High Energy Particle Physics

A special reference frame is the center of mass or zero momentum system frame. It is very useful when discussing high energy particle reactions.

We consider a collision between two particles with rest masses m_1 and m_2. We assume that particle 1 is moving with velocity $\vec{u}$ in the laboratory system and that particle 2 is at rest in that system. We have the energy-momentum 4-vectors

$$\tilde{p}_1 = \left(\frac{E_1}{c}, p_1, 0, 0\right) \quad \text{and} \quad \tilde{p}_2 = \left(\frac{E_2}{c}, 0, 0, 0\right)$$

and the total energy-momentum

$$\tilde{P} = \tilde{p}_1 + \tilde{p}_2 = \left(\frac{E_1 + E_2}{c}, p_1, 0, 0\right)$$

In a new frame moving along the x-axis with speed V we have

$$P'_1 = \Gamma\left(p_1 - \frac{V}{c}\frac{E_1 + E_2}{c}\right) \quad , \quad P'_2 = 0 \quad , \quad P'_3 = 0$$

where $\Gamma = \left(1 - \frac{V^2}{c^2}\right)^{-1/2}$.

In the center of mass system, $V = V_{CM}$ and $\vec{P}' = 0$. This says that

$$V_{CM} = \frac{p_1 c^2}{E_1 + E_2}$$

The energy available for physical processes such as the production of new particles or inelastic events is the total energy in the center of mass system, E'. In the center of mass system the total energy-momentum 4-vector is

$$\left(\frac{E'}{c}, 0, 0, 0\right)$$

We can find E' by using the fact that the norm of the energy-momentum 4-vector is invariant

$$\left(\frac{E'}{c}\right)^2 = \left(\frac{E_1 + E_2}{c}\right)^2 - p_1^2$$

or

$$E'^2 = E_1^2 + E_2^2 + 2E_1E_2 - p_1^2c^2 = E_1^2 + E_2^2 + 2E_1E_2 - \left(E_1^2 - m_1^2c^4\right)$$

$$= m_1^2c^4 + 2E_1E_2 + E_2^2$$

We have

$$E_1 = \gamma m_1 c^2 \quad \text{and} \quad E_2 = m_1 c^2 \qquad , \qquad \gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

Therefore

$$E = \left(\gamma m_1 + m_2\right)c^2 = \text{total energy in laboratory system}$$

and

$$E' = \left(m_1^2 + m_2^2 + 2\gamma m_1 m_2\right)^{1/2} c^2$$

The fraction of energy available for physical processes is

$$\frac{E'}{E} = \frac{\left(m_1^2 + m_2^2 + 2\gamma m_1 m_2\right)^{1/2}}{\gamma m_1 + m_2}$$

For the special case $m_1 = m_2 = m$ we have

$$\frac{E'}{E} = \sqrt{\frac{2}{1+\gamma}}$$

At low velocity or low energy of the incident particle (the one that is moving), we have

$$\gamma \approx 1 \to \frac{E'}{E} = 1 \to \text{all energy available}$$

In this case, most of the energy is rest energy and kinetic energy is unimportant. In the high speed or high energy limit we have

$$\frac{E'}{E} = \sqrt{\frac{2}{1 + \frac{E_1}{mc^2}}} \to \sqrt{\frac{2mc^2}{E_1}}$$

Thus, the useful fraction of energy decreases as $E_1^{-1/2}$. For example, in a 300 GeV accelerator (1 GeV = $10^9\ eV = 10^9 \times 1.6x10^{-12}\ J = 1.6x10^{-3}\ J$) an accelerated proton ($mc^2 \approx 1\,GeV$) colliding with a hydrogen target(protons) has

$$\frac{E'}{E} = \to \sqrt{\frac{2}{300}} = 0.082$$

or only 25 GeV is available for reactions!!! We will show how to fix this up shortly.

Let us look at **production reactions** in another way. Suppose that we have two particles that interact with each other(one is at rest -- the target) and produce N final particles. The high energy available from the incident particle is converted into mass of newly created particles. We ask the question: What is the minimum energy needed by the incident particle in order to produce the final state of N particles?

In the initial state we have

$$\left(\frac{E_{inc}}{c}, p_{inc}, 0, 0\right) + \left(m_{\text{target}}c, 0, 0, 0\right) = \left(\frac{E_{inc}}{c} + m_{\text{target}}c, p_1, 0, 0\right)$$

$$E_{inc}^2 = p_{inc}^2 c^2 + m_{inc}^2 c^4$$

In the final state we have

$$\left(\frac{\sum_{i=1}^{N} E_i}{c}, \sum_{i=1}^{N} \vec{p}_i \right) \quad \text{where} \quad E_i^2 = p_i^2 c^2 + m_i^2 c^4 \quad , \quad \text{i} = 1,2,3,4,........,\text{N}$$

Now, the norm of the energy-momentum 4-vector is invariant in time and across different frames. Therefore

norm in laboratory before = norm in center of mass after

This gives

$$\left(\frac{E_{inc}}{c} + m_{\text{target}} c \right)^2 - p_1^2 = \left(\frac{\sum_{i=1}^{N} E_{i,CM}}{c} \right)^2 - \left(\sum_{i=1}^{N} \vec{p}_{i,CM} \right)^2$$

By definition, however, $\sum_{i=1}^{N} \vec{p}_{i,CM} = 0$. After some algebra we have

$$E_{inc} = \frac{\left(\sum_{i=1}^{N} E_{i,CM} \right)^2 - \left(m_{\text{inc}} c^2 \right)^2 - \left(m_{\text{target}} c^2 \right)^2}{2 m_{\text{target}} c^2}$$

This is a minimum when $\sum_{i=1}^{N} E_{i,CM}$ is a minimum or when

$$\sum_{i=1}^{N} E_{i,CM} = \sum_{i=1}^{N} m_i c^2$$

or all the particles are at rest in the center of mass system after the collision (what are they doing in the laboratory system).

Therefore the minimum energy needed by the incident particle (this is called the **threshold energy**) is

$$E_{inc,threshold} = \frac{\left(\sum_{i=1}^{N} m_{\text{i}} c^2 \right)^2 - \left(m_{\text{inc}} c^2 \right)^2 - \left(m_{\text{target}} c^2 \right)^2}{2 m_{\text{target}} c^2}$$

For example, consider the reaction $p + p \rightarrow p + p + \pi + \pi + \pi$ where a proton is incident on another proton producing two protons and three pi mesons. The threshold energy is

$$E_{p,threshold} = \frac{\left(2m_p + 3m_\pi \right)^2 - 2m_p^2}{2m_{\text{p}}} c^2 = \left(m_{\text{p}} + 6m_\pi + \frac{9}{2} \frac{m_\pi^2}{m_{\text{p}}} \right) c^2$$

Clearly, this is a very non-intuitive answer!!!

Now let us consider the difference between a particle accelerator where one particle is accelerated and collides with a second particle at rest (as above=laboratory system) and two particle accelerators where each particle is accelerated in the same way (colliding beams=center of mass system). We have

Single Accelerator

$$\left(\frac{E_{\text{total lab}}}{\text{c}}, \vec{p}_{\text{total lab}}\right) = \left(\frac{E_1 + m_2 c^2}{\text{c}}, \vec{p}_1\right) \quad , \quad E_1^2 = p_1^2 c^2 + m_1^2 c^4$$

Colliding Beams

$$\left(\frac{E_{\text{total cm}}}{\text{c}}, \vec{p}_{\text{total cm}}\right) = \left(\frac{2E}{\text{c}}, 0\right) \quad , \quad E = \text{energy of each particle}$$

In the first case the accelerator must produce energy E_1 and in the second case each accelerator must produce energy E.

The two accelerators are equivalent (same energy available for physical processes) if

$$\left(\frac{E_1 + m_2 c^2}{\text{c}}, \vec{p}_1\right)^2 = \left(\frac{2E}{\text{c}}, 0\right)^2$$

Algebra gives the result

$$E = \frac{1}{2}\sqrt{m_1^2 c^4 + m_2^2 c^4 + 2 m_2 c^2 E_1}$$

If we consider the case of very high energy accelerators where $E_1 >> m_i c^2$ we have

$$E = \frac{1}{2}\sqrt{2 m_2 c^2 E_1}$$

Suppose we want to build a single 10 TeV accelerator ($1\,TeV = 10^3\,GeV$) so that $E_1 = 10^4\,GeV$. This is very difficult to design and requires the development of significant new equipment ($$$$$$$).

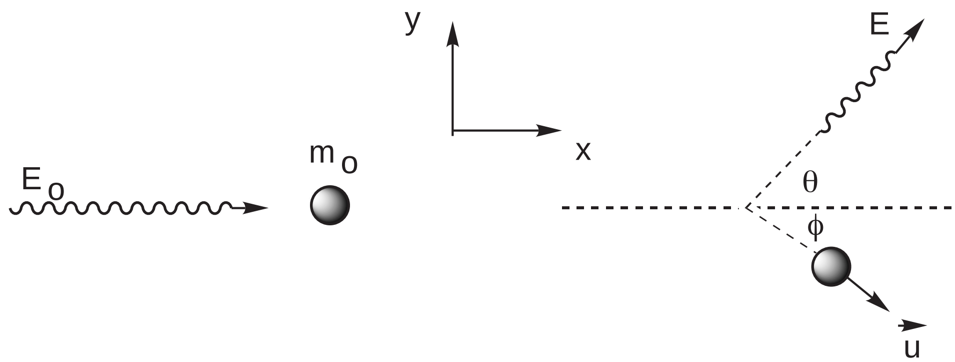
If instead we build two smaller accelerators and use them in the colliding beams configuration, then we get the same available energy with

$$E = \frac{1}{2}\sqrt{2E_1} = \sqrt{5000} = 71\,GeV$$

which we already know how to build. In fact, if we use an old single accelerator of this size that already exists, we then only have to build one small new accelerator ($$).

High Energy Collisions

Earlier we discussed low energy collisions between particles using conservation of energy and momentum. Let us look at the same processes at high energy. We consider a collision in which the incident particle has zero rest mass (photon) and the target particle is at rest. If the target particle is an electron, then this if the so-called **Compton Effect**. The process looks like



The photon momentum is $\frac{E_0}{c}$. After the collision the photon is scattered through an angle θ with energy E and the electron recoils at an angle ϕ with velocity $\vec{u}$. The final electron energy is

$$E_e = \gamma(u)m_0c^2 = \frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

Conservation of energy gives $E_0 + m_0c^2 = E + E_e$. Conservation of momentum gives (x and y directions)

$$\frac{E_0}{c} = \frac{E}{c}\cos\theta + p\cos\phi$$

$$0 = \frac{E}{c}\sin\theta - p\sin\phi$$

where

$$\vec{p} = \gamma m_0\vec{u} \qquad \text{or} \qquad E_e^2 = p^2c^2 + m_0^2c^4$$

We want to eliminate reference to the electron and find the new photon energy(that is what is detected in the experiment).

$$\frac{E_0}{c} = \frac{E}{c}\cos\theta + p\cos\phi \to p\cos\phi = \frac{E_0}{c} - \frac{E}{c}\cos\theta \to p^2\cos^2\phi = \left(\frac{E_0}{c} - \frac{E}{c}\cos\theta\right)^2$$

$$0 = \frac{E}{c}\sin\theta - p\sin\phi \to p\sin\phi = \frac{E}{c}\sin\theta \to p^2\sin^2\phi = \frac{E^2}{c^2}\sin^2\theta$$

Adding these equations we get

$$p^2c^2 = E_e^2 - m_0^2c^4 = E_0^2 - 2E_0E\cos\theta + E^2$$

Using the energy conservation equation we have (after algebra)

$$E = \frac{E_0}{1+\left(\frac{E_0}{m_0c^2}\right)(1-\cos\theta)}$$

The first thing to note is that $E > 0$. This means that a free electron cannot absorb a photon completely; there will always be a scattered photon of some energy. If we convert to wavelengths using

$$E = h\nu = h\frac{c}{\lambda}$$

we get

$$\lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\theta)$$

The shift in wavelength at a given angle is independent of the incident photon energy. You will do this experiment in Physics 14.