Some Thoughts on the Collapse Postulate

Based on unitary time evolution postulate, a system consisting of a quantum system (Q-system) and a measurement system (M-system), would necessarily evolve in this way

$$|initial\rangle = \left(a|+\rangle_Q + b|-\rangle_Q\right)|0\rangle_M \rightarrow |final\rangle = a|+\rangle_Q|+1\rangle_M + b|-\rangle_Q|-1\rangle_M$$

which is a superposition of Q-states and M-states. The M-states represent macroscopic pointer locations on some meter.

What we are saying is that if the meter was turned on when the system was in the $|+\rangle_Q$ state then the system evolves to

$$|+\rangle_Q|0\rangle_M \rightarrow |+\rangle_Q|+1\rangle_M$$

that is the meter (assuming it is a good meter) reads +1.

Similarly, if the meter was turned on when the system was in the $|-\rangle_Q$ state then the system evolves to

$$|-\rangle_Q|0\rangle_M \rightarrow |-\rangle_Q|-1\rangle_M$$

that is the meter (assuming it is a good meter) reads -1.

This says that **measurement**, within the framework of the standard four postulates in most texts, **CORRELATES** or **ENTANGLES** the "**dynamical**" variable (Q-system) to be measured and the "**macroscopic**" (M-system) indicator that can be directly (macroscopically) observed.

Derivation: Suppose that the meter has eignvectors (labelled by eigenvalues)

$$|+\rangle_M \Rightarrow meter\ on : reading\ +1$$
$$|-\rangle_M \Rightarrow meter\ on : reading\ -1$$
$$|0\rangle_M \Rightarrow meter\ off$$

and the system has eigenvectors (labelled by eigenvalues)

$$|+\rangle_Q \Rightarrow value = +1$$
$$|-\rangle_Q \Rightarrow value = -1$$

The initial state is

$$|initial\rangle = \left(a|+\rangle_Q + b|-\rangle_Q\right)|0\rangle_M$$

which represents the system in a superposition and the meter off.

We are interested in the evolution of this state according to quantum mechanics.

If, instead of the above initial state, we started with the initial state

$$|A\rangle = |+\rangle_Q |0\rangle_M$$

or

$$|B\rangle = |-\rangle_Q |0\rangle_M$$

and then turn on the meter, the states must evolve into

$$|A'\rangle = |+\rangle_Q |+\rangle_M$$

or

$$|B'\rangle = |-\rangle_Q |-\rangle_M$$

respectively, indicating that the meter has measured the appropriate value (that is the definition of a "good" meter) since the system is in an eigenstate and has a definite value with certainty.

If the system is in the initial state corresponding to a superpositon, however, then the linearity of quantum mechanics says that it must evolve into

$$|final\rangle = a|+\rangle_Q |+1\rangle_M + b|-\rangle_Q |-1\rangle_M$$

which is equation (1).

Interpreting the state vector: Two models....

(A) Pure state $|\psi\rangle \rightarrow$ a **complete** description of an individual Q-system. This corresponds to the statement that a dynamical variable $\hat{P}$ has the value p in the state $|\psi\rangle$ if and only if $\hat{P}|\psi\rangle = p|\psi\rangle$.

(B) Pure state $|\psi\rangle \rightarrow$ the **statistical** properties of an **ensemble** of **similarly prepared** systems.

Interpretation (A) is the standard interpretation espoused by 90% of all physicists. It assumes that, because the state vector plays the most important role in the mathematical formalism of QM, it must have an equally important role in the interpretation of QM, so that

Properties of world $\Leftrightarrow$ **Properties of** $|\psi\rangle$.

Interpretation (A) by itself is not consistent with the unitary evolution postulate, that is, the state $|final\rangle$ is not equal to an eigenvector of any indicator (macroscopic pointer) variable. This means that the pointer (of the meter) will "flutter" since the $|\pm 1\rangle$ states could be macroscopically separated in space. Since we never observe this flutter, any interpretation of $|final\rangle$ as a description of an individual system cannot be reconciled with both observation and unitary time evolution.

Interpretation (B) has no such difficulties. $|\psi\rangle$ is just an abstract mathematical object which implies the probability distributions of the dynamical variables of an ensemble. It represents a **state of knowledge.**

Physicists that believe interpretation (A) are forced to introduce a new postulate at this point to remove these difficulties. This is the so-called **reduction/collapse** of the state vector postulate, which says that during any measurement we have a **new real process** which causes the **transition**

$$|\,final\rangle \to a|+\rangle_Q|+1\rangle_M \;\; or \;\; b|-\rangle_Q|-1\rangle_M$$

or we end up with an eigenvector of the indicator variable and thus there will be no flutter.

Various "**reasons**" are put forth for making this assumption, i.e.,

"measurements are repeatable"

Since this experiment(where the repeated measurement takes place immediately after the first measurement) is rarely realized in the laboratory, I do not know what to make of a requirement like this one. In addition, in many experiments (like those involving photons), the system is destroyed by the measurement (photon is absorbed) making it silly to talk about a repeatable measurement.

In addition, the "**reduction**" process has never been observed in the laboratory, so I do not undertsand in what sense it can it be thought of as a real physical process.

It is important to note that this difficulty only arises for interpretation (A) where statements are made about state vectors representing individual systems.

Some Proposed Mechanisms for the Reduction

(1) The reduction process is caused by an unpredictable and uncontrollable disturbance of the object by the measuring apparatus (a non-unitary process).

This means that the Hamiltonian of the system must take the form

$$\hat{H} = \hat{H}_Q + \hat{H}_M + \hat{H}_{QM} \quad , \quad \hat{H}_{QM} \to disturbance$$

which means, however, that it is already built into the standard unitary time evolution via $\hat{U} = e^{-i\hat{H}t/\hbar}$ and, thus, the disturbance terms can only lead to a final state that is still a superposition of indicator variable states. **IT DOES NOT WORK** unless you do not tell us what is meant by **unpredictable and uncontrollable disturbance**!

(2) The observer causes the reduction process when she reads the result of the measurement from the apparatus.

This is just a variation of (1). Here, the observer is just another indicator device. The new final state becomes

$$|\,final\rangle = a|+\rangle_Q|+1\rangle_M|sees+1\rangle_O + b|-\rangle_Q|-1\rangle_M|sees-1\rangle_O$$

which is still a superposition and thus is **NO HELP.** It also introduces "**consciousness**" into QM and that is just silly! All that happens is the observer gets entangled also.

(3) The reduction is caused by the environment (called decoherence), where by environment is meant the rest of the universe other than the Q-system and the M-system.

In this model, the environment is a very large system with an enormous number of degrees of freedom. We do not have any information about most of the degrees of freedom and thus must **average** over them. This causes pure states to **change** into nonpure or mixed states in a **non-unitary** process.

Why do many physicists think an individual Q-system must have its own state vector or wave function and then assume the collapse postulate?

IT WORKS for doing calculations!

This view has survived so long because it does not lead to any serious errors in most situations. Why?

In general, predictions in quantum mechanics are derived from $|\psi\rangle$ which gives the wave function and which, in turn, gives the probabilities. The operational significance of a probability is a relative frequency so that the experimentalist has to invoke an ensemble of similar systems to make any comparisons with theory that is independent of any particular interpretation of the wave function. So that interpretation (B) is being used in the end anyway.

Does this mean that we should stop worrying about the interpretation of the wave function? NO! But that is the subject of another (more advanced) seminar.

What about interpretation (B)? It says that

A pure state describes the statistical properties of an ensemble of similarly prepared systems.

This means that we must use the density operator $\hat{\rho}$ as the fundamental mathematical object of quantum mechanics instead of the state vector.

It turns out that some systems only have a density operator $\hat{\rho}$ and do not have a "**legitimate**" state vector $|\psi\rangle$.

For example, consider a box containing a very large number of electrons, each having spin = 1/2. As we shall see later, this means the spin can have a measurable component = ±1/2 along any direction. A Stern-Gerlach device measures these spin components.

Now, suppose the box has a hole so that electrons can get out and go into a Stern-Gerlach device oriented to measure z-components (an arbitrary choice). We will find the results

+1/2 50% of the time and -1/2 50% of the time

We then ask the question - what are the properties of the electrons in the box?

There are two possibilities

(1) **Each individual** electron has the state vector

$$|\psi\rangle_Q = \frac{1}{\sqrt{2}}|+1/2\rangle + \frac{1}{\sqrt{2}}|-1/2\rangle = |\psi\rangle_{BOX}$$

which is a superposition.

or

(2) **1/2** of the electrons have +1/2
1/2 of the electrons have -1/2

so that

$$|\psi\rangle_Q = |+1/2\rangle \; or \; |-1/2\rangle$$

$$|\psi\rangle_{BOX} = \frac{1}{\sqrt{2}}|+1/2\rangle + \frac{1}{\sqrt{2}}|-1/2\rangle$$

which is the seems to be the same state $|\psi\rangle_{BOX}$ as in (1), but it is really NOT a superposition state in this case.

Therefore, it seems that we will not be able to tell which possibility is the correct one!

It turns out, however, that

$$|x - comp = +1/2\rangle = \frac{1}{\sqrt{2}}|+1/2\rangle + \frac{1}{\sqrt{2}}|-1/2\rangle$$

so that, in case (1), if we orient the Stern-Gerlach device to measure x-components we would find **all** the electrons are in the same state $|x - comp = +1/2\rangle$, that is, they are all the same!

On the other hand, in case (2) since

$$|z = \pm 1/2\rangle = \frac{1}{\sqrt{2}}|x = +1/2\rangle \pm \frac{1}{\sqrt{2}}|x = -1/2\rangle$$

we would find that

1/2 give the $|x = +1/2\rangle$ result
1/2 give the $|x = -1/2\rangle$ result

Therefore, the **states are not the same!** If we try to write a state vector for case (2) we have to write

$$|\psi\rangle_Q = \frac{1}{\sqrt{2}}|+1/2\rangle + \frac{e^{i\alpha}}{\sqrt{2}}|-1/2\rangle$$

instead of

$$|\psi\rangle_{BOX} = \frac{1}{\sqrt{2}}|+1/2\rangle + \frac{1}{\sqrt{2}}|-1/2\rangle$$

where α is a **completely unknown relative phase factor**, which must be averaged over during any calculations since it is different for each separate measurement (each member of the ensemble). With that property for α, this is not a legitimate state vector in my opinion.

If we use density matrices we have a different story. For a pure state we can always write $\hat{\rho} = |\psi\rangle\langle\psi|$ for some state vector $|\psi\rangle$.

In fact, case (1) gives

$$\hat{\rho} = \frac{1}{2}\left(|1/2\rangle\langle 1/2| + |1/2\rangle\langle -1/2| + |1/2\rangle\langle -1/2| + |-1/2\rangle\langle -1/2|\right) \Rightarrow \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

where the diagonal matrix elements represent probabilities and the off-diagonal matrix elements imply that we will observe quantum interference effects in this system.

Clearly, any pure state density operator cannot be written as the sum of pure state projection operators.

In case (2), however, we have

$$\hat{\rho} = \frac{1}{2}\left(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle -1/2|\right) \Rightarrow \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which clearly is the sum of pure state projection operators. This corresponds to a **nonpure or mixed** state. Note that the off-diagonals are zero so that this density operator cannot lead to any quantum interference effects.

If we treat case (2) as a pure state with the extra relative phase factor we would obtain

$$\hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & e^{i\alpha} \\ e^{-i\alpha} & 1 \end{pmatrix}$$

which becomes

$$\hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

when we average over α.

The decoherence process has this effect

$$\hat{\rho} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \underset{\substack{environment \\ reduction}}{\Rightarrow} \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$$

or a pure state turns into a mixed state! Here, environment reduction means averaging over the unknown degrees of freedom.

Are any of these ideas correct? This is not yet determined!

In the two-path experiment discussed earlier we had the following discussion:

What if we put a wall into the soft path at the point (x_3, y_1) ?

The wall stops the time evolution of part of the state, so that the state at t_4 would be

$$\frac{1}{\sqrt{2}}|h\rangle|x_5, y_4\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_3, y_1\rangle$$

In this case, the state remains nonseparable with respect hardness/color and coordinate-space properties at t_4 .

If a measurement of position of this electron were to be carried out at t_4 (if, say, we were to look and see whether the electron had emerged from the black box), the probability of finding it at (x_5, y_4)would be 1/2, and if it were found there it would be hard, and if its color were measured, it would be **equally likely to be green or magenta.** That is exactly what we said in our earlier discussions.

The difference here is the the state of the system is a mixture and not a superposition. The reason is that a measurement has been performed and this collapses the original superposition state into a mixture state.