Locality, EPR and Bell - Introduction

Before proceeding into this discussion we must define the properties of quantum states (vectors) when **more than one particle** is present in a particular state. In this seminar, we will only need to look at the case of two particles.

Imagine two particles (1 & 2) in states $|a_1\rangle$ (eigenstate of operator $\hat{A}$) & $|b_1\rangle$ (eigenstate of operator $\hat{B}$) respectively, We write the state describing this situation as

$$|a_1\rangle_1 |b_1\rangle_2 \to |a_1, b_1\rangle$$

and these symbols represent a vector in the state-space of the two-particle system.

What are the properties of these vectors, especially with respect to probabilities?

If the two particles **do not interact** with each other, then their probabilities are independent. That means that the probability that particle 1 has $\hat{A}$ measured as a_3 and particle 2 has $\hat{B}$ measured as b_4, is given by

$$\left| \left({}_1\langle a_3|_2\langle b_4| \right) \left(|a_1\rangle_1 |b_1\rangle_2 \right) \right|^2 = \left| {}_1\langle a_3|a_1\rangle_1 \right|^2 \left| {}_2\langle b_4|b_1\rangle_2 \right|^2$$

or that the **joint probability** that $A = a_3$ **and** $B = b_4$ is simply the product of the probability for $A = a_3$ and the probability for $B = b_4$.

Our basis set of vectors for the 2-particle universe is given by

$$|a_i\rangle_1 |b_j\rangle_2 \to |a_i, b_j\rangle \to |\mathrm{i}, \mathrm{j}\rangle$$

for all possible a_i, b_j values, which means that the dimension of the space is

(dimension of space of 1-basis) x (dimension of space of 2-basis)

The basis set is orthonormal,i.e.,

$$\left({}_1\langle a_i|_2\langle b_k| \right) \left(|a_j\rangle_1 |b_m\rangle_2 \right) = {}_1\langle a_i|a_j\rangle_1 {}_2\langle b_k|b_m\rangle_2$$
$$= \begin{cases} 1 & i = j \;\; and \;\; k = m \\ 0 & i \neq j \;\; and/or \;\; k \neq m \end{cases}$$

This means that any linear combination of basis vectors

$$|Q\rangle = c_{11}|1,1\rangle + c_{12}|1,2\rangle + c_{21}|2,1\rangle + c_{22}|2,2\rangle + \dots\dots\dots$$

will also be a vector, and hence another physical state, in the two-particle state space.

Now for something very interesting!

It turns out that a two-particle state $|Q\rangle$ (one of many such examples) as given below

$$|Q\rangle = \frac{1}{\sqrt{2}}|1,2\rangle - \frac{1}{\sqrt{2}}|2,1\rangle$$

which is a superposition of two-particle state vectors where particle 1 is in a superposition of states $|1\rangle_1$ and $|2\rangle_1$ and particle 2 is **simultaneously** in a superposition of states $|1\rangle_2$ and $|2\rangle_2$, **cannot be decomposed** into a state of the form $|r,s\rangle$, that is, a separately well-defined state of particle 1 and a separately well-defined state of particle 2.

That means that states like $|Q\rangle$ cannot possibly be described by propositions of the form

"the state of particle 1 is such-and-such"

and

"the state of particle 2 is so-and-so"

In other words, in such a state, no measurable property of particle 1 alone, and no measurable property of particle 2 alone, **has any definite value.**

These are called **nonseparable or entangled** 2-particle states. **Nonseparability or entanglement** along with **superposition** and **incompatibility** are the ways in which quantum theory **differs** most from classical physics.

Now consider the state:

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|x_1 = 5\rangle|x_2 = 7\rangle - \frac{1}{\sqrt{2}}|x_1 = 9\rangle|x_2 = 11\rangle$$

where

$$\hat{X}_1|x_1\rangle = x_1|x_1\rangle \text{ and } \hat{X}_2|x_2\rangle = x_2|x_2\rangle$$

i.e., the particle states are eigenvectors of the position operators in each space.

In this state, neither the position of particle 1, nor the position of particle 2, nor anything else about them separately, has any definite value here.

On the other hand, the difference in their positions does have a definite value because:

$$\left(\hat{X}_2 - \hat{X}_1\right)|\alpha\rangle = 2|\alpha\rangle$$

i.e., it is an eigenvector of the $\left(\hat{X}_2 - \hat{X}_1\right)$ operator with eigenvalue 2, which implies that the difference in the positions (in this state) is equal to 2 with **certainty** or probability = 1.

Two interesting cases will illustrate how we deal with probability and collapse for these states.

First, suppose that the state of the 2-particle system is $|\beta\rangle$, and that $\hat{A}$ and $\hat{B}$, which are observables of particles 1 and 2 respectively, are measured. The probability that the outcomes of those experiments will be $A=a$ and $B=b$, is given by

$$\left|\langle a,b|\beta\rangle\right|^2$$

which is just one of our original postulates.

Now, suppose that **only** $\hat{A}$ is measured.

The probability that the outcome of the measurement will be $A=a_1$ is given by a sum over probability amplitudes

$$P=\left|\langle a_1,b_1|\beta\rangle+\langle a_1,b_2|\beta\rangle+\langle a_1,b_3|\beta\rangle+....\right|^2$$

where $\hat{B}$ is any observable of particle 2 and the sum ranges over all the eigenvalues $\{b_i\}$ of $\hat{B}$ (all possible measured values of $\hat{B}$), which is just another of our postulates.

We say it as follows:

> the probability amplitude that $A=a_1$ is equal to sum of the probability amplitudes of all the various different and indistinguishable ways it is possible for $\hat{A}$ to be a_1, i.e., independent of the values of $\hat{B}$ and the actual probability is the square of the resulting total amplitude

Classically we would have the very different result

$$P=\left|\langle a_1,b_1|\beta\rangle\right|^2+\left|\langle a_1,b_2|\beta\rangle\right|^2+\left|\langle a_1,b_3|\beta\rangle\right|^2+\left|\langle a_1,b_4|\beta\rangle\right|^2+......$$

that is, the sum of the squares of the individual amplitudes (individual probabilities) and the possibility of any interference would vanish (no cross-terms).

Now let us elaborate the principle(postulate) of collapse for a two-particle state.

Suppose that the state of a certain 2-particle system just prior to the time t_1 is $|\delta\rangle$ and suppose that at t_1 the observable $\hat{A}$ (of particle 1) is measured, and suppose that the outcome of that measurement is $A=a_5$ (one of the eigenvalues of $\hat{A}$).

Here is how to calculate how the state of the 2-particle system is changed(collapsed) by the $\hat{A}$ measurement.

Start with state $|\delta\rangle$ expressed in terms of eigenstates of $\hat{A}$ and $\hat{B}$

$$|\delta\rangle = d_{11}|1,1\rangle + d_{12}|1,2\rangle + d_{21}|2,1\rangle + d_{22}|2,2\rangle + \ldots\ldots\ldots\ldots$$

where

$$d_{ij} = \langle i,j|\delta\rangle \text{ and } |i,j\rangle = |a_i, b_j\rangle$$

Then, throw away all the terms where $A \neq a_5$ (that is what the measurement $A = a_5$ does according to the postulates). This leaves

$$|\delta\rangle = d_{51}|5,1\rangle + d_{52}|5,2\rangle + d_{53}|5,3\rangle + d_{54}|5,4\rangle + \ldots\ldots\ldots\ldots$$

Renormalize the new vector to 1 (one of our postulates) and the resulting vector is the state vector of the system after the measurement.

Note that it has only one value of $\hat{A}$, but **many, possibly all,** values of $\hat{B}$.

If the original state had only one component with $A = a_5$ then $\hat{B}$ would necessarily have the value in that state. So if the state is

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|x_1 = 5\rangle|x_2 = 7\rangle - \frac{1}{\sqrt{2}}|x_1 = 9\rangle|x_2 = 11\rangle$$

and we measure $X_1 = 5$, then we **necessarily** have $X_2 = 7$(particle 2 must be in this state) due to our collapse postulate, **even though we did not measure $\hat{X}_2$. This is a most important point!** We state that $X_2 = 7$ even though we have not measured it; this is called a **counterfactual** statement.

With these properties of 2-particle states, we can continue our discussions.

E(instein)P(odolsky)R(osen) - Version #1

A famous attempt to escape from the standard (Copenhagen) way of thinking about quantum mechanics was initiated in the 1930s by Einstein, Podolsky and Rosen, and had a surprising aftermath, in the 1960s, in the work of John Bell. We first discuss the escape attempt itself and then delve into the most fundamental work of Bell. In 1935, Einstein, Podolsky and Rosen (EPR) produced an argument, which was supposed to open the way to an escape from the standard way of thinking about quantum mechanics.

First, they define **completeness:**

> **A description of the world is complete (for EPR) if nothing that is true about the world, nothing that is an element of the reality of the world is left out of the description.**

EPR never actually presented a prescription for determining what all the elements of reality are (made it very difficult to challenge them because they could always say....."you must have missed one").

Instead, they did something much narrower (which was sufficient for the purposes of their argument). They wrote down a condition for a measurable property of a certain system at a certain moment to be an element of the reality of that system at that moment.

The condition is that:

> **if, without in any way disturbing a system, we can predict with certainty (with probability = 1) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity**

Let us see what this condition means. Consider the following question:

If a measurement of a particular observable $\hat{O}$ of a certain particular physical system S were to be carried out at a certain particular future time T, what would the outcome be?

Suppose that there is a method available to me so that I can, prior to time T, answer that question with certainty.

Also, suppose that the method I used involves no disturbance of the system S whatsoever.

Then(according to EPR) there must **now** already **be** some definite information(**hidden away somewhere**) about what the outcome of the future $\hat{O}$ measurement on S would be at time T.

Some Examples:

Suppose we have just measured the color of some particular electron. Having done that (and since measurements of color are repeatable) we are in a position to predict with certainty what the outcome of a later color measurement will be, if such a measurement were to be carried out. Making such a prediction need not involve any further interaction with the electron at all. So the EPR reality condition says that color must, at present, be an element of the reality of this electron.

This is identical to any statements we might make in the standard way of thinking!

Suppose, on the other hand, that I had just now measured the hardness of an electron. In order to be able to predict with certainty what the outcome of a future measurement of the color of that electron would be (if we made such a measurement), I would need to measure the color of **that** electron (I would need to **interact** with it and potentially disturb it).

The EPR reality condition does not say that the color of the electron is an element of the reality of **this** electron at present.

Again this agrees completely with the standard way of thinking!

So what EPR want to argue is the following:

> if the predictions of quantum mechanics are correct, then there must be elements of reality of the world which have no corresponding elements in the quantum-mechanical **description** of the world

They are attempting to use quantum mechanics against itself. They are saying that quantum mechanics is missing something!

Their argument goes something like this:

Consider a system consisting of two electrons. Electron 1 is located at position 1, and electron 2 is located at position 2.

Assume that the color-space state of these two electrons is (note that it is a nonseparable or entangled state)

$$|A\rangle = \frac{1}{\sqrt{2}}|green\rangle_1|magenta\rangle_2 - \frac{1}{\sqrt{2}}|magenta\rangle_1|green\rangle_2$$

The state $|A\rangle$, like any state in the space, is necessarily an eigenstate of some observable (Hermitian operator) of this pair of electrons, say $\hat{O}$, where $\hat{O}|A\rangle = +1|A\rangle$ (eigenvalue = +1). Now we have written $|A\rangle$ in the color basis. Let us convert it to the hardness basis.

Remember

$$|green\rangle = \frac{1}{\sqrt{2}}|hard\rangle + \frac{1}{\sqrt{2}}|soft\rangle$$

$$|magenta\rangle = \frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

Substituting we get

$$|A\rangle = \frac{1}{\sqrt{2}}|soft\rangle_1|hard\rangle_2 - \frac{1}{\sqrt{2}}|hard\rangle_1|soft\rangle_2$$

It takes the **same nonseparable** form in both bases(in fact, it would do so in **any basis** we might use!).

Now suppose we carry out a measurement of the color of electron 1 (in the state in the color basis). The outcome of the measurement will be either **green** or **magenta** with equal probability (using our two-particle state probability rules).

Moreover, quantum mechanics says(and it is experimentally confirmed) that in the event that the outcome of the measurement is **green,** then the outcome of any subsequent measurement of the color of electron 2 will necessarily be **magenta** and in the event that the outcome of the measurement is **magenta,** then the outcome of any subsequent measurement of the color of electron 2 will necessarily be **green.**

Both of these statements follow directly from the collapse postulate for two-particle states.

EPR assumed (this is the **only assumption** (it is called **locality**) they make on top of the basic assumption that the predictions of quantum mechanics are correct) that things could in principle be set up in such a way as to guarantee that the measurement of the color of electron 1 produces no physical disturbance whatsoever in electron 2.

To them it was a **self-evident** statement!

What influence could there be???

There seemed to be any number of ways for them satisfy this condition.

You could separate the two electrons by some immense distance (nothing we have said so far says that any of the properties of quantum mechanics changes with electron separation). Then the two measurement events could be made spacelike and according to special relativity could not influence each other.

Or you could insert an impenetrable wall between them (nothing we have said so far say that any of the properties of quantum mechanics depends on what happens to be located in between the electrons).

Or we could set up any array of detectors you like in order to verify that no measurable signals pass from one of the electrons to the other in the course of the experiment (since quantum mechanics predicts that no such array, in such circumstances, whatever sort of signals it may be designed to detect, will ever register anything).

This is a very important point.

The locality assumption says that

> **I cannot punch you in the nose unless my fist gets to the place where your nose is (in space and time)**

Of course, something I do with my fist far from where your nose is can cause some other fist which is near your nose to punch your nose (i.e., something I do with my fist might signal somebody else to punch you in the nose). Their seemingly obvious assumption is just this:

> **if my fist never gets anywhere near your nose then I cannot punch you in the nose directly. If you got punched in such an arrangement, then it cannot be my fist that punched you**

If something I do with my fist far from your nose is the cause of your getting punched in the nose, then necessarily some **causal sequence of events at contiguous points in space and at contiguous points in time** (the propagation of some signal for instance) stretches all the way without a break from whatever it was I did with my fist to your being punched in the nose.

The important thing about the sequence of events is that it must necessarily require some finite time(exact amount depends on what is along the path in space and time) to completely unfold. The shortest time possible would occur if the signal(s) travel with the speed of light in a straight line (the maximum speed of propagation of information).

So summarizing their assumption:

Locality

> **measurement on color 1 has no effect on measurement of color 2 if measurements spacelike separated**

Returning to the color basis entangled state

$$|A\rangle = \frac{1}{\sqrt{2}}|green\rangle_1 |magenta\rangle_2 - \frac{1}{\sqrt{2}}|magenta\rangle_1 |green\rangle_2$$

it is clear that we can predict with certainty, if **locality** is true, without disturbing electron 2, what outcome of subsequent measurement of color 2 will be.

Measure color 1 $\rightarrow$ know outcome of measurement of color 2 **opposite** of outcome of measurement of color 1

or **we know color 2 without measuring it!**

The reality condition then says that color is element of the reality of electron 2. The color of 2 has a definite value when in state $|A\rangle$.

So both 1 & 2 have definite **color** values!

Now switch to the hardness basis so we can talk about hardness measurements.

$$|A\rangle = \frac{1}{\sqrt{2}}|soft\rangle_1 |hard\rangle_2 - \frac{1}{\sqrt{2}}|hard\rangle_1 |soft\rangle_2$$

Using the same arguments in hardness basis we find that both 1 & 2 have definite **hardness** values!

Since we actually can prepare states like $|A\rangle$ in the real world, EPR say the standard interpretation must be false.

EPR conclude both color and hardness are elements of reality of electron 2, even though the two observables are **supposed to be incompatible** according to quantum mechanics.

So the formalism must be **incomplete**, since some elements of the physical reality of world have no corresponding elements in Quantum Mechanics.

There must exist **hidden** facts(**not in the postulates**) about color and hardness of 2, when in state $|A\rangle$.

Is there way out of EPR's proposed dilemma?

Nothing in QM formalism allows both the color/hardness of electron 2 to be predicted with certainty simultaneously.

Similar arguments hold for electron 1.

EPR were clearly very clever!

If true, the statement that system is in state $|A\rangle$ then constitutes an incomplete description of state of pair of electrons ---there are some **hidden variables** somewhere.

EPR say QM predicts **everything** correctly, but is **wrong.**

EPR noticed something very odd about collapse postulate for two-particle systems.

It was **nonlocal.**

If two particles are initially in nonseparable state, then a measurement carried out on one can cause changes, **instantaneously**, in the quantum mechanical description of the other, no matter how far apart two particles are or what lies in between.

Suppose that a pair of electrons is initially in state $|A\rangle$ and a measurement of color 1 carried out. The outcome of the measurement is either green or magenta, with equal probabilities. The collapse postulate for a two-particle systems says as soon as the measurement is over, the state of 2 will be either $|magenta\rangle$ (if 1 was green) or $|green\rangle$ (if 1 was magenta) **depending on what happened in the measurement.**

EPR said nonlocality is **disposable artifact** of particular mathematical formalism, of a particular procedure for calculating statistics of outcomes of experiments and that there must be other (**as yet undiscovered**) procedures, which give rise to some statistical properties, but are **local** (no infinite speeds necessary).

30 years later, Bell showed that their suspicion was wrong.

Bell's work, as we shall see, is taken as a proof that any attempt to be realistic about values of observables of pair of electrons in state $|A\rangle$, must necessarily be nonlocal.

Things are actually even more serious than that!!

Bell actually gives a proof that there is genuine nonlocality in the actual workings of nature, **however** we attempt to describe it.

Nonlocality is feature of quantum mechanics, and via Bell's theorem is necessarily a feature of every possible manner of calculating (**with or without superpositions**) which produces same probability predictions (which are experimentally correct) as quantum mechanics.

What is this quantum nonlocality?

First, in state $|A\rangle$, statistics(probabilities) of outcomes of **measurements** on 2 depend nonlocally on outcomes of measurements on 1, and vice versa. But do statistics of outcomes of measurements on 2, when system in state $|A\rangle$, depend nonlocally on **whether** a measurement is **actually carried out on 1** (and vice versa)?

Let us figure it out.

Suppose system in state $|A\rangle$, and suppose we measure color 2. Using $|A\rangle$ in the color basis plus the standard quantum-mechanical rules for calculating probabilities of measurement outcomes implies the outcome of the measurement is **equally likely** to be green or magenta.

Suppose the system in state $|A\rangle$ and we measure color 1, and **then** measure color 2. The measurement of color 1 is **equally likely** to be green or magenta. If green, the collapse postulate says a subsequent measurement of color 2 will be magenta, and if magenta, the collapse postulate says that subsequent measurement of color 2 will be green.

So, when system in state $|A\rangle$, the outcome of a measurement of color 2 is equally likely to be green or magenta **whether or not** measurement of color 1 carried out first.

Suppose system in state $|A\rangle$ and we measure hardness 1, and then measure color 2. Using $|A\rangle$ in hardness basis plus probability rules says the outcome of hardness measurement on 1 equally likely to be hard or soft.

If the outcome of first measurement is soft, the collapse postulate plus probability rules say outcome of second measurement (color 2) is equally likely to be green or magenta. The same result is true if outcome of first measurement is hard.

So here is where we are:

When system is in state $|A\rangle$, the outcome of a measurement of color 2 is equally likely to be green or magenta, whether

measurement color 1 carried out first

or

measurement of hardness 1 carried out first

or

no measurement on 1 is carried out

The probabilities of various outcomes of measurement on 2 do not depend in any way on whether or not a measurement is made on 1 first.

Since the predictions of QM are correct, there must be non-local influences in nature and they must be of a particularly **subtle** kind.

The outcomes of measurements do sometimes depend non-locally on outcomes of other, distant measurements, but outcomes of measurements invariably **do not** depend non-locally on **whether** any other distant measurements actually get carried out.

Another way (tricky...must think about this one) to say this:

> **Non-local influences are so subtle (they surely exist) that they cannot be used to transmit any signal containing information, non-locally, between two distant points. They cannot encode information you want to send in decision to make a measurement or not to make one, or in a decision about which measurement to make, since no such decisions can have detectable non-local effects.**

Let us look at a simple experiment (actually done in 1980 with photons) that refutes EPR and agrees with Bell's results. We will discuss this experiment again later in more detail and mathematical rigor.

Consider a pair of electrons in an entangled state like $|A\rangle$ where we use the observable spin(up/down in any direction).

$$|A\rangle = \frac{1}{\sqrt{2}}|z-up\rangle_1|z-down\rangle_2 - \frac{1}{\sqrt{2}}|z-down\rangle_1|z-up\rangle_2$$

The electrons separate in physical space without changing this state vector. We end up with two separated electron beams. Each beam has equal numbers of z-up and z-down electrons (since probability = 1/2 for each in each component).

Each electron in one beam is correlated with a partner in other beam since if measure electron in one beam z-up, then partner in other beam is z-down due to entanglement and vice versa.

This nonseparable state remains nonseparable no matter what basis (direction we measure spin) we use.

We define the direction of measurement by the angle θ it makes with an arbitrarily chosen z-axis direction. The original state is ($\theta = 0$, i.e., we chose the z-axis). So we can write

$$|A\rangle = \frac{1}{\sqrt{2}}|z(0)-up\rangle_1|z(0)-down\rangle_2 - \frac{1}{\sqrt{2}}|z(0)-down\rangle_1|z(0)-up\rangle_2$$

A state in an arbitrary direction basis is

$$|A\rangle = \frac{1}{\sqrt{2}}|z(\theta)-up\rangle_1|z(\theta)-down\rangle_2 - \frac{1}{\sqrt{2}}|z(\theta)-down\rangle_1|z(\theta)-up\rangle_2$$

where (in original basis)

$$|z(\theta)-up\rangle = \cos\frac{\theta}{2}|z(0)-up\rangle + \sin\frac{\theta}{2}|z(0)-down\rangle$$

$$|z(\theta)-down\rangle = -\sin\frac{\theta}{2}|z(0)-up\rangle + \cos\frac{\theta}{2}|z(0)-down\rangle$$

Does this make sense? Stand on your head $\rightarrow$ θ = 180 $\rightarrow$

$$|z(180)-up\rangle = |z(0)-down\rangle$$

$$|z(180)-down\rangle = |z(0)-up\rangle$$

as expected.

Suppose we measure beam 1 in a direction θ_1 and beam 2 in a direction θ_2 and count number of times

or	beam 1 up **and** beam 2 up beam 1 down **and** beam 2 down	= a **MATCH**

and count the number of times

or	beam 1 up and beam 2 down beam 1 down and beam 2 up	= a **MISS**

Quantum mechanics says the experimental results depend only on the angle difference

$$\phi = \theta_2 - \theta_1$$

and **not separately** on θ_1 and θ_2. These results are confirmed experimentally no matter where two detectors are located...in same room, or on opposite sides of city or wherever!!!!.

Suppose we code the results coming from two detectors:

If $\phi = 0$, z-axes of the 2 detectors are in the same direction and we get

$$beam\,1 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow ....$$

$$beam\,2 \rightarrow \downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow ....$$

i.e., **ALL MISSES** (remember if 1 is up, 2 is down).

We get two opposite **binary data** sequences in the two detectors

1010010001101001.......
0101101110010110.......

If $\phi = 180$, the z-axes of two detectors are in opposite directions and we get **ALL MATCHES**, i.e.,

$$beam\ 1 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow ....$$

$$beam\ 2 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow ....$$

(up in 1 is in opposite direction to up in 2 or up in 2 is down in 1).

Quantum mechanics predicts the number of **MATCHES** varies like

$$[\#MATCHES] = [\#ELECTRONS]\sin^2\frac{\phi}{2}$$

$$[\#MISSES] = [\#ELECTRONS]\cos^2\frac{\phi}{2}$$

$$[\#MATCHES + \#MISSES] = [\#ELECTRONS][\sin^2\frac{\phi}{2}+\cos^2\frac{\phi}{2}] = [\#ELECTRONS]$$

as it should!

We now set the angle between the detector axes to be $\phi = 120°$. Quantum mechanics predicts

[#MATCHES] = 3/4 [#ELECTRONS]

What would EPR say?

If $\phi = 60°$, then

[#MATCHES] = 1/4 [#ELECTRONS]

(this is same as the quantum mechanical prediction - check it!).

So now EPR align the detectors ($\phi = 0°$).

They the rotate detector 1 such that $\phi = 60°$ and get

1/4 MATCHES.

So now EPR re-align the detectors ($\phi = 0°$).

They then rotate detector 2(opposite direction) such that $\phi = -60°$ and also get

1/4 MATCHES.

The EPR version of reality (**locality**) says rotating 1 detector can **ONLY** change sequence of ups/downs at 1 detector and rotating 2 detector can **ONLY** change sequence of ups/downs at 2 detector.

That is their LOCALITY assumption

Now they rotate detector 1 by $\phi = 60°$ from the aligned direction and rotate the detector 1 by $\phi = -60°$ from the aligned direction(total angle between = $120°$).

EPR then predict

1/4 + 1/4 = 1/2 MATCHES (**not 3/4**)

Experiment says answer is 3/4 !!. Quantum mechanics is correct!

The local view of reality due to EPR cannot be correct.

Somehow information is flowing between detectors 1 and 2 **NO MATTER WHAT WE DO.** The speed of information flow might even be infinite(best measurement so far $\approx 10^7 c = 3\times 10^{15} m/\sec$).

It is important to emphasize that we have not observed any non-local interactions directly, but only indirectly demonstrated the need for them.

Each measurement is separately local, i.e., we measure any spin direction in either detector and always get 50-50 up and down no matter what the other detector is doing.

It is the sequences that differ(in a random way) as we change the angles. This means, as before, that we cannot transmit any messages.

We only find out the sequences(messages) when we bring them together.

So, it seems in QM, that my QM fist can punch your QM nose without being at your QM nose!!