

Assignment 4

Special and General Relativity

Readings

Kogut ----- Chapter 5: Spacetime Measurements

Ellis/Williams - Chapter 3.5-3.6: Measurements in Flat Spacetime

Appendix B1-B3: 4-Vectors

Problem Assignments

Everyone Problem Assignments(hand in at start of seminar)

Ellis 3.20 Car in the garage paradox?

Ellis 3.21 Causal paradox?

Ellis B2 Apply transformations

Ellis B7 4-velocity to velocity addition

EP 5 Can you save them?

EP 6 Star Wars

Individual Problem Assignments

Ellis 3.18 Crossing the chasm? _____

Ellis 3.19 Pion decay _____

Ellis 3.22 Tied rockets _____

Ellis B1 Inverses _____

Ellis B3 Inverse transformations _____

Ellis B4 Inverse transformations _____

Ellis B6 4-velocity _____

Kogut 5.2 Tricks your eyes can play _____

EP 7 Strange World of Relativity _____

EP 12 Pole in the Barn Paradox _____

EP 13 Faster than light? _____

Subject Presentations (5 minutes)

Hole in the ice _____

Tied rockets _____

EP-5 Can you save them? In 2095 a message arrives at earth from the growing colony at Tau Ceti (11.3 y from earth). The message asks for help in combating a virus that is making people seriously ill (the message includes a complete description of the virus genome). Using advanced technology available on earth, scientists are quickly able to construct a drug that prevents the virus from reproducing.

You have to decide how much of the drug can be sent to Tau Ceti.

The space probes available on short notice could either boost 200 g of drug (in a standard enclosure) to a speed of 0.95, 1 kg to a speed of 0.90, 5 kg to a speed of 0.80, or 20 kg to a speed of 0.60 relative to the earth.

The only problem is that a sample of the drug in a standard enclosure at rest in the lab is observed to degrade due to internal chemical processes at a rate that will make it useless after 5.0 y.

Is it possible to send the drug to Tau Ceti? If so, how much can you send?

EP-6 Star Wars

The Federation space cruiser Execrable is floating in Federation territory at rest relative to the border of Klingon space, which is 6.0 light-minutes away in the +x direction. Suddenly, a Klingon warship flies past the cruiser in the direction of the border at a speed of $v=0.6c$. Call this event A and let it define time zero in both the Klingon and cruiser reference frames. At $t_B=5.0$ min according to the cruiser clocks, the Klingons emit a parting disrupter blast (event B) that travels at the speed of light back to the cruiser. The disrupter blast hits the cruiser and disables it (event C) and a bit later (according to cruiser radar measurements) the Klingons cross the border into Klingon territory (event D).

- (a) Draw a space-time diagram of the situation, taking the cruiser to define the rest frame and the Klingon warship to define the moving frame. Draw and label the worldlines of the cruiser, the Klingon territory boundary, the Klingon warship, and the disrupter blast. Draw and label events A, B, C, and D as points on your diagram.

- (b) When does the disrupter blast hit and when do the Klingons pass into their own territory, according to clocks in the cruiser's frame. Answer by reading the time of these events directly from the diagram AND from using Lorentz transformations.
- (c) The Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up in interstellar court, the Klingons claim that they are within the letter of the law: according to measurements made in their reference frame, the damage to the Execrable occurred after they had crossed back into Klingon territory: thus they were not in Federation territory at the time. Did event C (disrupter blast hits the Execrable) really happen after event D (Klingons cross the border) in the Klingon's frame? Answer this question using the spacetime diagram and confirm the result using Lorentz transformations.

EP-7 The Strange World of Relativity (problem 2.7 again)

Solve this problem with the Lorentz transformation equations and with a spacetime diagram.

At noon a rocketship passes the earth with a velocity of $0.8c$. Observers on the ship and on earth agree that it is noon.

- (a) At 12:30 PM as read by a rocketship clock, the ship passes an interplanetary navigational station that is fixed relative to the earth and whose clocks read earth time. What time is it at the station?
- (b) How far from earth (in earth coordinates) is the station?
- (c) At 12:30 PM rocketship time the ship reports by radio back to earth. When (earth time) does the earth receive the signal?
- (d) The station on earth replies immediately. When (by rocket time) is the reply received?

EP-12 Two farmers have a barn which is 10 meters long in their rest frame (unprimed). The farmers are standing at the left and right doors of the barn (the doors are open).

A pole carrier has a pole of length 12 meters in her rest frame and is carrying it horizontally while she runs towards the barn with a speed given by $\beta=0.80$. This means that .

If we believe all this relativity and length contraction stuff, then the farmers think the pole is $\gamma=1.67$.

$$L_{pole} = \frac{L'_{pole}}{\gamma} = 9.8 \text{ meters}$$

However, the pole carrier thinks the barn is only

$$L'_{barn} = \frac{L_{barn}}{\gamma} = 8.0 \text{ meters}$$

This means that, according to the farmers, the pole should be able to fit into the barn. The pole carrier, however, say no way, the barn is much too small.

Use a spacetime diagram to show that **both** sets of observers are correct; that they just tell different stories about the same set of events. Why are their stories different?

EP-13 Faster than Light

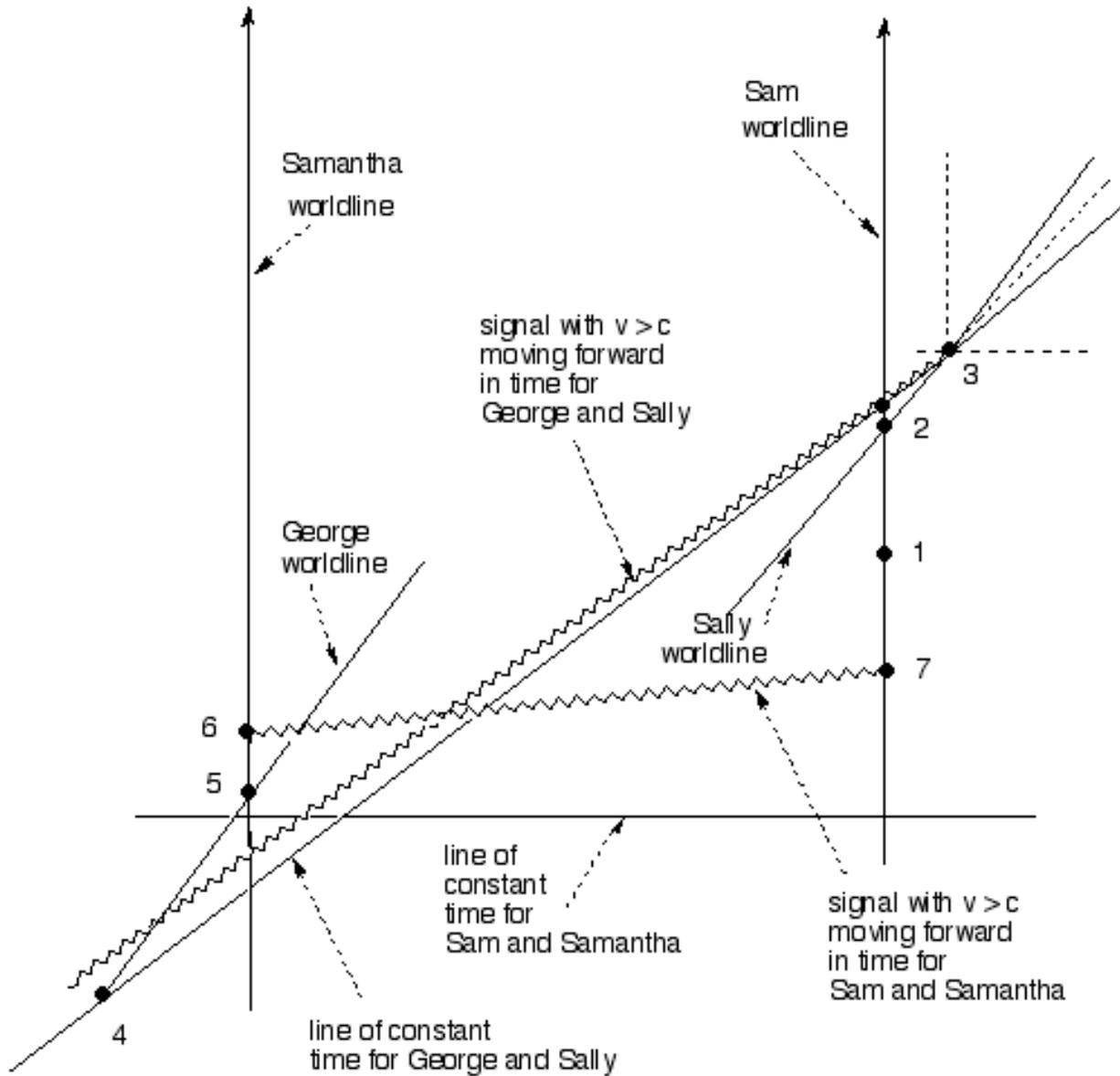
What happens if we allow some signal to go faster than the speed of light?

Consider the following story.

Sam is walking down the path towards Sharples. As he passes near Clothier tower a stone block falls off the tower and lands on his head, killing him. So Sam is now lying in heap at the base of Clothier tower. Soon after that incident, Sally comes along. Sam is Sally's good friend and she is distraught when she sees Sam lying in a heap. Sally is walking past Sam with some speed (she is in a different frame of reference). Now, Sally understands Special Relativity. Sally has in her possession a special device that can send a signal to someone on the other side of the universe at a speed $> c$ if they are in the same frame of reference. So Sally sends out a signal indicating what happened to Sam. The signal is received on the other side of the universe by George (in the same frame of reference as Sally). He is now desperate to tell Sam so he can avoid the stone block, but Sam is in a different frame and cannot receive his signal. So he tells the story to someone in Sam's frame, namely,

Samantha. Samantha also happens to have one of those devices that sends the speedy signal and she sends a signal to Sam.

The entire sequence of worldlines with the associated events is shown in the diagram below:



The events are:

- event #1 - Sam gets killed
- event #2 - Sally sees Sam
- event #3 - After patiently waiting Sally sends a $v > c$ signal to George

- event #4 - George receives the signal
- event #5 - George tells Samantha
- event #6 - Samantha patiently waits and then send a $v > c$ signal to Sam
- event #7 - Sam receives the signal from Samantha, realizes he is about to die and stops walking, thus avoiding the block and subsequent death

Questions:

If Sam is not dead, why would Sally send any signal?

If Sally does not send a signal making all the other stuff happen,
then why would Sam stop?

If Sam has no reason to stop, he then gets killed and Sally has a reason to send the signal.

Which is it?

We have what is called a **closed causal loop** here. There is no logical way out of this loop.

Does that mean it cannot occur, i.e., that no signal can travel faster than light?

or

Is there some other explanation?