality for the arrow of time later in this paper.

Finally, to gain a better understanding of Anandan's work, I will compare and contrast his theory of relational reality to other interpretations of QM. Anandan states that he believes his interpretation to be somewhere between the canonical Copenhagen interpretation of QM and Everett's "many worlds" theory. With respect to Copenhagen, Anandan agrees with all of the physical results of the theory and applauds it for doing away with the notion of absolute reality. However, unlike Anandan, the authors of the Copenhagen interpretation do not supply an alternative perspective on reality to replace absolute reality, a fault which Anandan posits as the cause of the difficulty that the collapse problem has traditionally posed for the Copenhagen interpretation. Also, Anandan's interpretation does not draw an arbitrary distinction between the workings of the quantum world and the workings of the classical world, thus setting it apart from the Copenhagen interpretation.

Anandan also sees himself as close to Everett's "many worlds" interpretation in that, like Everett, he believes that there are numerous worlds that "exist" with the various symmetries describing them. Since there are no laws essential to the character of the universe, Anandan dismisses the necessity of our space-time structure and thus

³⁴Anandan does not give any explicit reason for why he changed his perspective on the symmetry of time, and as such I will not proceed to speculate on why he might have done so.

posits a “polyverse” in which many different space-time structures exist. This does indeed seem similar to Everett’s interpretation in which each event of collapse splits the universe into two branches, each of which corresponds to a different outcome for collapse.

However, Anandan differs from Everett in that he does not believe in the actual existence of each of the universes in his “polyverse”, for such a view would correspond to the absolute realism that Anandan rejects. Since this universe does not directly interact with any other universe, such another universe cannot be considered real for Anandan, and thus, although it is important to consider and allow for other universes, such universes cannot be taken as physically real if one takes relational realism seriously.

Finally, Anandan draws parallels between his work on relational reality and the relational reality perspectives presented by Mermin and Rovelli. Again, Anandan states that he generally agrees with both of these perspectives on relational reality but that disagrees with them on a key point. While Rovelli takes relational reality to result from relations of information and the correlations of Mermin come in the form of system observables that relate subsystem observables, Anandan’s relations are physical interactions between particles and waves. Such interactions are considerably more concrete and physical and the suggestions of Rovelli and Mermin, and as such relational reality for Anandan looks considerably different from Rovelli’s and Mermin’s conceptions of relational reality.

4.2.3 Comparison of Anandan’s Work with GF

Having addressed Anandan’s conception of relational reality in the previous section, I will now proceed to discuss how Anandan’s view compares with GF. I will first discuss the similarities between these two interpretations and then proceed to analyze some important differences between them. First of all, it is important to note that both GF and Anandan’s relational reality dismiss the concept of fundamental natural laws. What is perhaps more surprising, however, is that both Anandan and Bohr and Ulfbeck, though working independently, appeal to space-time symmetries as the fundamental quantities of interest once laws have been discarded. The fact that both sets of authors utilize basically the same derivation of the CCR from the Poincaré group is perhaps not surprising in light of this similarity in perspectives.

Another important similarity shared by relational reality and GF is that both abandon causality. This is, no doubt, a consequence of the aforementioned abandonment of dynamical laws by both sets of authors. It is interesting to note, however, that Bohr and Ulfbeck do not analyze the consequences of such lawlessness and abandonment of causality for the nature of time whereas this is one of the essential features of relational reality upon which Anandan comments at length. One might conjecture that Bohr and Ulfbeck were simply unconcerned with the problem of the arrow of time, but this conclusion seems specious since Bohr and Ulfbeck’s first paper on GF[16] mentions the asymmetry of time while none of the following papers comments on it. It is my impression, therefore, that Bohr and Ulfbeck were more reticent in abandoning their attachment to the asymmetry of time than was Anandan, though both sets of authors draw essentially the same conclusions concerning causality.

Despite these important similarities, however, there are some essential differences

between relational reality and GF that should be addressed. The first difference lies in the fact that Bohr and Ulfbeck refuse to grant reality to particles at any point, whether they interact with each other or not. In GF it is events, not objects, that have fundamental reality. For Anandan, however, particles certainly can have reality, and though it is events like interactions among particles that impart reality to the particles, particles themselves are capable of becoming fundamentally real in a way that they cannot in GF. Thus, though Bohr and Ulfbeck take a step in dismissing particles and waves as fundamental entities by replacing them with events in a manner analogous to dismissing laws as fundamental relations in favor of symmetries, Anandan continues to assert that particles can have reality as well as events and that, since events are events among particles or waves, that it may even be these quantities that are fundamentally real.

This disagreement as to the reality of particles is the major difference between GF and relational reality, but there is another important realist question that may or may not be a point of contention between the two theories. Anandan states explicitly that he is not a structural realist, meaning that he does not believe that the mathematical space-time structure exists in itself. This makes sense since the symmetries must interact for them to be real and thus any “in itself” realism would be antithetical to Anandan’s views. Bohr and Ulfbeck, however, seem to be slightly on the fence about the realism of space-time symmetries. Although they posit that these entities are outside of space-time and manifest themselves on the space-time scene in fortuitous events, they include a throwaway comment in their latest paper [18] that states that they have refrained from using the phrase “world of space-time symmetries” because they do not want to suggest that such a world that cannot be observed can still exist. This seems like they are endorsing Anandan’s relational realism, but since this comment is the only one I could find out of the three papers on GF that seems to address the GF stance on the reality of space-time, it is certainly not conclusive.

5 Stuckey et. al.’s Relational Blockworld (RBW)

It is now time to examine Stuckey, Silberstein, and Cifone’s (SSC) work on relational blockworld (RBW). SSC’s work combines many of the physical results previously discussed. Their work utilizes arguments by Kaiser (as previously discussed in the section on the CCR), Anandan, and Bohr and Ulfbeck. However, before explaining how SSC combine these previously-derived results to arrive at their RBW theory, there is one missing piece to the RBW puzzle that I have not yet provided, specifically the blockworld (BW) view of time. Thus, in this section I will first explain the BW theory of time, following which I will explain how all of the various theories I have previously discussed come together in RBW. Finally, I will analyze RBW in terms of GF to determine how these two theories compare.

5.1 The Relativity of Simultaneity and the Blockworld View of Time

In the course of their papers on relational blockworld (or RBW)[52][53][54][55][56][57], SSC weigh in on the debate of presentism versus eternalism. As firm supporters of eternalism as per RBW, SSC utilize the relativity of simultaneity (RoS) to disprove presentism. My purpose in this section is to give a brief (and simplified) argument against presentism along the lines of the argument presented by SSC in their papers. I will keep the main features of their argument intact though I will change the method of argumentation to one I find more intuitive. My goal is to provide not only a simplified exposition of the SSC counter-presentist argument but to set the stages for possible arguments against the BW view of time that I will expound upon later in my section on the relationship between RBW and the philosophy of time³⁵.

5.1.1 General Outline and Definition of Terms

Before presenting our RoS argument against presentism, I will first provide a general outline of such an argument and give preliminary definitions for some relevant terms. The general form of the arguments against presentism utilized by Putnam, Rietdijk, and SSC goes as follows:

1. Assume presentism
2. Define the term “co-real”³⁶
3. Show that the consequences of the definition of the term “co-real” and RoS contradict presentism
4. Conclude that presentism is false from the combination of 1 and 3
5. Conclude that eternalism is true from the rejection of presentism.

³⁵This following section is essentially a reproduction of a section from my paper with Silberstein[43] on the BW view of space-time.

³⁶The actual term “co-real” appears only in the SSC papers, but since these present the most recent incarnation of the RoS argument against presentism, I follow their terminology here.

To begin with, then, we must define the terms that will form the foundation for much of the argument to come. The first necessary term to define is presentism. Presentism is a kind of realism that takes as real only those events which occur in the present. For instance, since we are sitting next to our friend Joe who is currently reading a paper, the event of his reading a paper and the event of our typing this paper are both real while the event of Joes leaving to eat dinner is not real because it has not happened yet and the event of our leaving to eat lunch is not real because it has already happened. In terms of simultaneity, then, one can define presentism as the view that the only real things are those which are simultaneous with a given present event. Eternalism, by contrast, is the view that all things that are past, present, and future have equal reality. Thus, Joes reading, our typing, Joes leaving for dinner, and our leaving for lunch are all equally real despite the fact that one of these events has already occurred while another has yet to occur. Thus, eternalists hold that all events are equally real, regardless of whether or not said events are simultaneous.

There are two elements, then, that are important for establishing both presentism and eternalism: reality and simultaneity. The debate presupposes that there is a unique (non-equivocal) sense of the term reality that both sides share. The dispute therefore is over whether or not present events have some ontologically privileged status qua their property of existing at time some time t where t is in the present. For the purposes of this discussion, two events which share reality as share a single, unique feature (i.e., the same ontological status with respect to realness); this uniqueness seems to be the absolute minimal criterion an event would have to satisfy for it to be considered real in any meaningful sense of the word.

To better understand the minimal sense of reality at work here, one must posit two separate principles: the reality value and reality relation. Reality values or R-values can be thought of as the ontological status of any given event. Within space-time, every event can be assigned an R-value that represents its ontological status, and there is a one-to-one and onto mapping of possible R-values onto ontological statuses. In the interests of defining reality generally, I will not attempt to enumerate how many R-values exist, but one could easily take reality to be binary and thus assert that, for any event, if its R-value is 1, that event is real, and if its R-value is 0, that event is not real. One could use higher values like 2 and higher to denote other states, such as possibly real, real in the future, etc., but, as previously stated, all such possible R-values will not be elaborated here. It should be pointed out that our uniqueness criterion on reality translates into this system simply as the claim that every event has a single unique R-value. This seems intuitive since an event with an R-value of both 1 and 0, on our scheme, would be both real and unreal, making it contradictory.

Our other sense of reality as expressed in the reality relation will be essential to our discussion of co-reality. The reality relation can be recast as the idea of equal reality and exists between any two or more events that can be considered equally real. Translated in terms of R-values, a reality relation exists between any two events that must have the same R-value. For instance, if events A and B are equally real, then the R-value of event A is the same as the R-value of event B . One should notice here that our definition of equally real does not assume that two equally real events are both real; equally real events A and B may have whatever R-value you please as long as the R-values are the same for both of them. This explains what a presentist means

when she says, The present is the only thing that is real since the presentist will hold that events in the future and the past will have different R-values from events in the present³⁷. Thus, our purposefully limited characterization of the equally real relation has been defined so as to be useful in a definition of co-reality.

As for simultaneity, if it is possible for one to construct a hyperplane of simultaneity (i.e. a manifold in space-time that connects two space-like separated observers or events) between any two or more events, then these events are said to be simultaneous. Such simultaneous events are required to be space-like separated. Light-like and time-like separated events cannot have a hyperplane of simultaneity constructed between them in any sub-luminal reference frame. Also, a hyperplane of simultaneity may be drawn between any two space-like separated events, meaning that the space-like separation of events A and B is necessary and sufficient for their simultaneity.

Combining the criteria of equal reality (equally real means that two events have the same R-value) and simultaneity (simultaneous means that two events are space-like separated such that a hyperplane of simultaneity can be constructed between the two events in some frame) gives us the relation of co-reality, which refers to, as the name suggests, two events that are equally real simultaneously. The presentist perspective can be restated in terms of this co-reality as the stance that co-reality between events is a necessary and sufficient condition for the reality (that is, for both events sharing the R-value 1 corresponding to real) of these events if at least one of these events occurs in the present. This restatement of presentism in terms of co-reality is the assumption alluded to in step 1 above.

Our previous examples should make our notion of co-reality more explicit. For instance, Joes paper reading and our paper typing are co-real events as per this criterion because they are space-like separated, meaning that there exists some frame in which these two events are simultaneous. However, our paper typing and our leaving for lunch are time-like separated, so there is no frame in which these two events are simultaneous and they are therefore not co-real. These two criteria of reality and simultaneity as have been defined here are necessary and sufficient for our use of co-real, and so I turn next to our RoS argument that utilizes this definition of co-real to reveal the contradictory nature of presentism when combined with relativity.

5.1.2 RoS Argument

Consider the following situation: our friends John and Josephine stub their toes at the same time in my stationary reference frame. The event of John stubbing his toe is labeled A in Figure 2 and the event of Josephine stubbing her toe is labeled as B in Figure 2:

³⁷This is clearly not a fleshed-out definition of some rich sense of reality; the characterization of reality has been left as general as possible such that no presentists may find grounds for disagreement in it. What has been characterized is thus not a *full* characterization of reality but merely a *minimal* definition.

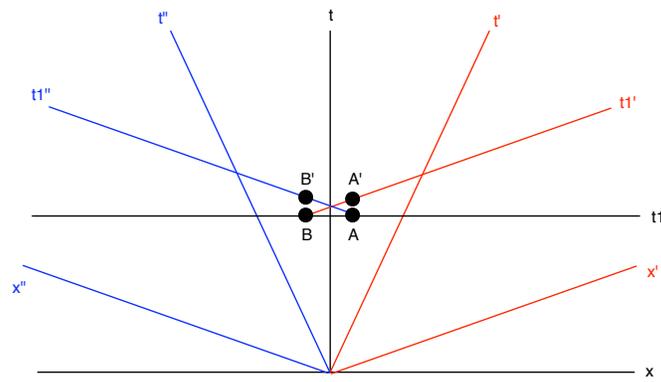


Figure 2: RoS Proof Space-Time Diagram

At a later time (but again, simultaneously in my frame), both Josephine and John shout in pain from stubbing their respective toes. John’s shout of pain is labeled A' while Josephine’s shout of pain is labeled B' in Figure 2. I note that in my frame, both toe-stubs occur at time $t1$ in Figure 2. Thus, events A and B are co-real as per the previously-established criteria.

Now, some time before this the alien battlecruisers P and D pass each other directly over my head. The primed axes refer to the frame for battlecruiser P and the double-primed axes refer to the frame for battlecruiser D. Both of these battlecruisers tell a different story from mine. For battlecruiser P events B and A' occur at the same time, and thus B and A' are co-real. For battlecruiser D, however, events B' and A occur at the same time, and thus B' and A are co-real.

I will now introduce the symbol \diamond to stand for “is co-real with”. Thus, the following three statements are true:

$$A \diamond B \tag{219}$$

$$B \diamond A' \tag{220}$$

$$B' \diamond A \tag{221}$$

From the previously established criteria for co-reality, one can establish two important facts about co-real events α , β , and γ . First, if $\alpha \diamond \beta$ is true, then $\beta \diamond \alpha$ is true. Thus, the operator \diamond is commutative. This fact must be true since co-reality is an equivalence relation. The second important fact about co-reality is that the co-real operator is transitive, even across frames. That means that if $\alpha \diamond \beta$ is the case and $\beta \diamond \gamma$ is the case, then $\alpha \diamond \gamma$ must also be the case. This follows directly as consequence of our definition for equal reality³⁸. Thus, applying the properties of transitivity and commutativity to the above relations, we arrive at the result that:

$$A \diamond A' \tag{222}$$

$$B \diamond B' \tag{223}$$

Generalizing from this result, then, one can conclude that a prior event (the stubbing of a toe) is as real as a later event (a shout of pain). If the first event (A , for instance) occurs in the present, then A' occurs in the future and the RoS argument suggests that the future is as real as the present. Likewise, if A' occurs in the present, then A occurs in the past and the RoS argument suggests that the past is as real as the present. Both of these conclusions contradict the presentist assertion that the present is real while the past and future are not since past, present, and future must share reality equally by the above argument. Thus, since presentism in conjunction with

³⁸This feature of co-reality is perhaps not intuitive, but a simple conceptual argument can show why equal reality, as it has been defined here, must be a transitive property. If two events A and B are co-real in a given frame, this means that they share an R-value. Likewise, events B and C must also share a unique R-value. Since the uniqueness criterion on reality implies that the R-value shared by A and B must be the same as the R-value shared by B and C , it then follows that A and C must have the same R-value as well, and thus they must be equally real.

relativity and our other basic assumptions leads to a contradiction, presentism must be false given our assumptions. Finally, since variations of this argument would answer equally well anyone who would argue that only the past is real or only the future is real, the only conclusion left for a realist is that eternalism must be correct since both presentism and possibilism must be discarded.

The SSC argument for eternalism holds up extremely well as long as their definition for co-reality is correct; however, if this term is defined alternatively, it is possible for presentists to logically refuse to endorse the conclusions of SSC's argument. This possibility will be discussed in more detail in later sections of this paper dealing specifically with the philosophy of time and realism in RBW. For now, it suffices to have arrived at SSC's conclusion: the BW view of time is the proper one to take in light of the special theory of relativity.

5.2 Relational Blockworld Interpreted

With all of the pieces finally out on the table, I now turn to the manner in which SSC combine the blockworld (BW) perspective previously elaborated with the relationality proposed by Bohr and Ulfbeck, Kaiser, Anandan, and others, to yield a unified interpretation of space-time entitled "relational blockworld" (RBW). Though all of the SSC papers I have previously cited deal with this interpretation, I will primarily draw my examples in this section from one of the most recent papers, [57], which does the best job of explaining how all of the pieces of the RBW puzzle fit together into a unified theory that is capable of accounting for both the results of quantum mechanics (QM) and special relativity (SR).

Before launching into an explanation of this interpretation, however, a word should be said about RBW. Unlike Bohr and Ulfbeck's GF, RBW does not currently lend itself to observable predictions that deviate from those of currently-accepted QM. As RBW is still developing (the first paper on RBW was, after all, written less than five years ago), its experimental use and predictive power have not yet been fully explored. The best way to view RBW at the present is as an interpretation of QM and SR rather than a physical theory in its own right. RBW carries with it a great number of philosophical commitments, which I will explore later, but it currently lacks the physical predictive power to make it any more than an interpretation of QM and SR.

The three most important pieces of evidence utilized by RBW in SSC's papers are the BW result of the relativity of simultaneity (RoS) argument previously explained, Bohr and Ulfbeck/Kaiser's derivation of the CCR, and Anandan's derivation of the density matrix from non-dynamical entities. What these three pieces have in common is an emphasis on the explanatory role of some fundamental background structure of space-time. In the RoS argument, it is the structure of RoS that reveals eternalism as an important philosophical consequence of space-time structure. The CCR derivation likewise shows that a background structure of K4 allows one to derive the CCR from symmetry relations. Finally, Anandan's derivation of the density matrix shows that dynamical "laws" are not necessary for the prediction of quantum results; rather, one can utilize the physical geometry of a given system to derive all of the necessary quantum behavior of that system.

It makes sense, then, that the foundation of RBW lies in the assertion that space-

time symmetries in a 4-dimensional Minkowski BW are fundamental to all of the material constituents of this BW. All previous theoretical devices (particles, waves, laws, etc.) are abandoned in favor of structure as the primary explanatory tool in a RBW view. The results of Anandan and Bohr and Ulfbeck/Kaiser show that space-time structure is sufficient as the basis for all quantum mechanical predictions, and the RoS argument helps to explain the nature of the BW as a 4D entity, thus adding explanatory power to the model by adding temporal structure to the BW.

One of the first conclusions from SSC’s “ontological structural realism” (15) is, in many ways, the same as Bohr and Ulfbeck’s in GF: the so-called “constructive objects” like particles, waves, etc. that have been traditionally used to describe the world are, in fact, a result of relations among various space-time structures. All currently-held beliefs in metaphysical, material entities are wrong, according to RBW, since these entities only emerge as a manifestation of the background space-time geometry. What is more, SSC utilize Anandan’s result to argue that even dynamical laws boil down to relations among space-time structures³⁹. Every law of the universe, then, and every speck of matter in the universe is but a reflection of the geometry of space-time.

Before going on, it is necessary that I explain SSC’s conception of space-time structure in more detail. As previously stated, SSC are realists about M4, the space-time geometry best-supported by the results of SR. One might then ask about the Hilbert space wherein QM is traditionally taken to reside. SSC abandon Hilbert space realism in favor of M4 realism due to the CCR result. What is more, the RBW view explains how QM fits into the picture of space-time. Rather than receiving the traditional “law” treatment of other theories that would place the rules of QM alongside physical descriptions of the world like Newtonian Mechanics and electromagnetism, RBW views QM as a necessary feature and consequence of M4. QM is a “probabilistic rule by which new trajectories are generated...(QM) provide(s) *constraints on the distribution of events in spacetime*” (6, original emphasis). SSC go on to describe the emergence of “strange” quantum phenomena like non-locality and entanglement as “geometric features of the spacetime structure just as gravity is taken to be a feature of the geometry in general relativity” (6).

As SSC point out, this novel approach to QM succeeds in doing away with several of the interpretational problems plaguing QM. The first of these, the measurement problem, is explained away by an appeal to the relationality first invoked by Bohr and Ulfbeck. The measurement problem itself “collapses” if one views detector clicks not as representations of physical particles traversing a detector but as manifestations of the symmetry of the system. Adding a detector to a photon path changes the symmetry of a given experimental setup, and as such it makes sense that the relation among all pieces of experimental equipment changes when the relation is forced to reflect the addition of an extra detector. For example, the system prior to the insertion of the measuring device could be described as a set $M = (a_1, a_2, \dots, a_n)$ and the system after the insertion of the measuring device could be described by the set $M' = (a_1, a_2, \dots, a_{n+1})$. Clearly, M' and M are not the same set, so there is no reason why one should find

³⁹I should briefly note here, however, that SSC do not deny the explanatory power of dynamical laws. These laws, while not fundamental, are still good descriptions of physical phenomena, and thus they should not be disregarded as useful physical models for making predictions; however, realism regarding such laws should be abandoned in the face of the primacy of structure in RBW

it troublesome that they are described by different quantum mechanical laws. Yes, if one views the systems M and M' as the same system, what Copenhagen describes as “collapse” still occurs, but it occurs as a reflection of changes to an experimental setup, not as a “spooky”, discontinuous physical process.

Another problem that RBW dismisses is the problem of non-locality in QM. Two entangled particles share a connection that collapses one particle “faster than the speed of light” when the other particle is measured, thus posing a challenge to SR as I discussed in the “Background” section; however, when the story becomes one of space-time structures as opposed to particles and the clicks that signal measurement are seen as reflections of space-time symmetry instead of records of particle states, one views entanglement as a global constraint on symmetry in the system, forcing distributions of detector click events to behave in a certain way. Whereas a space-like separation between two particles that seem to be in instantaneous communication with each other may be a troubling concept, the idea of a global constraint operating on two space-like separated parts of a system is not nearly as worrisome. After all, to borrow SSC’s analogy, is it really that troubling to have one planet gravitationally attracted to a body that is space-like separated from it if gravity is just a reflection of the curvature of space-time?

SSC’s RBW thus presents a union of SR and QM that lies in the fundamental nature of space-time relations. As an interpretation of SR and QM, RBW seems a viable alternative to the Copenhagen interpretation in that it explains the “strange” quantum phenomena of measurement and non-locality without needing to invoke never-before-seen, “spooky” physical entities or other various (and dubious) conceptual baggage.

SSC’s goal in proposing the RBW interpretation is to provide the necessary perspective for a new inquiry into the nature of quantum gravity. Thus, SSC hope that RBW will yield important physical results that will make uniting QM with gravity easier. As Stuckey has recently shown[58], the results of the two-slit experiment can be derived by an appeal to RBW, but this most recent paper is as far as RBW has carried SSC into the field of physical applications up to this point. The usefulness and validity of RBW in the absence of further physical predictions cannot be fully evaluated, but there is still enough in the RBW interpretation of QM and SR to merit a deeper discussion into its physical and philosophical stances.

5.3 Does GF Imply a Relational Blockworld?

The final question I should address in this section before moving on to a “counter-argument” to RBW is what relationship exists between Bohr and Ulfbeck’s GF and SSC’s RBW. As I explained in the previous section, RBW is essentially the synthesis of two ideas: a “relationality” provided by Bohr and Ulfbeck, among others, and a blockworld provided by the RoS argument. Thus, as expected, the previous explanation of RBW invoked some of the relational explanations regarding the existence of matter that Bohr and Ulfbeck utilize in GF. SSC frequently cite Bohr and Ulfbeck to describe the very positions SSC take themselves with regard to the reality of matter and the nature of space-time relations. Does this mean, then, that RBW is essentially just GF with the RoS argument tacked-on, or that RBW is, in some way, the fulfillment of the project that Bohr and Ulfbeck started?

I do not believe either of these possible interpretations of RBW is correct, for there are several important features of GF that SSC do not commit to in their explanation of RBW. The first important feature of GF that is not found in RBW is, in fact, genuine fortuitousness itself. Nowhere in RBW is there any suggestion that quantum events emerge randomly and acausally. Though SSC do reject the notion of temporal, causal laws, they do not attach themselves to completely fortuitous clicks. SSC state that the laws of QM put conditions on click patterns and do not express the belief that these individual clicks are fundamentally fortuitous, so it seems that QM is the final word on click constraints. As a side note, Silberstein⁴⁰ has expressed the view that the idea of genuine fortuitousness itself is “explanatorily empty”, and thus that, instead of adopting the entirety of GF to use in RBW, SSC opted to cite Bohr and Ulfbeck’s mathematical results and leave much of their physics and metaphysics at the door.

Another aspect of Bohr and Ulfbeck’s GF that is not found in RBW is the idea of the matrix variable. Though much is said in RBW about symmetry relations, there is no distinction that leads to Bohr and Ulfbeck’s primary and secondary matrix variables. In RBW particles are treated as emergent from relations of space-time structure just like dynamical laws are. There is no metaphysical difference between relations that emerge “directly” from space-time structures and those that emerge from other space-time relations, and thus the primary/secondary matrix variable distinction is meaningless in the context of RBW. Since SSC do not explicitly endorse Bohr and Ulfbeck’s matrix variables or even hint at the existence of such entities, it seems that, once again, SSC did not deem the conception of matrix variables relevant or useful enough to include in RBW.

With matrix variables and fortuitousness gone, much of the philosophical and metaphysical structure of GF is missing in RBW, save the conception of relationality and the fundamental role of symmetries and structure in space-time. It is important, then, that even though RBW does use the results of Bohr and Ulfbeck as well as some of their interpretation regarding relationality, in no way should one construe the inclusion of GF’s results in RBW as an across-the-board endorsement of all of GF by SSC. Bohr and Ulfbeck and SSC seek different goals; Bohr and Ulfbeck seek to explain the relationship between SR and QM as well as the indeterminacy of QM while SSC attempt to find a unified background structure for space-time from which to build a new interpretation of QM and SR. These two projects, while similar, are different enough that Bohr and Ulfbeck and SSC have produced importantly distinct theories, and though I believe that the two can be reconciled (a belief I will attempt to vindicate in a later section), one must appreciate each of the theories on their own terms.

⁴⁰Personal correspondence, July 16, 2007

6 Counter-Argument to RBW

Stuckey et. al.'s conception of a non-dynamical relational blockworld (RBW) flies in the face of those who posit a dynamical view of both space and time. Such a dynamical perspective is found in the writings of Elitzur and Dolev[25] who argue that non-dynamical accounts of time and space do not capture all that is necessary in a theory of space and time. Their first objection is that time is inherently directional due to entropic considerations and a fundamental quantum indeterminacy, and their second objection is that a static view of space-time cannot explain the phenomenon of the quantum liar paradox. In the following pages, I will first describe Elitzur and Dolev's objections and then answer them, revealing that Elitzur and Dolev's argument based on entropy and indeterminacy begs the question and utilizing SSC's[52][53][54][55][57] result to show that the framework of RBW is sufficient to explain the quantum liar paradox.

6.1 The ED Argument for Time Asymmetry

Elitzur and Dolev (hereafter "ED") construct an argument against non-dynamical theories of space and time by invoking several physical quantities, the first of which is entropy. ED construct a thought experiment involving a game of billiards. At the beginning of the game, the system is in a state of low entropy; the cue ball is in its proper place, and the other balls are lined up appropriately. However, after some initial momentum is imparted to the cue ball, the balls in the system fly about until the system reaches a final state of higher entropy.⁴¹

What ED want to imagine, however, is the system running in reverse; that is, they imagine the system moving from a state of high entropy to a state of low entropy. Such a system is certainly imaginable, and thus it would seem as if time could run backwards or forwards in one picture or the other. ED use a computer program to model this situation and then begin varying the initial condition parameters randomly. What they find is that, in the first situation of normal time ordering, the random perturbations in the initial conditions always lead to a state of increased entropy compared to the initial state. Thus, when time is running forward, even with a fundamental indeterminacy factored in, the entropy of the system will always increase.

From this result one might expect randomness or indeterminacy to have an equally marginal effect on the entropy of the reversed system, but this is not the case. ED show that the system run backwards becomes *extremely sensitive* to initial conditions, so much so that most random variations in the initial conditions lead to a final state of increased entropy rather than decreased entropy. Thus, ED posit that, if there is a fundamental indeterminacy in the initial conditions of a closed system, the entropy of the system will increase whether the system is run backwards or forwards. This invariance of entropy increase under time directionality would thus reflect a fundamental asymmetry in spacetime.

From here, ED turn to quantum mechanics, specifically to the ideas of collapse and Heisenberg's uncertainty principle. First, ED argue that, since the collapse of the

⁴¹It is important to note here that, unlike normal pool balls, the balls used in the thought experiment must be indistinguishable for the entropy of the system to increase.

wave function upon the measurement of a particle is not causally determined by the state of the particle prior to the measurement, the act of measurement introduces a fundamental indeterminacy to any quantum property of the particle. Likewise, ED interpret the uncertainty principle

$$\Delta x \Delta p \geq \hbar/2 \tag{224}$$

as evidence of a fundamental “*ontological indeterminacy*” rather than simple epistemological ignorance. These two quantum results, then, lead ED to believe that spacetime has the inherent fundamental indeterminacy necessary to prove the asymmetry of time.

6.2 Why ED’s Argument for Time Asymmetry Begg the Question

ED’s argument proceeds basically as follows:

1. If initial conditions are fundamentally indeterminate, then time is asymmetric in that it always moves in a direction of increasing entropy.
2. Initial conditions are fundamentally indeterminate
3. Therefore, time is asymmetric in that it always moves in a direction of increasing entropy.

The argument is clearly valid, and thus to argue against it one must take issue with either the first or second premise. I will argue that ED have ignored essential entailments of their second premise and, as such, that their argument begs the question.

This missing entailment in the second premise regards the nature of measurement. ED take the position that an understanding of the measurement process is less critical than an understanding of the “fundamental indeterminacy” that results from measurement. This might seem to align well with Bohr and Ulfbeck’s views of indeterminacy in GF; however, Bohr and Ulfbeck discuss indeterminacy on a fundamental, quantum scale while the indeterminacy in ED’s setup is macroscopic since entropy considerations, which are most valid in large samples, are central to the argument. Thus, while Bohr and Ulfbeck’s indeterminacy is fundamental and estranged from processes like measurement, ED’s setup necessitates a description of the measurement process by which small-scale indeterminacy translates to a scale of lower resolution. By focusing so intensely on the indeterminacy that results from measurement and its implications for entropy and time, ED do not address the nature of the measurement process itself which is ontologically prior to any interpretation of indeterminacy they can put forth. There is no analysis of what the measurement process entails, and thus it is up to the reader to understand exactly how indeterminacy arises from measurement.

Without an interpretation of measurement from ED, I turn to Michael Nauenberg’s[42] interpretation of what constitutes a measurement. He writes:

...a Geiger counter, a photographic film, the retina of the eye and associated neural connections, or any other detector creates a more or less permanent record by means

of physical and/or chemical processes that are *irreversible*. (11)

Nauenberg's conception of a measurement hinges in the irreversible character of the measurement process. If the process used to create a measurement were reversible, by contrast, it would be fairly easy to erase whatever measurement had been made, thus leaving no record of the measurement. A measurement must be at least somewhat permanent or else the word "measurement" loses meaning.

It seems that irreversibility is then essential for a measurement to be made. It is this measurement which is ontologically prior to the fundamental indeterminacy of the initial conditions explained earlier by ED. Thus, indeterminacy arises if and only if irreversible measurements are being used. Yet irreversible measurements are those which increase the entropy of a system. For indeterminacy to arise, then, the system is assumed to be one of increasing entropy in which the second law of thermodynamics must hold. But this condition is exactly the conclusion that ED are trying to prove! The second premise of the ED argument is only correct if the conclusion (proposition 3) is correct, and thus ED's argument begs the question and does not establish time as fundamentally asymmetrical.

Therefore, the argument that:

1. If initial conditions are fundamentally indeterminate, then time is asymmetric in that it always moves in a direction of increasing entropy.
2. Initial conditions are fundamentally indeterminate
3. Therefore, time is asymmetric in that it always moves in a direction of increasing entropy.

does not follow since proposition 2 is contingent on the following proposition:

- 2'. Entropy must increase for the fundamental indeterminacy of measurement to be meaningful.

Thus, since proposition 2' entails proposition 3 just as it entails proposition 2, and since ED must assume proposition 2' to be able to support proposition 2, ED's argument has been shown to beg the question and therefore must be dismissed.

6.3 The Quantum Liar Paradox (QLP)

In the same paper, ED posit another problem for a non-dynamical worldview: the quantum liar paradox (QLP). The QLP can be seen as the result of an experiment involving a Mach-Zehnder Interferometer (MZI), the apparatus shown in Figure 3. The photon source in the MZI emits light which then encounters a beam-splitter, sending part of the light along the upper path and part of the light along the lower path. The two beams are then recombined at the second beam-splitter, and then, depending on

the state of the photon(s), the beam will either encounter detector A or detector B. The mirrors between the beamsplitters allow for quantum interference.

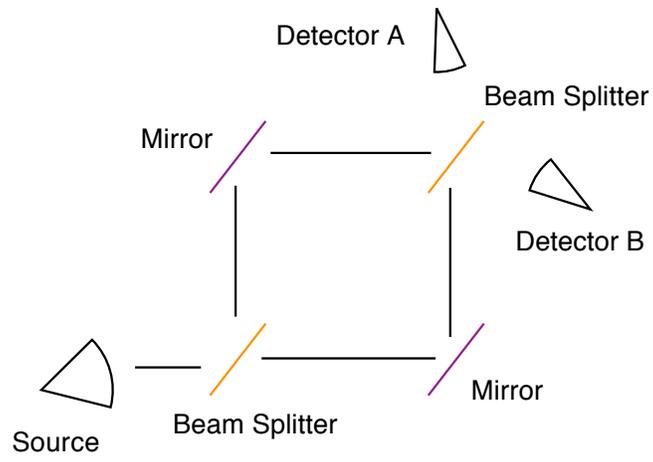


Figure 3: The Mach-Zehnder Interferometer (MZI) Experimental Setup

One can easily predict that, if the light is in the eigenfunction $|X+\rangle$ ⁴², when it leaves the MZI as shown in Figure 3, it will be in the same eigenfunction $|X+\rangle$ and thus all of the photons will be incident on detector B, which we will call the X+ detector. This phenomenon is due to constructive interference and still obtains even when only one photon is emitted from the source. In dynamical terms, this interference is possible because the light is split by the beam-splitter into two paths which interfere with each other in the second beam splitter. Constructive interference forces the photons on the path towards detector B while destructive interference prohibits photons from impinging upon detector A.

⁴²Here, $|X+\rangle$ is used to denote a $+1/2$ x-spin state of a spin-1/2 particle. Similar notation to that used for $|X+\rangle$ here will be used in the rest of this section to denote the spin in other directions as well.

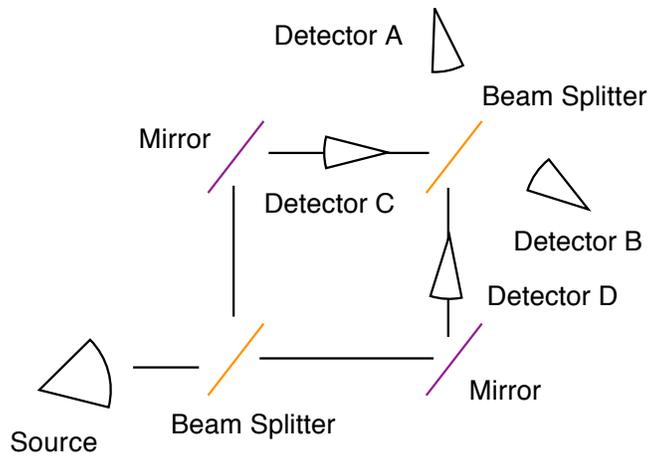


Figure 4: The Quantum Liar Paradox (QLP) Experimental Setup

The situation changes when detectors C and D are added to the setup along the legs of the MZI (see Figure 4). We now take the source to produce atoms instead of photons so that the spin analogue will follow SSC's explication of the problem more closely. Detectors C and D can measure either Z1+ or Z2- respectively or nothing at all. ED posit these detectors as atoms in the Z1+ and Z2- states respectively. Thus, there are four logical combinations of the states of the atom along each path of the MZI:

1. +Z1 along the C path, +Z2 along the D path.
2. +Z1 along the C path, -Z2 along the D path.
3. -Z1 along the C path, +Z2 along the D path.
4. -Z1 along the C path, -Z2 along the D path.

In the first of the above combinations, the atom will be blocked along detector C's path but not along detector D's path. Thus, it is uncertain whether or not detector A or detector B will click. In the second combination, the atom will be blocked along detector C's path and along detector D's path, meaning that neither detector A nor detector B will click since no atoms will leave the MZI. In the third combination, neither detector C nor detector D will block the path of the atom, and thus the result of the MZI with no detectors, a detector B click, will obtain. Finally, in the final combination, detector D will block the atom but detector C will not, leading to another indeterminate state like combination 1.

Thus, if detector A clicks, only combinations 1 and 4 are possible, and thus the state of the system prior to a detector A detection event can be represented by the following wave function:

$$\frac{1}{\sqrt{2}}(|Z+\rangle_1 |Z+\rangle_2 + |Z-\rangle_1 |Z-\rangle_2) \quad (225)$$

This state represents the entanglement of atoms 1 and 2 in detectors C and D respectively. Thus, measuring the Z-spin in one of the boxes gives the value of the other box given an A detector click. This establishes a "matter of fact" [57] as to the Z-spin of the atom in the system. However, after the results for a number of trials have been gathered and analyzed, one finds that the correlations between the measurements violate Bell's inequality [25]. The violation of Bell's inequality means that there is *no* way for the atom's spin to be determined as a "matter of fact" *before* the measurement takes place. Thus, to quote SSC [57], "*there must be a fact of the matter concerning the Z spins in order to produce a state in which certain measurements imply that there is no fact of the matter for the Z spin*" (25, original emphasis). The determination of a state leads to its indeterminacy, therefore, making this situation analogous to the liar's paradox presented by such statements as "This sentence has never been written".

After positing this paradox, ED go on to propose a backward-causal theory of spacetime that allows for future events to rewrite past events. Such a spacetime would certainly be able to explain the phenomenon of the QLP since it incorporates the very paradox into its structure. However, it is not readily apparent how a non-dynamical theory of spacetime could do away with this problem. In the QLP, a measurement (which is an event in time) changes a prior space-time event. Thus, a single event (the

existence of a certain state of the wavefunction prior to measurement) in terms of its position and its time is really two events, one of which makes sense in a context prior to measurement and one of which makes sense in a context after the measurement has been made. Such a static spacetime would seem to necessitate double-valued structures, which RBW does not and should not have to incorporate. Thus, the QLP presents a problem that those in favor of RBW need to take seriously.

6.4 SSC's Solution to the QLP

SSC avoid the trap presented by QLP in two ways: first, they are able to mathematically reproduce the results of the QLP using the formalism created specifically for RBW, and second, they qualitatively produce an interpretation of the QLP which turns the paradox into a pseudo-problem. I shall describe their attack of the QLP via RBW mathematics first.

SSC's first task in using RBW mathematics to describe the QLP is to describe the setup in Figure 3 using spacetime symmetries. To do this, they introduce the entity $Q(a_0)$ which is the operator representing spacetime symmetries of the beam-splitter. Using the results of Bohr and Ulfbeck[16], SSC determine a matrix representation of $Q(a_0)$ for this system. Their result is:

$$Q(a_0) \equiv \frac{1}{\sqrt{2}}(I - iS(a_0)) \quad (226)$$

Where

$$S(a) = \begin{pmatrix} 0 & e^{-2ika} \\ e^{2ika} & 0 \end{pmatrix} \quad (227)$$

is the matrix representation of a reflection provided by Bohr and Ulfbeck and $a_0 = \pi/4k$. Equation 226 is obtained from the physical description of the beam splitter. Upon encountering the beam splitter, half of the beam will travel through the splitter unchanged while the other half will be rotated at an angle of 90 degrees. This rotation is represented by a matrix of the form:

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \quad (228)$$

Since θ is 90 degrees here, $R(\theta)$ can be written as:

$$R(\pi/2) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (229)$$

Thus, the beam-splitter's symmetry can be described by the equation:

$$Q \equiv \frac{1}{\sqrt{2}}(I + R(\pi/2)) \quad (230)$$

Thus, since $-iS(a_0) = R(\pi/2)$, the definition of $Q(a_0)$ presented by Equation 230 is valid. Whether it is calculated using $S(a_0)$ or $R(\pi/2)$, however, the matrix representing this symmetry appears as

$$Q(a_0) = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad (231)$$

So, for the MZI setup shown in Figure 3, the final state of the system (given the initial state of $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$) is given by

$$|\psi\rangle = Q(a_0)^+ Q(a_0) |1\rangle = |1\rangle \quad (232)$$

Since $Q(a_0)$ is a Unitary matrix. The mirrors do not contribute any symmetry to the calculation because the mirrors (which are represented as S rotation matrices only) commute with the $Q(a_0)$ matrix, and thus $Q(a_0)$ and $S(a_0)$ share a common set of eigenvalues. Thus, the mirrors will not change the state vector in any measurable way. The above result agrees with the result given in the previous section since our basis of $|1\rangle$ and $|2\rangle$ represents a detector B click and a detector A click respectively.

This non-dynamical account, it should be noted, makes no mention interference or even any physical quantities; rather, the only mathematical objects utilized in this proof are the initial state of the system, the symmetries of the beam-splitters and mirrors, and characterization of the system of two detectors, thus establishing a 2D system whose eigenstates correspond to clicks in the respective detectors⁴³. SSC next consider the QLP setup shown in Figure 4; however, since they are concerned only with the system in the event of a final click in detector A, they need only add one extra detector to the system.

The extra detector adds an extra dimension to the system, meaning that the apparatus must be described by 3D manifestations of symmetry now. The basis used will correspond to

$$\begin{aligned} \text{a click in detector B } (|1\rangle &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}), \\ \text{a click in detector A } (|2\rangle &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}), \\ \text{a click in detector C } (|3\rangle &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}). \end{aligned}$$

Unfortunately, however, the beam-splitter symmetry from the 2D case has now been broken. Depending on whether detector C or detector D from Figure 4 is present, the click at either detector A or detector B will be ruled out by the presence of the new detector. Thus, the first beam-splitter $B(a_0)$ (depending on which path the detector is placed in) is represented by the matrices:

⁴³It should be noted here that there are dynamical ways of solving the QLP using Schrödinger dynamics; however, these solutions lead to interpretational problems about what is happening as the events “unfold in time”, and thus the task of RBW is to provide a non-dynamical solution to the QLP that does not run into the interpretational difficulties of the dynamical solution.

$$B(a_0) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \text{ or } B(a_0) = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \quad (233)$$

The first of these instantiations of the matrix variable $B(a_0)$ represents a detector blocking the route to detector A while the second of these matrices represents a detector blocking the route to detector B. The fact that the symmetry of the system is reflected in these matrices (and the fact that, for two different placements of the new detector, the symmetry is broken in two different ways) supports Stuckey et. al.'s and Bohr and Ulfbeck's claim that it is the symmetries of the system, not any physical properties of it, that result in observed phenomena.

Unlike the first beam-splitter, the second beam-splitter has only possible value. Its matrix in this representation is

$$B^+(a_0) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (234)$$

This result is rather straightforward since the obvious generalization of the 2D case to the 3D case is

$$B^+(a_0) = \begin{pmatrix} Q^+(a_0) & 0 \\ 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (235)$$

The fact that there is only one matrix value assumed by the matrix variable reflecting the symmetry of the second beam-splitter shows that the breakage of symmetry already appears in the first beam-splitter and the fact that detectors A and B keep their symmetry (that is, their positions have not been moved since the MZI case in Figure 3). So, to calculate the final state of the system, there are two calculations to do corresponding to the two separate paths on which the detector can be present. Solving for $|\psi\rangle$ in the first case obtains, by symmetry to the 2D case:

$$|\psi\rangle = B(a_0)^+ B(a_0) |1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} |1\rangle \quad (236)$$

So

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} |1\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad (237)$$

And likewise, for the detector on the other path,

$$|\psi\rangle = B(a_0)^+ B(a_0) |1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} |1\rangle \quad (238)$$

So

$$|\psi\rangle = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} |1\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad (239)$$

Comparing the results for both paths, one can conclude that the path in which the detector is placed has no effect on the probabilities of detector clicks since a sign difference is the only discrepancy between the two results. Both of these results reveal that, given a click in detector A, no matter which path the new detector C was added to, the state vector of the entire system introduces ED's indeterminacy since the state vector is now a superposition between two states. Thus, the results that produced the QLP in the first place have been reproduced.

Is there the same problem in this explication of the QLP as in ED's? The answer is a resounding "no", for where ED needed to explain temporal inconsistencies, SSC were able to produce the same result *without any of ED's dynamical formalism!* What ED have to account for is why the state vector changes as it does; for SSC, there is no "change" in the state vector at all. All of the problems in the QLP arise from its characterization as a dynamical process, and for this reason many like ED are forced to resort to backwards causal processes to explain this phenomenon⁴⁴. Thus, when SSC do away with the dynamical picture of spacetime, they also turn the QLP into an interpretational pseudo-problem⁴⁵.

What accounts for the behavior of the QLP, then, if not a dynamical process? The calculations above are carried out solely by analyzing the symmetries of the given experimental setup. The only interactions involved are relations between the detectors, beam-splitter, mirrors, and "photon" source. Yet, even though a "photon" source is involved, there is no need to mention an object called the "photon" (or, for that matter, the "atom") at all in the calculations above. The entire process can be calculated in a static RBW that relates symmetries instead of describing the dynamical evolution of particles.

SSC's derivation, while illuminating, is still not without its flaws. First of all, many of the matrices representing the symmetries of the system depend on the results of dynamical experiments (and possibly dynamical thought in general) for the matrix values assumed by the matrix variables of the system. Thus, even though RBW does seem to be an interpretation of quantum mechanics that is less problematic and open to paradoxes like QLP than the standard view, it is not a view of physics that can be

⁴⁴See the next section for more details on backward-dynamical accounts

⁴⁵Again, it should be emphasized that SSC are not trying to produce a novel result in their derivation here but rather account for the QLP in non-dynamic language that does not lead to the interpretational difficulties that dynamical accounts of the QLP lead to.

completely isolated from the dynamical theories that preceded it. This may make it difficult for RBW to make experimental predictions different from those of the currently dominant interpretation of quantum mechanics since many of RBW's predictions seem ad hoc. While I will not directly address the issue of how RBW may be provided with confirming or disconfirming evidence, the penultimate section of this paper will address future prospects for both RBW and GF.

6.5 Backwards Causality: Another Possible Solution to the QLP?

At the very least, what the QLP has shown is that the standard interpretation of time and causality is not suited to describing certain phenomena on the quantum scale. One might reasonably argue that when one is confronted with the QLP, the only proper recourse is to change one's conception of time and causality. The most obvious way, perhaps, to modify the standard interpretation of time is to use the interpretation that ED and Price[44] resort to: the idea of a backwards-causal theory of time.

This idea of a backwards-causal theory of time must not be confused with SSC's RBW view of space-time. While both theories of time allow for causal and temporal motion in either a forwards or backwards direction, SSC's account is fundamentally atemporal and non-dynamical while both Price and ED view time as a flowing, continuously changing medium. The difference can be viewed as the following: SSC's hope is to abandon the traditional conception of time's arrow as a fundamental part of the symmetry of the universe. Price and ED, on the other hand, choose to extend the arrow of time such that it is allowed to point in both directions at once. There are other differences between the two theories; for example, RBW does away with the notion of particles like photons and electrons as real particles while backwards-causal theories, in general, do not. However, the main difference pertinent to my analysis is the difference in the two rival theories' attitudes towards time.

The backwards-causal picture of the QLP, then, would go as follows: the measurement of the photon in the future sends information back to the past of that photon, forcing it to "act as a particle" as it hits the beam splitter instead of "acting as a wave". This change is mediated by some causal connection between the particle's past and its future that can travel backwards along a light cone. This signal is not "sent" by any sort of particle but is a property of the particle itself. The past particle's properties change to reflect the properties of the particle in the future. An interaction, then, causes a backwards fork that changes the past that led to the interaction. The acceptance of such a backwards-causal fork essentially builds the strange results of the QLP in as a mechanism for quantum processes and makes the very natures of such a "paradox" a fundamental theorem of quantum mechanics.

The backwards-causal picture may not seem especially appealing given that it runs so contrary to intuition and builds a conceptual problem into the framework of quantum mechanics, but Price⁴⁶ suggests another merit of the backwards-causal picture. The EPR paradox may seem spooky because the two entangled photons seem to be communicating with each other instantaneously across their space-like separation and such

⁴⁶[44], pp 251-2

space-like communication seems a violation of special relativity; yet, if the backwards-causal picture is used to describe the scenario, the results are not so troubling (that is, not any more troubling than the initial premise of backwards-causality). The measurement of one photon sends a signal backwards in time to the two particles when they interacted with each other originally to become entangled, and thus each one continues in space in a determined state afterwards. The communication in this description is all within the forward light cone of the two entangled particles, and thus there is no spooky space-like communication between particles. Thus, the backwards-causal picture, aside from describing the QLP, seems to be helpful at disentangling other quantum mysteries as well.

However, there are several important questions to ask of the backwards-causal picture of time, the first of which is why non-local connections are “spookier” than backwards-causal connections. Granted, if one accepts backwards causality, one need not accept non-locality, but non-locality can describe the EPR paradox just as well without defying traditional conceptions of time. There is no reason to prefer the backwards-causal picture description of EPR to the non-local description of EPR, and so backwards-causality only seems to be a useful interpretation of the quantum world if its description of the QLP is useful.

SSC[57] suggest that the backwards-causal picture of the QLP is inadequate. Their argument is that one can use interaction-free measurements (IFM) to construct the QLP experimental setup. The virtue of IFM for this description is that measurement interactions are necessary for the backwards-causal picture to hold up. ED state that it is the direct interference of the wave function of the particle with the wave function of the measurement apparatus that leads to the backwards-causal signal while Price’s description of backwards-causality necessitates a future interaction between particles for a signal to travel to the past. Thus, if IFM is a feasible option for a QLP apparatus, then the backwards-causal picture’s efficacy for dealing with the QLP will be greatly reduced.

SSC’s argument for the existence of IFM follows:

...nothing in the theory of quantum mechanics precludes the creation of a singlet state
 from particles that might have *never* intersected in the past *or* future, or have even emerged from a common source. And, this interaction-free measurement might even
 be achieved using not one but two photons coming from entirely independent sources...

(27)

According to SSC, it is the “mere fact that two atoms enter into the right sort of experimental relationships” (28) that allows for a measurement to take place, not an interaction between the two particles. One can make proper inferences about the properties of a particle in a given situation without interacting with it directly. As an example, SSC invoke ED themselves who propose IFM as an example of the inconsistencies of quantum mechanics. This example shows how the phenomenon of IFM can and does occur.

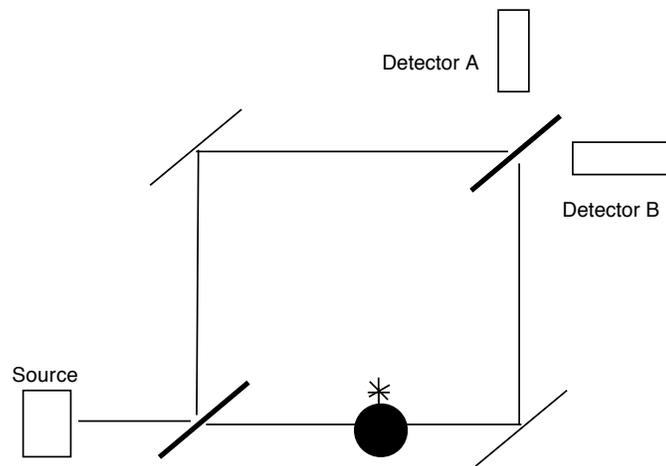


Figure 5: Interaction-Free Measurement (IFM) Experimental Setup for an MZI

Consider Figure 5. This is a tweaked version of the MZI used in the QLP. Instead of having one detector along any number of paths, there is one incredibly sensitive detector (ED use a bomb like the one in Figure 5 in their description of the apparatus) that is triggered by the presence of a single photon. Now, consider two states of a system: with a bomb in the MZI and without a bomb in the MZI. How can one determine whether or not there is a bomb in the MZI without sending a photon down the path in which the bomb might exist and thus risk detonating the bomb in the process?

The calculation for an apparatus without a bomb is fairly straightforward: it is simply the MZI described previously, and if the photons in the source are all prepared such that they would enter detector A, they will all enter detector A. However, if a bomb is placed in the path, then the previous calculations suggest that any photon sent through the apparatus will have a 50% chance of triggering the bomb, a 25% chance of the photon hitting detector A, and a 25% chance of hitting detector B. Now, sending a single photon through the MZI (if the MZI has a bomb in it) requires that one of these three results obtains. If the bomb is triggered, then clearly an interaction with the bomb has taken place and it explodes. The information as to whether there is a bomb in the setup is obtained, but at the cost of the bomb itself. If detector A is triggered, no information can be gathered since a single triggering event is possible for both the plain MZI and the MZI with a bomb in it.

The final option is that detector B is triggered, and it is this result that is the most interesting. For such a result to occur, the “photon” must travel a path separate from the bomb; yet, though the bomb does not interact with the photon, the bomb does change the result of the experiment somehow since now detector B can click though it could not in the bomb’s absence. Thus, a click in detector B is a kind of interaction-free measurement; it evinces the existence of the bomb without interacting with the bomb at all. Of course, such measurements only occur in 25% of the armed MZI apparatuses, but when they do occur, they certainly allow for IFM.

The IFM in this example poses a dire problem for Price’s and ED’s backwards-causal picture of the quantum world because the photon does not interact with the bomb, yet the photon’s actions are changed by the bomb. The only recourse left to such a backwards-causal theorist would be to posit some unknown, non-local interaction between the bomb and the photon that causes this change, but the non-locality of the interaction would lead to the same problem that backwards-causality did away with in the EPR experiment. Thus, the existence of IFM casts doubt upon the veridicality of the backwards-causal picture.

One might ask, then, how RBW would respond to IFM in its interpretation of the QLP if backwards-causality is to be rejected for not accounting for IFM. RBW deals with this phenomenon in two ways: first, by discarding the notion of actual, existing photons, RBW does not require interactions between such “photons” to have any sort of physical meaning, and thus the problem of IFM does not hinder the RBW interpretation. The second way is that RBW describes the entire experimental apparatus as a series of space-time symmetries made manifest in blockworld whose relations constitute the phenomena observed. These relations are allowed to be non-local, and thus the addition of a bomb to the setup changes the symmetries in space-time even if it does not “directly interact” with any part of the experimental setup via “photon” or any

other “particles”.

Thus, though backwards-causality does present an interesting alternative to RBW as an interpretation of the QLP, because of the existence of interaction-free measurements, backwards-causality is rejected as a viable alternative to RBW.

7 Philosophical Implications of GF and RBW

Up to this point, I have been generally concerned with the physics involved in the formulation of GF and RBW and some of their consequences; however, as I have hinted throughout, both of these theories involve important philosophical ideas as well as physical ones. Hopefully, a discussion of the philosophy involved in both GF and RBW will be able to shed new light on their physical projects. As Barry Stroud writes in his *The Quest For Reality*[51]:

We want to understand not only what gives rise to our perceivings and believings but also whether *what* we perceive or believe or come to think about the world represents it as it really is. In which respects does the world correspond to the way we think it is, and in which not? The philosophical quest for reality is an attempt to answer some such question. It is a question not only about the world, and not only about our perceptions, thoughts, and beliefs about it, but also about the relation between them.
(6)

As Stroud points out in this passage, we tend to place our perceptions in boxes that suggest either that our perceptions in some way relate directly to the way the world is or that our perceptions are misleading in a given situation. GF and RBW, with their claims about fundamental and relational realities, are engaged in a project of this type, and thus both theories allow for a great deal of potential philosophical work that can help clarify and embellish both the GF and RBW interpretations.

In this section, I will proceed as follows: first, I will characterize the philosophical stances of GF and RBW with regard to the reality of matter and structure, the reality of time, and the fundamental nature of uncertainty. Then, I will attempt two original philosophical projects: first, I will try to explain how one might come to GF's assertion regarding the fundamental nature of uncertainty, and finally, in a much larger project, I will show how a synthesis of RBW and GF which I call the "Fortuitous Relational Blockworld" (FRBW) disproves any sort of asymmetry in time. Such a position, I believe, is one that is amenable to Stuckey, Silberstein, and Cifone (SSC) in particular, and thus my purpose in proposing the FRBW is to show how the incorporation of more than just GF's physical arguments into RBW can further SSC's project.

7.1 Philosophical Characterization of GF and RBW

The first task in examining GF and RBW from a philosophical perspective is to explicate exactly to which philosophical stances both of these theories are fundamentally committed. Specifically, I will examine the issue of realism as regards matter and structure, the reality of time, and the fundamental nature of indeterminacy and randomness in both of these theories.

7.1.1 Realism in Matter and Structure

The first claim that one can make about both GF and RBW is that both theories hold structural realist positions. At its simplest, structural realism claims that the most

fundamental and true object of science is the form or structure of physical theories, specifically the mathematical form. This structural realist position has been viewed by Worrall[64] and French and Ladyman[29] as one of the potential saviors of scientific realism; though science may get underlying explanations wrong, the mathematical formalism of the “right” answer is usually similar to the formalism of the theories that preceded the “right” answer regarding the phenomenon in question. For example, though theories that invoked the ether were later deemed fundamentally flawed, the mathematics of the correct, non-ether theories was similar to that of the ether-invoking theories. Though a scientific theory is not always completely right and does not always describe reality as it actually is, as long as the mathematical form of this theory has been experimentally supported, one can say that the mathematical structure of the theory properly accords with reality.

The structural realism of GF and RBW comes across in two different ways: the idea of the fundamental reality of space-time structure and in the reality of relations but not relata. Space-time structure, if regarded as fundamentally real, requires a structural realist position since space-time is a metric that provides mathematical constraints on objects in space-time. Treating the idea of such a geometry as fundamentally real, even if the current characterization of this geometry is wrong, constitutes a form of structural realism. Likewise, to treat the mathematical relations of QM as fundamentally real again requires a form of structural realism; though the state vector that describes the combined spin of two electrons may prove to be mathematically insufficient in future generations, the formalism of quantum mechanics that treats these relations as fundamental will continue to be real. Thus, since both the reality of relations and space-time structure are fundamental to both GF and RBW, both of these theories take structurally realist stances.

I am not concerned in this section with evaluating the promise of structural realism⁴⁷; rather, my goal is simply to determine the stance of GF and RBW with respect to the realism of structure and matter as accurately as possible. To this end, I believe that the distinctions among several types of realism in a forthcoming paper by Stathis Psillos[45] might be helpful⁴⁸. In this paper, Psillos distinguishes among four different types and two degrees of structural realism. The four types of structural realism he cites are Eliminative, which claims that there are no objects and that the mathematical structure is all that there is; Reconstructive, which concedes that there are “real” objects in the world but that these objects must be understood in terms of the fundamental structure; Formal, which states that objects in the world are mathematical entities and should be recast as such; and Semi-formal, which posits that only unobservable objects need to be recast in mathematical terms and the rest of the objects can be taken as is. Eliminative structural realism is the most ontologically extreme form: not only is the structure all we can know, the structure is all that exists. Reconstructive

⁴⁷For those who are interested in such a discussion, please consult [64] for a historically important argument in favor of structural realism, [29] for a contemporary reformulation and evaluation of structural realism, and [45] for a contemporary attack on structural realism

⁴⁸French and Ladyman also make a distinction between ontic structural realism (OSR) and epistemic structural realism (ESR) that is helpful for evaluating structural realist interpretations; however, since Psillos’s taxonomy is more nuanced than the OSR/ESR distinction, I have chosen to use his taxonomy over French and Ladyman’s.

structural realism still takes an ontological stance, but it emphasizes the fundamental nature of the structure and allows for other objects to exist. Formal and Semi-formal structural realism could perhaps be seen as versions of reconstructive structural realism since both allow that objects in the world exist but that some of them can and should be recast in terms of the fundamental structure, again emphasizing that objects can exist as we perceive them but that it is the mathematical formalism of these structures that is really “real”.

Along with these four positions (or, in my formulation of them, two positions with two more refined positions of reconstructive structural realism), Psillos also characterizes two degrees of structural realism: mild and radical. The mild form of structural realism states that the structure is ontologically basic but that it does not supervene on the properties of objects. Mild structural realism seems to characterize the reconstructive structural realist position fairly well. The second degree of structural realism, radical structural realism, states that the structure is ontologically basic and that, without this structure, there are no properties or objects at all. This radical form seems to best characterize the eliminative structural realism described above. However, though I have cast mild and radical structural realism in terms of reconstructive and eliminative structural realism here, it is still possible for one to be a radical reconstructive structural realist⁴⁹. A radical reconstructive structural realist might state that the way in which the structure underlies the existing objects is that the structure supervenes on the properties of those objects. Such a view logically holds together, and thus one should not assume that a reconstructive structural realist position necessarily implies mild structural realism.

The question, then, is where RBW and GF fall in this taxonomy⁵⁰. The theory of GF seems simple enough to categorize in terms of this taxonomy. As Bohr and Ulfbeck state in their second paper on GF[17], matter does not exist; it is the relations that exist, just like it is the space-time symmetries that give rise to these relations. It seems, then, insofar as Bohr and Ulfbeck would be unwilling to cede the existence of particles or any entities such as this, that they are radical eliminative structural realists, for all that exists in their view is structure. However, it should be noted that Bohr and Ulfbeck do not spend much time writing about the reality of macroscopic particles. Their attention is focused on flatly denying the existence of particles and waves on the quantum level, not going into too much detail on the macroscopic events they take as real. Thus, one is forced to conclude that their position is one of radical eliminative structural realism since they do not present a single type of object whose existence they would support without reducing it to structure.

For RBW, the question is slightly trickier. Though in general SSC present a position similar to Bohr and Ulfbeck’s regarding the fundamental reality of space-time relations and quantum mechanical relations via wave function, they also seem to make an effort

⁴⁹It seems to me, however, that a mild eliminative realist would be an impossibility since the non-existence of objects would seem to imply the non-existence of their properties. It seems absurd to proclaim that structure exists and matter doesn’t but that the properties of this structure do not *come* from structure

⁵⁰I should mention at this point that it is not clear that Psillos’s taxonomy is completely comprehensive, and thus RBW and GF may not fit nicely into the boxes he provides for structural realist interpretations. However, since his is one of the best structural realist taxonomies currently available, I believe his taxonomy serves as a decent starting point for a discussion of structural realism.

to “save the appearances” by stating that macroscopic objects, insofar as they are made up of a large, almost uncountable number of mathematical relations, are real as well⁵¹. Thus, their structural realism seems to be of the reconstructive variety since macroscopic objects are real but are not assimilated directly into structure. Specifically, RBW suggests a formal structural realism since all objects in some way are constructed from the very structure of space-time and its mathematical relations. This criterion specifies exactly how the geometry of space-time is to be viewed as underwriting the entirety of nature. RBW also seems to be a radical form of structural realism since it views everything about macroscopic phenomena as somehow emergent from the mathematics of space-time. Thus, it seems logical to conclude that RBW is advocating a radical formal structural realism.

This discussion of structural realism helps to pin down the philosophical position of GF and RBW with respect to other structural realist theories, but it also helps one understand GF’s and RBW’s stances on structural realism with respect to each other. As the above footnote pointed out, the structural realist stances advocated by GF and RBW are not that different from each other. Both are radical forms of structural realism, and GF is pigeonholed into the eliminative structural realist camp only because it does not address the ontological status of the macroscopic world as well as RBW does. The structural realism found in GF could be viewed as a subset of the structural realism advocated by RBW in a way, and so, while the theories of structural realism found in both interpretations may not directly coincide, they are certainly not incompatible with each other.

This discussion of structural realism has brought out the two features of space-time that GF and RBW both take as fundamentally real: space-time geometry and space-time relations. The former can be viewed as a mathematical description of the world as it is, in itself; however, the treatment of space-time relations introduces a radical relationality into both interpretations that suggests that objects themselves are not real while relations between these things are. It is easy to see how these two “real” elements of the universe can combine to make the entities that exist within space-time “real” as well. As long as Bohr and Ulfbeck’s “primary manifestations of symmetry” form the basic building block relations from which the universe is built, then everything else can be defined as real in terms of relations between and among other relations. Both RBW and GF are more complete than any interpretation of QM or SR that posits simply a fundamental space-time geometry since such a theory fails to explain how things can be real that are not part of this mathematical formalism. On the flip side, a world where the only real things are relations sinks into an infinite regress since everything exists only in terms of relations among other smaller entities. Such a view would be “turtles all the way down”. However, by providing a ground for being in space-time and its geometry, RBW and GF succeed in accounting for a kind of realism (about objects now, not about scientific knowledge) concerning the world. Both are capable of “saving the appearances” while at the same time explaining the quantum world.

However, calling both space-time itself and relations within it “real” seems contra-

⁵¹It should be noted here that this idea in no way contradicts any of the beliefs Bohr and Ulfbeck have about the unreality of particles or waves on the quantum scale. SSC simply address macroscopic issues in a way that Bohr and Ulfbeck do not, and thus SSC’s stance on this issue could be viewed as a logical extension of Bohr and Ulfbeck’s.

dictory; if reality is relational, how can a “thing in itself” like space-time exist without relation to anything else? And if space-time is allowed to exist as a “thing in itself”, why should not certain entities within space-time be allowed to exist in the same way? There is a tension between these two views that neither GF nor RBW is fully able to resolve. I believe that such a tension is necessary given the clean solution that both of these views, when combined, gives to the reality of matter; however, since there are those to whom this tension must seem problematic, I believe a solution can be found within Bohr and Ulfbeck’s stance towards realism in general.

Bohr and Ulfbeck make a strange comment in the second paper on GF[17] when they state that there is no “world of matrix variables” that they will talk about; of course, up to this point, Bohr and Ulfbeck have made several strange comments about how the matrix variables “emerge” from somewhere outside of space-time to appear within space-time. Why would Bohr and Ulfbeck, then, add a throwaway comment about how such a world “outside of space-time” should not be treated as really existing? Are they simply instrumentalists such that none of their mathematical machinery is to be treated as real? Is their only goal is to make good predictions?

I believe that the reason behind Bohr and Ulfbeck’s statement about the “world of matrix variables” can be found from an examination of two competing views of reality. The first is explained by Putnam[46] in the following quotation: “Today material objects are taken to be paradigm mind-independent entities, and the correspondence is taken to be some sort of causal relation” (205). The “correspondence” referred to in this passage seems to suggest that there is a view of reality that anything which causes⁵² something to be real must, in itself, be real. Causal explanation, then, can be a reason to treat an entity as real. For any two entities a and b , then, if a leads to b and b is real, then a is real as well on this view of realism. I will call this perspective “causal realism” hereafter.

Another view, however, may be more familiar to modern audiences. This is the “realism” of the verificationists who claim that the only things that are meaningful and real are those things that can be distinguished from each other. Something must be distinguishable or observable to be real on this account. This is a different kind of criterion from a causal connection because it limits what we can say about reality a bit more completely. Imagine that a causes b just as before, but this time we are not able to observe a empirically. We then cannot make any statements about event a concerning its reality if we are to be good verificationists. However, the causal realist would argue that the mere existence of a causal relation between a and b is enough to establish the reality of b . Thus, the two would fundamentally disagree on the reality of a in the above example.

However, there is one important point to note about the verificationist perspective: it does not say that, in the above example, a is unreal, it simply remains agnostic as to the reality of a . The difference between the causal realist and the verificationist, then, boils down to the causal realist making statements about unobservables about which the verificationist would not dare comment. The differences between the causal realist and the verificationist come down to differences in scope. Most contemporary

⁵²I am speaking about explanation and derivation in this sense of causality used here, not “cause and effect”. Thus, if a “causes” b in this sense, then a is the fundamental, underlying principle that leads to the emergence of b .

scientists have adopted the more conservative verificationist stance since it allows them to remain agnostic on certain tricky issues.

Bohr and Ulfbeck, I believe, follow in this verificationist tradition: the “world of matrix variables”, if it exists, exists outside space-time. For this reason, there would never be any way to directly observe this “world”. For this reason, Bohr and Ulfbeck decide to remain agnostic on the existence of this world. They do not explicitly deny its reality, but they are not causal realists who would be committed the reality of such a world simply because the matrix variables that appear in space-time are causally connected to this “world”. It is this verificationist stance, I believe, that causes the confusion in Bohr and Ulfbeck’s statement about the non-reality of the “world of matrix variables”: they simply endorse Wittgenstein’s famous maxim, “whereof one cannot speak, thereof one must be silent”.

I believe that this same verificationist stance helps one reconcile the difference between a more radical relationism that holds that everything only exists as a relation of other entities and the reality commitments of RBW and GF. It is not necessarily true that space-time exists “in and of itself” for the verificationist who endorses radical relationism, just that the relata that compose the relations of space-time are empirically inaccessible. Since the most fundamental thing anything inside space-time can access is the geometry of space-time itself, it does not make sense for the verificationist to try to reach beyond space-time to speak of the reality of anything we can never know. All the objects we can know about exist within space-time, and thus all of them can be viewed either as relations of space-time (or various relations of these relations) or as the geometry of space-time itself. One need not endorse the claim that space-time exists “in itself” fundamentally to accept the premises of RBW and GF; instead, one need only take a verificationist stance along with an endorsement of radical relationality to arrive at the very same realist conclusions.

Hopefully, this characterization of GF’s and RBW’s ontological commitments has made these commitments more palatable to the general audience and has explained where, in general, RBW and GF stand towards the philosophical community at large on issues of the realism of matter and structure as well as towards each other. Perhaps the most important conclusion reached in this section is that RBW and GF do not hold incompatible ontological commitments. Though Bohr and Ulfbeck seem to endorse the realism of space-time, they don’t believe they can talk about reality outside the context of space-time. This conclusion will allow for my project later in this section when I attempt to combine these two perspectives in a unique and meaningful way to respond to the issue of temporal asymmetry. Before attempting this synthesis, however, there are other important issues that I must examine, specifically the stances of RBW and GF regarding time and uncertainty.

7.1.2 The Nature of Time

The next important issue that arises in a philosophical discussion of GF and RBW concerns the status of time in these two theories. For RBW especially, time plays a central role since the very idea of a blockworld hinges on a rejection of presentism. For GF, however, the issue of time seems to be less important. Though GF explicitly rejects the idea that the future is in any way causally connected to the past because

of the principle of genuine fortuitousness, it does not weigh in on the issue of whether the past or future are ontologically different. In the first paper on GF[16], Bohr and Ulfbeck appear to take a stance on the issue of time by stating that there must be a fundamental temporal asymmetry that is accounted for in their list of fundamental space-time symmetries, but by the time of their second GF paper[17], they have backed off from their previous stance and prefer to remain agnostic on the issue. Because of this agnosticism towards the existence of time, then, the analysis in this section will primarily focus on RBW and its theory of time.

Philosophically speaking, the idea of eternalism or the blockworld is not particularly new. As I have pointed out elsewhere[43], Parmenides of Elea was one of the first philosophers to posit that the distinction between the future and the past is not well-founded. Instead, Parmenides held that all of space-time is essentially a unified whole, thus suggesting a primitive notion of blockworld. Today, the 4D blockworld picture of space-time has found acceptance among many philosophers such as Sklar[49] among others.

However, there has still been much resistance to the idea that the past, present, and future are all equally real. Philosophers such as Dorato[24] and Savitt[48] have attempted to show that the difference between the presentist and eternalist amounts to a misuse of language while others like Stein, who argued for a view of presentism that constituted incorporates a “here” as well as a “now”, have attempted to change presentism in such a way that it still retains the character of the present while refuting the relativity of simultaneity argument invoked first by Putnam[46] and then later by SSC. However, as I have already shown elsewhere[43], these attempts to save presentism or to dismiss the presentist-eternalist argument do not pan out. The relativity of simultaneity argument against presentism both establishes an ontologically fundamental blockworld and refutes any meaningful interpretation of presentism.⁵³

This eternalist perspective on time holds a great deal of philosophical and physical promise. As Richard Healy points out[31], many formulations of quantum gravity are much more tenable if one treats time in a 4D blockworld instead of the 3D moving world of presentism. The blockworld setup also seems to be a more tractable one for the determinist, though most formulations of presentism still allow for determinism. In short, there are many reasons why a physicist or a philosopher might be motivated to view space-time as a 4D blockworld, and many of these reasons, as I will show later in this paper, provide a great deal of promise for solving some of the problems plaguing contemporary physics.

To conclude, then, the eternalist position of RBW seems extremely feasible and plays a central role in making RBW a tractable interpretation of quantum mechanics and space-time. The RBW stance is that there is no ontologically privileged position in time with respect to past, present, and future, and that space-time is best viewed as a 4D whole. This is, once again, compatible with GF since GF is unconcerned with the issue of time as a theory. Because GF is agnostic on the issue of time, then, it seems that there is nothing in the philosophy time endorsed by GF or RBW that could

⁵³The exception to this might be Arthur[9], who presents a view of the present in terms of light cones that can be reconciled with the relativity of simultaneity argument; however, this view of presentism involves lumping in parts of what has been traditionally treated as the past or the future in with the present, thus seeming to leave the “present” as an *ad hoc* definition that saves presentism in name only.

be considered an irreconcilable difference. Again, this compatibility suggests that my future project of combining GF and RBW will not lead to any contradictions regarding the treatment of time.

7.1.3 The Fundamental Nature of Uncertainty

The issue of fundamental uncertainty in GF and RBW poses the exact opposite problem from the issue of time in these two theories: on this subject, RBW remains silent while GF incorporates fundamental uncertainty as a cornerstone of its theory. Fundamental uncertainty, in terms of GF, states that the randomness and unpredictability of certain phenomena on the quantum scale reveal a more deeply-seeded indefiniteness that pervades all phenomena on a fundamental level. At energy scales currently inaccessible, GF suggests that all events happen completely lawlessly. This stance has already been discussed as the idea of genuine fortuitness, the central doctrine of GF.

RBW, however, does not take a stance as to how fundamental the indeterminate character and lawlessness of events is. In the GF perspective, indeterminacy is taken as a brute fact about the universe that emerges from the existence of matrix variables, and it is this brute fact that accounts for most of the features of the rest of GF. RBW, however, does not need to appeal to such a feature of the world to reach its conclusions; the RBW treatment of time is sufficient for one to establish the relational reality that both GF and RBW hold as a necessary feature of the universe. For this reason, it makes sense that RBW would not need to invoke indeterminacy in the same manner as GF does, while GF does not need to invoke a time argument like RBW does.

Still, it is important to note that both RBW's treatment of time and GF's treatment of indeterminacy may be compatible with each other. What would it mean for something to be "genuinely fortuitous" in a blockworld? That is the question I now seek the answer. I wish to reinterpret the doctrine of genuine fortuitousness in light of RBW to show that GF and RBW are, in fact, compatible, and to suggest how someone advocating RBW might be led to the same conclusion regarding "lawlessness" as someone advocating GF. This project will, I believe, accomplish two goals: first, it will show what RBW adds to GF by giving GF a new argument for the fundamental nature of lawlessness in the natural world, and secondly, it will explain how the doctrine of genuine fortuitousness can be translated into the language of RBW, thus allowing for the synthesis of GF and RBW I call "FRBW" that will be invoked in the next section.

I begin with the example used by Bohr and Ulfbeck: that of a photon source and detector setup. The detector and the source are both made up of some extremely large number of particles. The behavior of the detector and source, on both the GF and RBW accounts, is determined by the behavior of the atoms that compose the apparatus since the macroscopic phenomenon exists as a relation of the microscopic entities. However, if one follows this chain down to the absolute lowest constituent parts both temporally and spatially, one finds that even these relations are best described (once again, for both GF and RBW) as a series of events. These events are the ones both treated in the RoS argument for blockworld by RBW and explained as "genuinely fortuitous" in GF. RBW, then, is concerned with the implications of these events for relativity, whereas GF is more concerned with the origin of these events. However, it seems clear that both interpretations depend on these instantaneous and infinitesimal

events for descriptive power⁵⁴.

These events seem to pose a problem for RBW; after all, if there are extremely small events from which macroscopic phenomena can be built, it seems that the “ground of being” is not space-time symmetry at all but these “building block events”. Bohr and Ulfbeck answer the question of “which is more fundamental, the events or the symmetries?” by saying that, in fact, events are actually manifestations of space-time symmetries; the two are one and the same. Likewise, RBW follows GF and collapses the structure-matter dichotomy itself. However, Bohr and Ulfbeck’s next claim is that the matrix variables that contain within themselves the seeds of space-time events and the information and relationality of space-time symmetries enter the space-time scene in a completely lawless manner, and it is not clear that supporters of RBW would be required to endorse this claim. Why could the “matrix variables” not originate from within space-time itself? In fact, it is possible that the exact opposite of genuine fortuitousness is true in a blockworld: all events may arise as a direct consequence of the blockworld structure and not in some indeterminate manner as both GF and the Copenhagen interpretation would suggest. Thus, there may be some tension between GF and RBW that is not easily resolvable.

However, consider the interpretation of realism as verificationism from the section on reality in RBW and GF. In the verificationist interpretation, only the blockworld can be considered real since only causes within a blockworld can be verified; however, this does not preclude the existence of causes outside of the blockworld which could lead to the structure of the blockworld itself, it only suggests that such causes would be incomprehensible to any observer inside the blockworld. The fundamental ontology of space-time, then, could simply be a result of a combination of the verificationist approach with the radical relationality of reality.

Since objects outside the realm of blockworld must necessarily remain uncertain, then, it is possible (if one is willing to abandon the verificationist approach) that the phenomenon of genuine fortuitousness can be reinterpreted as what results from extra-blockworld causes. It may not be that there are “no fundamental causes” or “no laws” that operate in the genuinely fortuitous character of space-time events; rather, it may just be that the cause of these events lies outside the framework of space-time. The existence and non-existence of the “building block events” of space-time may not be a direct consequence of laws *within* the blockworld, but this does not mean that internal laws cannot be the result of laws *outside* the blockworld. If extra-blockworld laws are the ones dictating the properties of blockworld events, such uncertainty as the principle of genuine fortuitousness embodies certainly emerges.

The question of genuine fortuitousness in RBW, then, boils down to whether the “building block events” of space-time obtain their character solely from the nature of space-time or whether it is possible that some extra-blockworld cause leads to their origination. This question leads finally back to the question of quantum measurement.

⁵⁴I use the terms “instantaneous” and “infinitesimal” here not to suggest that the duration and spatial extent of the events is effectively zero but to suggest that these events are those than which nothing shorter, spatially or temporally, can exist. In this way, I am suggesting a kind of “atomizing” of the world based on these events (i.e. breaking down everything into the world into these smallest parts and using them to describe the fundamental constitution of the world), but I believe that such an atomization is consistent with and even implied in the language of GF if not RBW

The Copenhagen interpretation of quantum measurement states that it is a fact about nature that no one can predict the result of any single quantum measurement if the system is prepared in a superposition with regard to the quantity being measured. To contradict Copenhagen’s claim on this matter would be to suggest a “hidden variable theory”, and though RBW does purport to be similar to such a theory, the best GF and RBW can do is to show where the *laws* governing quantum mechanics come from, not where their *lawlessness* does. Bell’s inequalities throw doubt upon the very idea of the existence of hidden variables. All experiments that I am aware of up to this time have perfectly confirmed the Copenhagen interpretation’s stance towards the unpredictability of the results of any given mixed-state measurement. Thus, it seems safe to say that, at this point in time, the best evidence we have regarding quantum theory suggests that some uncertainty in quantum theory needs to be accounted for.

Thus, RBW has two options for dealing with the uncertainty suggested by the measurement problem: it can either suggest that this uncertainty reflects some unknown variables that we have not accounted for and thereby endorse one of the many currently unpopular and still unverified hidden variable theories that exist today, or it could abandon a search for causality only within the blockworld framework and endorse the GF perspective⁵⁵. The issue of whether any particular hidden variable theory is easily incorporated into the blockworld framework cannot be addressed here since supporters of RBW have several choices of hidden variable theories were they to take that route; however, a new interpretation of genuine fortuitousness allows for the random character of events in blockworld to be explained just as well as the random character of the geometry of space-time that blockworlders support. So, while at this point it seems obvious why SSC would choose to remain agnostic on the issue of fundamental uncertainty in space-time, I have at least proven that GF is not incompatible with RBW. To make a stronger claim and show that RBW necessitates GF, one would have to disprove all current and potential hidden variable theories, and that is a task too grand for the scope of this paper.

However, though I have shown that, on the issues of reality, time, and uncertainty the RBW and GF perspectives are not incompatible, I have not yet given any impetus for the RBW supporters to accomodate GF into their theory. The purpose of the next section is to provide such a motivation by utilizing a combination of GF and RBW to solve the problem of time’s asymmetry.

⁵⁵Thus far, I have been speaking in fairly abstract terms, which can sometimes be unhelpful on the issue of explaining anything about the structure of space-time. I want to pause for a moment to suggest a new potential view of the blockworld within a “genuinely fortuitous” universe in more visual terms. Instead of the “static spacetime jewel” of blockworld that is often invoked by eternalists to help their readers conceptualize of what a blockworld would “look like” from the outside, now imagine that a picture on a slide is being projected onto the surface of this space-time jewel. From the perspective of one inside the jewel, one might ask “Why is this section blue while this section is black?”, and from within the jewel, one could not formulate an answer since one could not see the entire picture projected on the jewel; however, from outside the jewel, an observer (some analogue of Newton’s God, perhaps, looking down on his “sensorium” from the 5th dimension) could easily see the pattern and understand that all of the “genuinely fortuitous” events inside the space-time jewel are, in fact, completely determined by the pattern in the projector.

7.2 The Asymmetry of Time and the Fortuitous Relational Blockworld (FRBW)

In this section, I will invoke a combination of GF and RBW which I call the “Fortuitous Relational Blockworld” (FRBW) theory to solve the problem of the asymmetry of time. I will first explain the problem of the asymmetry of time and why RBW would like to find a convincing solution to it. The problem of the asymmetry of time is that many phenomena in the world seem to distinguish between either “the past” and “the future” or “before” and “after”. These “asymmetrical” phenomena can be divided into two parts: physical temporal asymmetries and personal temporal asymmetries. Physical temporal asymmetries seem easy enough to understand. Imagine looking at two photographs, one of a baseball in the air in front of a glass window and one of the same window broken, shattered glass littering the area around it, with a baseball on the other side of the window frame. One would argue that the first picture came before the second picture, and one could invoke several different facts about the natural world to do so. For instance, one could claim that the baseball “caused” the window to break, or that, by the second law of thermodynamics, entropy tends to increase in closed systems, and since the final picture evinces more entropy than the first, the time ordering suggested above is the best inference. Physical temporal asymmetry describes differences in the past, present, and future, or among the before, during, and after, in terms of physical processes in the natural world. If there exists physical temporal asymmetry, then, it would seem that temporal asymmetry is an objective feature of the natural world.

However, the above example of the two pictures and the time-ordering could be explained in terms of personal temporal asymmetry instead. For instance, one could appeal to the time-ordering as a result of experience: I have often seen a baseball smash through a window, but I have never seen a number of broken pieces of glass arrange themselves into a window pane of their own accord. Another justification of the time-ordering could come from the nature of explanation itself since I can tell a story that “makes sense” if I time-order the photographs as described above but am unable to tell a plausible story about the opposite ordering of these photographs. These justifications all appeal to aspects of human nature to justify the asymmetry of time. Mentalistic phenomena like experience and social constructions like language become the ultimate justification for personal temporal asymmetry instead of the natural phenomena that justify time-ordering in physical temporal asymmetry.

The problem of temporal asymmetry, then, boils down to what one should hang the various phenomena that point to asymmetries in time upon. For physical temporal asymmetry, the traditional response is that the objective, mind-independent nature of time is itself asymmetric, but for personal temporal asymmetry, the traditional response is that the asymmetries in time emerge due to the subjective nature of human experience. One could easily endorse eternalism if one takes the personal temporal asymmetry approach since the asymmetry of time only reveals something about human minds and not about the external world. The conclusion that time itself is asymmetric poses a potential problem to many eternalists. If asymmetries in our perceptions of time are not due to some human faculty like experience or explanation, the past and the future could have different characteristics which could, theoretically, single out the

present frame as the preferred one. In much presentist literature, the idea of temporal asymmetry is deeply connected to the idea of “time’s arrow” (that is, time singling out a single direction in which to move) and a “moving present”, both of which are hallmarks of presentist position and both of which are inimical to eternalist thought. There are, of course, some eternalists like Maudlin who believe that eternalism is compatible with temporal asymmetry, but SSC themselves prefer to reject objective temporal asymmetry lest temporal asymmetry bring with it ideas of a preferred present frame or objective becoming⁵⁶. Thus, if one could find a way to hang physical temporal asymmetry on a natural phenomenon other than time, one would be furthering the project of eternalism and positing a theory that would be, in at least some respects, attractive to SSC, other supporters of RBW, and eternalists in general.

I believe that such a project is possible, but before I begin I will explicate the currently-held beliefs about temporal asymmetry. Dainton[22], on pages 46 and 336 of his book *Time and Space*, enumerates the nine different “types” of temporal asymmetry. These are⁵⁷:

1. Entropic Asymmetry
2. Radiation Asymmetry
3. Fork Asymmetry
4. Causal Asymmetry
5. Explanatory Asymmetry
6. Knowledge Asymmetry
7. Action Asymmetry
8. Experience Asymmetry
9. Counterfactual Asymmetry

These asymmetries can be explained as follows: Entropic asymmetry is the asymmetry in time that arises from the second law of thermodynamics. This law states that, for a closed system, entropy tends to increase with time. The temporal asymmetry that this phenomenon describes, then, is that the past will be a lower state of entropy than the future; the second form of asymmetry, radiation asymmetry, arises from the fact that radiation tends to have a point as its source, but radiation does not tend to converge on a single point; fork asymmetry, the third form of temporal asymmetry, points out that later events in time tend to be correlated with a previous temporal event, past events do not tend to be correlated with the same event in the future; the fourth form of temporal asymmetry, causal asymmetry, points out that causes precede effects in time; explanatory asymmetry, the fifth form, is the idea that one past event can explain many future events, but the opposite is not true; the sixth form is knowledge asymmetry, which is the fact that human beings have knowledge of the past but not the future; the seventh form is action asymmetry, which results from the ability of human beings to contemplate future action and act on it at a later time while one cannot act on an event that has already occurred; experience asymmetry, the eighth form, is the asymmetry that results from human beings living a life in a present that

⁵⁶Personal communication, Silberstein 2007

⁵⁷The numbering of these types of temporal asymmetry is my own, not Dainton’s

moves from the past towards the future; and finally, counterfactual asymmetry in the structure of counter-factuals since past events generally serve as possibilities in true counter-factuals whereas future ones do not.

It seems fairly obvious from the start that asymmetries 5-9 do not pose a great problem to the eternalist. In fact, eternalists might even be fervent proponents of these positions since all of these asymmetries are personal temporal asymmetries that hang on the human perception of time instead of some objective feature of time. SSC[56] have even published a paper that takes consciousness as the basis for temporal asymmetry. If the cause of the human perception of temporal asymmetry is some uniquely human feature of the world, such as can be found in explanations that rely upon the nature of explanation itself, knowledge, human action, experience, or the language of counterfactuals, then it can be explained as yet another human phenomenon that can mislead us into believing that the world is other than it is. Thus, it is only asymmetries 1-4 that the eternalist response to temporal asymmetry must address.

To provide this response, I propose a combination of RBW and GF that I call the “Fortuitous Relational Blockworld” (FRBW). FRBW is exactly like RBW in every way except that, instead of remaining agnostic on the nature of fundamental uncertainty in the natural world, FRBW explicitly endorses the claim that events in the world are “genuinely fortuitous”. Lawlessness and uncertainty are incorporated as essential features of the FRBW at its most basic levels. As has been shown previously, there is nothing essential in GF or RBW that necessarily precludes the other, so I believe that such a synthesis of RBW and GF will be seen as acceptable by proponents of both theories even if they do not believe it to be necessary.

Another observation at this point will be helpful as well. In his essay “Time and Reality” [23], P.C.W. Davies writes: “It is also important to realize that we have no physical evidence that time *itself* is asymmetric. Time asymmetry is a property which, as a matter of observational fact, exists in the material world. The *world* is asymmetric *in* time” (64, original emphasis). What Davis points out is that it is not just time that should be considered in inquiring into temporal asymmetry but the world itself. For a proponent of FRBW, of course, this means that all of blockworld is to be considered and implicated in considering this asymmetry, not just the single dimension of time. This kind of holism obtains because all of the asymmetries observed in space-time concern events which have positions in both space and time, and thus all of blockworld must be considered when one searches for a place to hang “temporal” asymmetry.

Now, from the idea of FRBW, I will first attempt to address the problem of entropic asymmetry. If the assertion that “entropy increases in time” is to be challenged, it seems that the second law of thermodynamics itself must be called into question. Since this seemingly “most physical” of all temporal asymmetries seems to pose the greatest problem to the eternalist, I believe it is the one that should be addressed first.

I will begin my response to entropic asymmetry by asking the reader to imagine a blockworld in three dimensions: one of these is time, and the other two represent space. The blockworld I have in mind is a cone with the time axis corresponding to the axis of the cone. Now, imagine that the inhabitants of this blockworld experience time as we experience time; that is, they believe that they move from the past towards the future and exist, at any given time, in a present that is given by one time-slice of the cone. One of the interesting features of this blockworld is that it shows that the blockworld

interpretation can explain the seeming “expansion” of the spatial component of the blockworld in time. Now, return to our four dimensional FRBW in which we have empirical evidence of the expansion of the spatial universe in time from the red shifting of galaxies. Such expansions can be viewed as consequences of the geometry of a blockworld just as easily as the “expansion of space” is for the three dimensional blockworld previously considered. This conclusion may seem irrelevant to the issue at hand for the moment, but bear with it.

Now, consider the true nature of entropy. Entropy in its most mathematical (and interpretively correct) form is simply the distribution of energy units over some interval of space. In the “microstate, macrostate” terminology, states with higher entropy are macrostates (states that correspond to potentially many different microstates) that correspond to the most microstates. For example, consider a system consisting of three boxes and three energy units. The macrostate of “three energy units in one box” corresponds to three microstates, one of which has all three energy units in the first box, one of which has all energy units in the second box, and one of which has all energy units in the third box. Now, consider the macrostate “two energy units in one box and one energy unit in another box”. This state corresponds to six microstates since, if the two energy units are in the first box, the third energy unit could be in box two or it could be in box three. Thus, the macrostate “two energy units in one box and one energy unit in another box” has a higher entropy than the state “three energy units in one box”. What this example also points out is that states of higher entropy are generally ones in which energy is evenly distributed as opposed to being clumped in one area or another.

In the FRBW view of an expanding universe, the number of boxes is growing with time. Thus, the statement made by the second law of thermodynamics that entropy increases with time simply reduces to the idea that a finite amount of energy in the universe spreads itself out over an ever-growing amount of space as time goes on. This phenomenon is similar to that of removing a partition in a box where one side of box contains energetic gas molecules while the other side of the box houses only a vacuum. As the partition slides in such a way as to enlarge the volume of the gas molecule side (the universe) and shrink the volume of the vacuum side (whatever the universe is expanding into), the entropy of the system rises. So, too, the expansion of the universe allows for “moving particles” to spread themselves out over a larger volume, thus increasing the entropy of the system.

The problem with this reinterpretation for blockworld, however, is that there is no mechanism in blockworld for change. There can be no motion, no movement of particles from one place to another. However, in FRBW, there is another guiding principle: fundamental lawlessness. There is no reason why any “clumping” of energy, be it kinetic energy, relativistic rest energy of matter, the potential energy of a field, or any other form of energy, should remain a clump in time. The randomness must operate within the laws of space-time, of course, since we are talking about a blockworld here, but it follows that if particles are moving randomly in a box of ever-increasing volume, the entropy of the system will increase in time as a consequence. In this analogy the randomness of such motion is due to the genuine fortuitousness of FRBW and the expansion of space is a feature of the geometry of our FRBW.

Thus, the asymmetry of time is not an essential characteristic of any space-time in

which the second law of thermodynamics must hold. As I have shown, the FRBW, with its features of a fundamental lawlessness of physical events and a static blockworld, is a sufficient explanation for entropic asymmetry. It may be objected that I have not proved that entropic asymmetry *must* be hung on FRBW instead of an objectively asymmetric time, which may be true, but I have diffused the threat to eternalism by proposing a story that accounts for the entropic asymmetry of time while keeping and even invoking all of the essential features of FRBW. This is enough for my purposes.

It seems to me that the second and third varieties of temporal asymmetry, radiation and fork asymmetry, are also explained away by this story. In an blockworld whose geometry invites the illusion of expansion, if uncertainty is taken to be an essential feature, then events tend to spread out from a point rather than come together. Radiation spreads out through space on this principle, then, as do events in the fork symmetry. Both of these phenomena can be explained by the FRBW just as easily as entropic asymmetry was, and thus neither pose a problem to the eternalist picture of blockworld as long as FRBW is taken as a viable picture.

The only temporal asymmetry left to be explained is causal asymmetry, which can also be explained by FRBW after a quick deflation. First, by the fundamental nature of fortuitousness inherent to a FRBW and its implied Humeanism about laws, all causal issues must be reinterpreted in one of two ways: causation must either be a uniquely human feature of the world, or it must simply be a misrepresentation of correlations in the natural world. If causation is taken in the first way, it becomes a personal temporal asymmetry and thus it poses no threat to blockworld. If causation is interpreted the second way, then, in terms of causation, the phrasing of causal asymmetry must be exactly the same as the problem of fork asymmetry, which was resolved by an appeal to FRBW. No matter which way causal asymmetry must be interpreted, FRBW is able to deflate the problem of causal asymmetry in such a way that it is no longer a problem.

Thus, FRBW has explained away all possible threats to the blockworld based on temporal asymmetry. However, even if I have shown that the blockworld is not necessarily threatened by issues of temporal *asymmetry*, I have not explained on what basis one can advocate the opposite view, temporal *symmetry*. This project is one currently in the works by Aharonov and Tollaksen[2]. Aharonov and Tollaksen claim that they have formulated a new time-symmetric version quantum mechanics which is consistent with all of the experimental results of traditional quantum mechanics while also making new predictions concerning not-yet-observed phenomena. An evaluation of their results is not within the scope of this paper, but these claims align themselves well with SSC's assertion that abandoning the presentist view of time and its inherent time asymmetries will lead to new and better physical theories.

FRBW provides a framework consistent with RBW and GF that also yields results favorable to both interpretations as well as proponents of a time-symmetric quantum mechanics. I do not believe that the power of FRBW is limited to arguing against an objective asymmetry in time, however. Perhaps the unification of these two interpretations will suggest other important results in both the realms of philosophy and in physics. For now, however, I will leave the promise of FRBW for another time to discuss some criticisms of GF and the problems they pose.

8 Potential Problems with GF and RBW

Though both GF and RBW have been shown to possess great promise as interpretations of QM, both interpretations still have their critics. Since both interpretations are relatively new (The two most accessible papers on GF, [17] and [18], were published in 2004 and 2005 respectively, and none of the RBW papers were written before 2005), there is relatively little literature critiquing either of these perspectives explicitly: there is but one paper devoted to attacking GF, and there are no papers at the current time that explicitly critique RBW. My purpose in this section is to address both potential and actual critics of GF and RBW, addressing a paper by Mohrhoff[40] against GF directly as well as some other more general concerns raised by others. I will end the discussion of each interpretation's critiques with my own concerns which will hopefully suggest ways in which GF and RBW can be improved in the future.

8.1 Problems for GF

The only paper in the scientific literature at this time dealing with GF directly is Ulrich Mohrhoff's paper "Making Sense of a World of Clicks"[40]. In this paper, Mohrhoff generally agrees with some of GF's points and conclusions since they align well with his own Pondicherry interpretation of QM, but he does point out several problems he sees in GF as a theory⁵⁸. I will proceed in this section to explicate exactly what issues Mohrhoff takes with GF, after which I will attempt to answer Mohrhoff's concerns before proposing some concerns of my own.⁵⁹

Mohrhoff's first objection to GF is that it does not properly identify what phenomenon is genuinely fortuitous. Mohrhoff argues that Bohr and Ulfbeck take the value of a measurement event as genuinely fortuitous. For instance, a spin- $\frac{1}{2}$ particle could have spin up or spin down in the x-direction after measurement if it was originally in a spin-y eigenstate. However, Mohrhoff believes that a more fundamental process is genuinely fortuitous: not simply *which* value a measurement yields, but whether or not a measurement yields a value at all. For Mohrhoff, the mere assumption of a value in space-time by a matrix variable, to use the language of GF, is genuinely fortuitous, not just what value that matrix variable "assumes" in entering the space-time scene at all.

This question of whether the manifestation of a matrix variable in the space-time is genuinely fortuitous does not suggest a contradiction between GF and Mohrhoff to

⁵⁸It should be noted, to Mohrhoff's credit, that he proposes solutions to most of the problems he reveals in GF. While most of these solutions do suggest an assimilation of GF into his own interpretation of QM, they still seek to maintain the core of GF. Thus, Mohrhoff's critique should not be viewed as one by an enemy of GF who holds a diametrically opposed position to that of Bohr and Ulfbeck but rather as one of an ally seeking to rectify mistakes within a theory to improve it and make it more viable.

⁵⁹I should note here that I am not interested in a complete analysis of Mohrhoff's work since it plays up his Pondicherry interpretation of QM, an interpretation about which an entire thesis could be written. Thus, I will not address the subtleties of Mohrhoff's interpretation itself here but rather how his concerns with GF point out problems that supporters of GF need to address. This task will require that I change the phrasing of some of Mohrhoff's objections so that they bear more directly on the task of critiquing GF, but the main position of Mohrhoff as I articulate it will be essentially the same in character as Mohrhoff's articulation of his views.

me. Bohr and Ulfbeck do not appear to claim that Mohrhoff’s position about the genuinely fortuitous character of the “appearance” of matrix variables in space-time is false. For instance, they write[17]: “The click, by which a matrix variable manifests itself in the world of experience is a physical event of a novel character” (761). The “novel character”, Bohr and Ulfbeck go on to explain, is exactly the “lawlessness” of genuine fortuitousness itself. The above passage attributes such genuine fortuitousness to events themselves, which exist qua the act of a matrix variable manifesting itself in space-time. Thus, Bohr and Ulfbeck seem committed to the view that not only is the assumption of a space-time value by a matrix variable genuinely fortuitous but the very manifestation of a matrix variable within space-time is genuinely fortuitous as well. Bohr and Ulfbeck thus incorporate both Mohrhoff’s claim and the claim Mohrhoff attributes to them into their theory easily, making Mohrhoff’s objection null and void⁶⁰.

Having dealt with Mohrhoff’s first objection to GF, then, I move to his second. Mohrhoff argues that Bohr and Ulfbeck reject the terminology of indeterminate and indefinite variables in their discussion of GF. This is the first of many objections based on language that Mohrhoff levels against Bohr and Ulfbeck. Indefinite and indeterminate variables have been traditionally used to describe QM, and there is no mention of them in Bohr and Ulfbeck’s writing. Mohrhoff thinks that it would not be much trouble for Bohr and Ulfbeck to incorporate such entities into their theory of GF and that doing so would make GF more appealing and understandable to their general audience of physicists.

I do not believe that the fact that indeterminate and indefinite variables can be redefined in GF terms, however, necessitates that Bohr and Ulfbeck should undertake such a project of redefinition. After all, it seems that ordinary language is one of the largest barriers to understanding quantum phenomena. Bohr and Ulfbeck seek to distance themselves from “classical” notions they feel have infiltrated the newer quantum theories, so it would make sense that they would abandon the terminology used by such physicists and the potentially “classical” imagery it tends to invoke. The idea of an entity that exists without having a value would confuse the discussion of matrix variables that manifest on the space-time scene without values, and so it would serve no purpose to use the terms Mohrhoff prefers except to confuse the readers and detract from the project of GF. Thus, Mohrhoff’s second objection to GF can easily be discarded.

The third objection that appears in Mohrhoff’s writing on GF is that Bohr and Ulfbeck use vague language when they talk about matrix variables. For instance, how do matrix variables “emerge”? Where do they “come from”? How do they differ from one another? What does it mean for something to “manifest” but not “be present”, or “come into being” in the world of experience from some other place that is not “real” in their terminology? All of these questions arise because of the vagueness of Bohr and Ulfbeck’s language and their strange characterization of matrix variables, and it leads to much confusion as to what these matrix variables actually are and what properties they have.

⁶⁰It may be that Mohrhoff’s confusion on this issue was a result of Bohr and Ulfbeck’s confusing language about matrix variables, which Mohrhoff faults later in his paper

I believe Mohrhoff has an excellent point here. Bohr and Ulfbeck are extremely imprecise with their use of language. I read through Bohr and Ulfbeck's papers literally dozens of times before I was able to write even somewhat cogently about their matrix variables. These entities Bohr and Ulfbeck call matrix variables are hard to pin down and explain, but, since they form the crux of GF, they should be explained in more detail lest the very foundation of GF sink into obscurity. This objection of Mohrhoff's is one I support against GF as it stands, and I believe that GF must propose a more fleshed-out conception of matrix variables before the theory itself can proceed.

Playing off of this previous interpretation, Mohrhoff argues a fourth point against GF: Bohr and Ulfbeck's notion of reality seems inconsistent since GF claims that the only world is the "world of experience" but that there is at least some sense in which matrix variables "exist" outside of space-time. How can matrix variables exist in any sense if they are outside of space-time? In fact, how can there be a meaningful predicate "is outside of space-time" since the word "is" implies existence while the phrase "outside of space-time" implies non-existence? Mohrhoff suggests that Bohr and Ulfbeck abandon their view that reality is just "the world of experience" and instead pose the "existence" of matrix variables in terms of a possible/actual dichotomy; that is, matrix variables "exist" outside space-time in that it is possible that they will come to exist in the future and "exist" inside space-time in that they are actualized.

Mohrhoff's problem, as I pointed out in the previous section on realism in GF, is one that supporters of GF must address in some way. I believe that Bohr and Ulfbeck essentially take a strange version of verificationism that takes only what is inside space-time as "real" since only that can be observed (reality in this picture becomes simply a different way of phrasing "what we could potentially observe") but which is forced to accommodate the existence of entities outside of space-time, leading to inconsistent terminology. I believe that both Mohrhoff's possible/actual dichotomy and my own inside/outside space-time dichotomy are equivalent since any event outside space-time may potentially enter space-time just like every event in space-time is actual. Mohrhoff and I are essentially gesturing at the same distinction that Bohr and Ulfbeck need to make within their theory for it to be internally consistent, and I believe that, while Bohr and Ulfbeck do not need to explicitly endorse my solution to the problem or Mohrhoff's, they must find some way of explaining their conception of reality or else GF will seem intractable.

Most of the rest of Mohrhoff's paper after this fourth objection addresses how his Pondicherry interpretation of QM could rework GF to make it better, but these objections are too specific to Pondicherry to be addressed here. Before finishing his paper, however, Mohrhoff raises one final objection to GF in that he claims that GF is still holding onto some "classical" notions of physics in their formulation of QM. I believe that most of these "classical" notions that Mohrhoff alludes to are, in fact, a result of our "classical" language which we traditionally use to understand the world. Bohr and Ulfbeck, as has already been discussed, run into problems with terms like "exists" which become muddled when Bohr and Ulfbeck do not clearly define their terms. I believe that this objection of Mohrhoff's, that there are "classical" aspects that linger in the theory of GF, in the end simply serves to point out Bohr and Ulfbeck's imprecise use of language. This final objection, then, is simply, upon my reading, a reinterpretation of Mohrhoff's third objection about the use of language in GF.

The main critique that emerges from Mohrhoff's paper is singular in theme: Bohr and Ulfbeck are not suitably precise with the language of their theory, and thus they make certain assertions that seem strange, contradictory, or confusing. Bohr and Ulfbeck need to precisely characterize what they mean by the terms "matrix variable" and "reality" for GF to make sense as a physical theory since right now their language use is so ambiguous that neither Mohrhoff nor myself are able to determine all of the entailments of these two ideas. Bohr and Ulfbeck may be on the right track with their GF interpretation of QM, but until they pin down the foundational ideas of their theory in such a way that seems non-contradictory and less obscure and obtuse, it is doubtful that GF will be able to do much physical or philosophical work.

There are objections of a different variety, however, that Mohrhoff has not mentioned, perhaps because his interpretation of QM relies upon them too: objections to the conclusions Bohr and Ulfbeck draw from their derivation of the CCR from relativity and their derivation of the density matrix. The criticisms I raise of these two derivations do not regard their mathematics or methodology but rather challenge the fact that they prove what Bohr and Ulfbeck take them to prove.

First, I will address the CCR derivation. It is clear that special relativity (SR) leads to the CCR, thus revealing the "footprint" of relativity in non-relativistic quantum mechanics (NRQM); however, one wonders if the nature of some of the approximations in this derivation do not suggest that the general claim that "NRQM comes from SR" is too general a conclusion to draw. First, it is worth noting that the framework of SR is obeyed by events on small scales; however, on larger scales, such as the size of the universe, SR is insufficient and the laws of GR are necessary to explain relativistic phenomena. SR, then, is a kind of local approximation of GR; what is more, the CCR proof utilizes not only SR but an approximation of SR that takes length contraction and time dilation to first order only⁶¹. This is a kind of "second order" relativistic approximation, the "first order" approximation being the one from GR to SR. With so many approximations, it seems that Bohr and Ulfbeck's claim that the laws of QM emerge from the "coarse-graining" of the relativistic laws of the universe makes sense. This all follows Bohr and Ulfbeck's conclusion nicely.

However, why should one not rather argue that it is the laws of QM which constrain the algebra of the Poincaré group and thus shape the geometry of space-time? Bohr and Ulfbeck, so ready to abandon causation, seem to be eager to read into Kaiser's link between QM and SR that one is ontologically prior to the other by stating that the laws of QM emerge as a result of SR. However, could there not be just as easily laws that govern the motion of particles on the quantum scale that likewise determine the geometry of space-time on that scale? Bohr and Ulfbeck may have proven their link between SR and QM, but they have not proven that either SR or QM necessarily implies the other. It could be that all of the approximations just discussed for SR are justified by the laws of QM; because the particles are described by NRQM, the low-velocity limit applies, and because quantum laws govern small scales and astronomical laws large scales, SR is preferred to GR. It is possible to reformulate the discussion, then, as the emergence of relativity from QM, and as such, Bohr and Ulfbeck should support their conclusion that the link between SR and NRQM implies that SR *leads*

⁶¹This is the low-velocity limit

to QM instead of merely asserting the latter⁶².

A similar argument may be leveled against Bohr and Ulfbeck's derivation of the density matrix. The density matrix is a mathematical object whose form can be derived (using suitable limits and approximations) from a general law of algebra as shown by Bohr and Ulfbeck; thus, the language of relativity, algebra, can be used to derive an object of QM, the density matrix. However, does this derivation show that these laws of algebra are prior to the mathematical object that can be derived from them, or does it simply show that the formulation of the algebra of Minkowski space is similar to the formulation of the algebra of Hilbert space? This is reminiscent of the argument wherein a mathematical object that is essential to a non-Euclidean space can be derived from the structure of Euclidean space if one takes the proper limits. Does this prove that Euclidean space is ontologically prior to non-Euclidean space? No. Rather, the derivation shows that there is some way of mapping a Euclidean space to a non-Euclidean space. Thus, the spaces must be connected in some way, but it is not clear that one space must be "prior" in some way to the other. Thus, the same argument leveled against Bohr and Ulfbeck's CCR derivation can be leveled against their density matrix derivation as well since the mere connection between QM and SR does not necessitate that one "causes" the other⁶³.

Thus, there are two important general criticisms one might levy against Bohr and Ulfbeck's GF: first, that their language needs to be more precise for their interpretation to be coherent, and secondly, they need to explain why the correlations between SR and QM in their derivations of the CCR and density matrix justify assigning some sort of ontological priority to SR over QM. I do not believe that either of these criticisms is fatal to GF since the language can be reworked while preserving the content of the theory and since I believe that some at least fairly basic reasons to believe that a connection between QM and SR implies SR's ontological priority exist (see above footnotes), but I do believe that these are two criticisms supporters of GF need to address if GF is expected to stand alone as an interpretation of QM.

8.2 Problems for RBW

One might suspect that RBW is even more susceptible to criticism than GF since the work of RBW builds upon GF; however, though I believe that some of GF's problems do carry over into RBW, I believe that RBW is less open to criticism than GF. I will analyze how RBW deals with the criticisms leveled against GF (that is, the imprecision of language and support for the ontological priority of SR) to find that RBW is more

⁶²I do not mean in this section to argue that there is any better reason to believe that QM leads to SR than that SR leads to QM; in fact, there are other reasons, such as the fact that geometry provides a more satisfying explanatory foundation than do the laws of QM with their tenuous interpretations, that suggest to me that SR should describe QM and not the other way around. All I am asking for is for Bohr and Ulfbeck to explain their reasoning and make their logic explicit.

⁶³Once again, I believe that Bohr and Ulfbeck could cite good reasons for why one should see such a connection as causal since there does not appear to be a proof of relativistic principles or objects from the laws of QM alone, and such asymmetry seems to suggest causation; however, once again my point here is only that Bohr and Ulfbeck do not make such an assertion explicit and need to do so to draw their conclusions

secure than GF with regard to these criticisms. I will conclude that, though RBW may still have issues it needs to work out, there are no major criticisms that need to be addressed by RBW before it can be taken as a consistent interpretation of quantum mechanics

The first question is whether or not RBW leads to the kinds of confusion with language that GF suffers from, and the answer seems at first to be “no”. Unlike GF, RBW does not need to posit such entities as matrix variables, and it is difficult to find anything in the writings of SSC (despite the fact that they wrote over twice as many papers on RBW as Bohr and Ulfbeck wrote on GF) that is presented as obliquely as GF’s matrix variables. However, RBW still runs the risk of imprecision when dealing with how one defines “reality” just like GF does. An imprecise definition of “reality” in RBW could be doubly deadly since the presentist/eternalist distinction hinges on such a definition, as pointed out by recent philosophers of language (See Dorato[24] and Savitt[48] for examples). Some philosophers of language like the ones just referred to go so far as to say that the presentist/eternalist distinction does not even make sense, though this claim clearly depends on how one defines “reality”. Though SSC do not define reality in their main RBW papers, Michael Silberstein and I have recently argued against the philosophers of language and presented a definition for reality and “what is real” in RBW in terms of definiteness and determinacy inside of space-time[43]. Thus, now that “reality” has been pinned down in the context of RBW, there are very few if any grounds upon which one could object to RBW due only to SSC’s language use.

One might more easily criticize RBW for another fault of GF’s: the failure to draw the connection between the ontological priority of SR and a link between SR and QM. SSC go through Bohr and Ulfbeck’s derivations of the CCR and the density matrix without pausing at the conclusion to make this connection explicit, and thus one seems as justified objecting to RBW on these grounds as to GF. There is, however, one reason that this jump from connection to causation can proceed unaccounted for in RBW but not in GF: because SSC are unapologetically Humean in their treatment of dynamical laws and because they invoke the blockworld view that posits the geometry of space-time as ontologically prior to the physical laws within space-time⁶⁴, there are plenty of reasons for SSC to reject the thesis that QM should be prior to SR. In fact, SSC could simply cite Anandan and his treatment of dynamical laws to establish that SR should be ontologically prior to QM and not the other way around. Thus, though SSC do not explicitly state why the SR/QM connection directly implies the ontological priority of SR in the derivations previously cited, they provide enough of a rationale for why they would make such a move in the rest of their work that RBW avoids the criticism that plagues GF on this issue.

Thus, RBW has avoided the criticisms that GF falls prey to. I do not believe that RBW is unassailable just because it has avoided GF’s pitfalls, but I do believe that it is in a good position to be taken as a consistent interpretation of QM that holds great promise for future formulations of quantum theory and relativity. There are no major criticisms that I can bring to the table against RBW at this point, and so I conclude that RBW’s foundations are on solid ground and that SSC’s reasoning in RBW follows

⁶⁴As opposed to the Neo-Lorentzian view of Brown and Pooley[19] who posit that the laws of space-time are ontologically prior to the geometry and remain agnostic as to the nature of space-time; see Peterson and Silberstein[43] for more details

excellently. I believe that GF, too, can attain this status should its issues be sorted out appropriately, and such a clarification of GF would perhaps make the assimilation of GF into RBW to form the FRBW even more favorable, but for the time being GF needs to sort out some of its language issues, among other things, before it can be taken as a fully consistent interpretation of QM.

9 Possible Research into and Consequences of GF and RBW

Logical analysis of the exposition and argumentation of GF and RBW aside, what comes to bear most importantly when deciding which interpretation of QM to adopt is how well an interpretation of QM conforms to experimental data. As has already been shown, both GF and RBW account for all of the most recent experimental results of NRQM; however, if one of these interpretations makes predictions which differ from those of other QM interpretations and which can be verified through experimentation, this interpretation will stand on even firmer ground. As pointed out by Czyz[21], GF makes such a prediction, though RBW does not seem to be equally verifiable. Both interpretations can also be analyzed in terms of their promise for the creation of other physical theories, however, and as such I will proceed to analyze both the predictive and theoretical promise of GF and RBW.

I will begin my discussion with the only interpretation here examined that yields experimentally verifiable predictions: GF. As Jerzy Czyz points out, GF takes uncertainty as a fundamental feature of the universe. The laws of QM in GF arise only because of the coarse-graining of the macroscopic world. Should this coarse-graining be broken down by increasing the resolution and quality of the instruments used in experimentation, Czyz argues, the fortuitous character of microscopic events should appear. If measurement events follow each other to fractions of a section (approximately 10^{-17} of a section, according to Czyz), the breaking of the laws of QM should be noticeable, thus supporting the claims of Bohr and Ulfbeck.

Despite the fact that it is possible to experimentally verify the predictions of GF in the future, however, there are still some loopholes that GF can escape through even if Czyz's predictions for GF don't turn out to obtain. First, Bohr and Ulfbeck can always argue that the microscopic world is much smaller than we ever considered it could be. For instance, if time scales of 10^{-17} do not give results that confirm GF, Bohr and Ulfbeck (and possibly Czyz) could always argue that these results will pop out at even smaller time scales, such as 10^{-18} of a second. In short, supporters of GF could always keep pushing back the resolution necessary to observe genuinely fortuitous behavior, meaning experimental results may only be able to confirm GF, not disconfirm it.

Another potential escape from bad experimental results open to GF is the fact that all means of reporting observational data about the quantum world require a process of macroscopic translation. For instance, Bohr and Ulfbeck point out that a detector click is itself composed of millions of tiny events within the detector. The "macro click" follows a certain law, but the "micro click", the genuinely fortuitous event, does not. Thus, a proponent of GF could argue that there is no way that we could ever observe a genuinely fortuitous event. Czyz assumes that the act of resolving a microscopic event can convey the necessary fortuitous information about this event to us without the genuine fortuitousness of the event being lost in translation, and he may be right; however, if GF's predictions according to Czyz are wrong, this may not put GF in any worse a position since all this experiment would "support" to Bohr and Ulfbeck is that genuinely fortuitous events are undetectable on a macroscopic scale since coarse-grain reporting ruins fine-grained phenomena.

GF, then, should not gloat in that it, unlike RBW, is experimentally verifiable

since GF can be supported but not refuted on the basis of experimental evidence. Both interpretations, however, can be judged on a different basis: how well thinking about the world in accordance with GF or RBW may help resolve some of the current issues in physics. As interpretations of QM and not physical theories in their own right, GF and RBW may be able to suggest different ways of approaching certain problems. If physical theories can be built from the framework of GF or RBW which help solve some of the issues currently plaguing physicists, this will lend credence to GF's and RBW's claims that these theories are the best way to think about the physical world.

There exists one major problem in modern physics that both GF and RBW may provide the correct framework for addressing: the problem of fine-tuning. The problem of fine-tuning begins with the fact that there are numerous equations and physical theories that rely on very specific values for certain parameters. One thinks of Planck's constant or the cosmological constant, for instance, as such important parameters. However, though experimental evidence shows that these parameters are correct, there is no reason physicists can derive from first principles for many of these parameters to have the values they do.

The general approach to solving fine-tuning problems has traditionally been to attempt to derive any one of these parameters from more general laws; however, to date, this approach has not produced any definitive results. GF and RBW, however, suggest a different approach to fine-tuning: instead of taking each parameter's value by itself and trying to motivate it alone from first principles, why not assume that these values are interconnected? Starting from nowhere and trying to derive a single parameter's value from first principles gets one nowhere, so why not start with the values of other parameters and go from there? To ignore the fact that other parameters have values would be to ignore the interconnectedness of the universe which both GF and RBW stress through relational reality. Thus, though such inquiries may never provide one with reasons to believe in the value of a parameter based solely on first principles, it may only be through the appreciation of the relationality of these parameters that one can come to any reason to believe that our values for these constants are correct aside from empirical evidence⁶⁵. It seems that, if such justification of parameter values exists and is the only one we can ever come to, that the relational reality espoused by both GF and RBW will be strongly supported.

RBW may yield promise in another important field, however: the field of quantum gravity (QG). For decades now physicists have searched for a way to quantize the gravitational field to no avail. Though string theory and other such popular contemporary physical theories may promise that they have solved QG, none of these theories can be experimentally verified with equipment available today. Though some have faith that the problem of QG will be solved by string theory or quantum loop gravity, others are

⁶⁵There seems to be an obvious analogue here between the fine-tuning problem and correspondence/coherence theories of truth and knowledge. A correspondence theory of truth holds that something is true if it agrees with the way that something is in the outside world. This theory is similar to the traditional approach to the fine-tuning problem since it involves looking outside of other "known truths" to the "things in themselves". A coherence theory of truth, however, holds that something is true if it agrees with other things one holds to be true. Likewise, the GF/RBW approach to the fine-tuning problem justifies parameter values by other parameter values, creating an interconnected web instead of a hierarchy of derivation.

less optimistic and are looking for places where physics has gone wrong to correct our mistakes in hopes of finding a solution to QG from such correction.

RBW proposes such a correction to our underlying assumptions about physics by challenging the way physicists think about time. If there is no special “present” moment or inherent temporal asymmetry to the universe, it is possible that, upon correcting our view of time, the solution to QG becomes apparent. Lee Smolin[50] holds this view. He writes:

“Time is nothing but a measure of change—it has no other meaning. Neither space nor time has any existence outside the system of evolving relationships that comprises the universe. Physicists refer to this feature of general relativity as *background independence*. By this we mean that there is no fixed background, or stage, that remains fixed for all time. In contrast, a theory such as Newtonian mechanics or electromagnetism is *background dependent* because it assumes that there exists a fixed, unchanging background that provides the ultimate answer to all questions about where and when.

One reason why it has taken so long to construct a quantum theory of gravity is that all previous quantum theories were background dependent. It proved rather challenging to construct a background independent quantum theory, in which the mathematical structure of quantum theory made no mention of points, except when identified through networks of relationships.” (24-5)

By recognizing the true nature of time as RBW portrays it, Smolin’s “background independence” becomes actualized. The RBW view removes space-time from Newtonian ideas of some “absolute” framework upon which the laws of the world are supposed to rest. If Smolin is right, then RBW may be the first step towards comprehensive theory of QG and the blockworld perspective will be vindicated.

Thus, while there may not be any hard, concrete method to test both GF and RBW such that either one is proven definitively right or wrong, we can examine what work emerges from these two interpretive frameworks of space-time to see whether they lead to proposals of viable solutions to the fine-tuning problem and QG. It is possible that, in confronting these problems, both GF and RBW will produce better physical theories which resolve some of the biggest problems in the physics of our day. There is also the possibility, however, that these interpretations of QM are both on the wrong track and the solutions to the fine-tuning problem and QG, if they even exist, must be found elsewhere. It is now up to the theoretical physicists to incorporate the ideas of GF and RBW into their more concrete physical theories and for the experimental physicists to test them, for such a process remains as the only way to further support or refute the claims of GF and RBW.

10 Conclusion

Following this in-depth inquiry into GF and RBW, what has become apparent about these two interpretations of QM and SR? The first conclusion one can draw from the discussion of this paper is that both GF and RBW stand on solid physical reasoning. As sections 3 and 5 of this paper show, the physical arguments both GF and RBW invoke to prove their position lead to the conclusions that both interpretations suggest. The reasoning of RBW is so sound that even a phenomenon that might be used to disprove the theory, the quantum liar paradox (QLP), can be explained by RBW and thus provides another reason to believe RBW instead of doubt it. Finally, section 7 shows that many of the philosophical views about reality, time, and indeterminacy espoused by GF and RBW are ones shared by modern philosophers of physics. What is more, my invocation of the FRBW in section 7 suggests that GF and RBW could be combined in such a way as to solve the problem of temporal asymmetry, one of the many hotly-debated issues in the philosophy of time. In short, it seems that the most resounding conclusion one can draw from this discussion of GF and RBW is that both interpretations seem to be not only well-supported but compatible.

Despite all of these reasons to support GF and RBW, however, section 8 shows that there is still some clarifying work to be done (by GF especially) before GF and RBW are likely to be accepted as viable reinterpretations of Copenhagen QM. Both GF and RBW may lead to some confusion because modern language, based on human perceptions of time and existence, is ill-suited to discussing physical theories that take the true nature of time and reality to behave differently than we are inclined to believe. What is more, both GF and RBW could provide more explicit support for the causal connection they see between the geometry of SR and the laws of QM. However, both of the flaws I see in GF and RBW may be corrected, allowing both of these interpretations of QM to stand as viable alternatives to the Copenhagen interpretation of QM.

Returning to the problem of QG and the fine-tuning problem previously explicated, will GF or RBW, well-founded as they are, provide the solution to these problems of contemporary physics? Section 9 shows that such GF and RBW solutions to these problem are indeed possible, but one cannot yet know for certain whether either interpretations potential will be fully actualized in the future. The weight of such support falls on the shoulders of more concrete and physical theories of physics than the interpretations of QM and SR discussed here. Thus, judgement cannot be passed on the GF or RBW interpretations until such concrete theories are proposed and tested at the earliest.

In conclusion, despite the fact that both of these interpretations involve seemingly nonsensical conclusions about the non-existence of matter, the collapse of the matter/structure dichotomy, and the nature of time and causality, both interpretations are well-reasoned and present promising re-conceptualizations of QM that may provide a viable alternative to string theory for solving the problems of QG and fine-tuning. We must await further evidence before casting final judgement on either interpretation, but for now it seems sufficient to conclude that both GF and RBW provide as promising and useful an interpretation of QM as can be found anywhere in contemporary physics.

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12 Glossary of Philosophical Terminology

I have recently been informed that, to the greater physics audience to whom this paper is meant to appeal, many of the philosophical terms I frequently use in this paper may be confusing. Thus, I have included this short glossary of philosophical terms to help those who may be unfamiliar of my terminology. These are only terms whose definitions do not appear in the text of my paper already.

Epistemic: Fundamental, but based on one's own state of knowledge. Sometimes, this term is conflated with the term "epistemological". It is contrasted with "ontic".

Epistemological: Adjectival form of epistemology. Having to do with the nature and scope of one's knowledge. Usually compared with and contrasted to ontological.

Epistemology: The study of the nature and scope of knowledge.

Humeanism: When in reference to laws, Humeanism refers to treating laws as not causally and fundamentally real. There are other more recent implications of Humeanism in the metaphysics of causality, but I will neglect those here.

Ontic: Both fundamental and real, in contrast to "epistemic".

Ontological: Adjectival form of ontology. Having to do with reality, the nature of being, or the way things "really are". Usually compared with and contrasted to epistemological.

Ontology: The study of reality and the nature of being.

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