

## Quantum Theory - How it works.....The Details

Suppose that we want to predict the behavior of a physical system using the QM algorithm. How, exactly, do we go about it?

### General Approach:

- [1] Identify the vector space associated with that system---this is the space where all possible physical states of the system can be represented -- this simply means that we must find a **suitable basis** set of vectors.
- [2] Identify the operators associated with various measurable properties of the system --- this means calculate **eigenvectors and eigenvalues** of appropriate observables.
- [3] Map out the specific correspondences between individual physical states and individual vectors -- this means to decide how to label the state vectors(or **what goes inside the ket**).
- [4] Ascertain the present state vector of the system by means of measurements --- this means to determine what are its **initial labels**. This is called **state preparation**.
- [5] The **time evolution** of the systems is then determined by the time evolution operator  $\hat{U}$  --- this is a **deterministic** equation until the next measurement.  $\hat{U}$  must be specified for each system.
- [6] The **probabilities of particular outcomes** of a measurement carried out at a future time can then be calculated by using Postulate #4 which says that the probability of measuring the eigenvalue  $b$  of the observable  $\hat{B}$  at time  $t$  when we are in the state  $|\psi\rangle$  at time  $t$  is given by  $|\langle b|\psi\rangle|^2$  .
- [7] The **effects of measurement** are taken into account using Postulate #5 -- this implies that the state **collapses** to an appropriate (determined by value measured) eigenvector of measured observable.
- [8] Then [5]-[7] are just **repeated** over and over again.....

### Remember:

Postulate #5 says that the state vector is "**collapsed**" into an eigenvector of the measured observable operator.

Postulate #3 tells us how a state evolves in time.

This is the standard or **Copenhagen interpretation** of quantum theory due to Bohr, Dirac, Born and von Neumann.

I can now state the standard way of talking, which students of physics are traditionally required to master, about what superpositions are.

The way to deal with the apparent contradictions we saw in our earlier discussion of color and hardness measurements boils down to this explanation .....

The right way to think about superpositions of, say, being green and being magenta is to think of them as situations where **color predictions** cannot be made, situations where **color talk** is unintelligible.

Talking and inquiring about the color of an electron in such circumstances (in the standard view) makes no sense whatsoever.

As long as you follow this rule all of our earlier contradictions go away.

In this view, it just is not so that hard electrons are not green and not magenta and not both and not neither, since color talk of any kind, about hard electrons, simply has no meaning at all and the same is true for all other incompatible observables.

That is the way it is .... that is the way the world works according to the quantum physicist!

Of course, once an electron has been "**measured**" to be green or magenta, then it "**is**" green or magenta (**color talk now applies**) according to this standard interpretation.

Measuring the color of a hard electron, then, is **not a matter of determining** what the color of that hard electron is ... **it has none - it only has probabilities to have color values if we measure color.**

Rather, it is a matter of "**collapsing**" the state of the measured electron into one where color talk applies, and "**then**" determining the color of the newly created, color-applicable state.

**Measurements** in QM (and particularly within the standard view) are very **active processes**.

They are not processes of **merely learning** something about the system ..... they are invariably processes which **drastically change** the measured system.

The most important rule that we can state is as follows:

If you are going to measure some observable  $\mathcal{O}$ , then choose as your basis vectors the eigenvectors of the  $\mathcal{O}$  operator since it is only

with these states that  $\mathcal{C}$ -talk makes any sense, i.e., if you are discussing color measurements than use the color basis where color-talk makes sense! The same holds true if you are going to measure anything else!

### Some Consequences

We are allowed to use any **orthonormal set** (provided their number = dimension of the space) as a basis. If such other sets exist, then they must be eigenvectors of some other observables (not color). For examples here are two other orthonormal sets.....

Set #1

$$|1\rangle = \frac{1}{2}|g\rangle + \frac{\sqrt{3}}{2}|m\rangle \quad \text{and} \quad |2\rangle = \frac{\sqrt{3}}{2}|g\rangle - \frac{1}{2}|m\rangle$$

Set #2

$$|1\rangle = \frac{1}{2}|g\rangle - \frac{\sqrt{3}}{2}|m\rangle \quad \text{and} \quad |2\rangle = \frac{\sqrt{3}}{2}|g\rangle + \frac{1}{2}|m\rangle$$

Either set can be used as a basis(they both are orthonormal pairs). **Prove that for yourself.**

Since all basis sets correspond to some operator, let us choose to call their associated observables **direction**  $\mathcal{D}$  and **truth**  $\mathcal{T}$  where set #1 are eigenstates of **direction operator**  $\mathcal{D}$  with associated eigenvalues +1(up) and -1(down) and set #2 are eigenstates of **truth operator**  $\mathcal{T}$  with associated eigenvalues +1(true) and -1(false)

So that we can write

Set #1

$$|1\rangle = |\text{direction} = +1\rangle = |\text{up}\rangle = \frac{1}{2}|g\rangle + \frac{\sqrt{3}}{2}|m\rangle$$

$$|2\rangle = |\text{direction} = -1\rangle = |\text{down}\rangle = \frac{\sqrt{3}}{2}|g\rangle - \frac{1}{2}|m\rangle$$

Set #2

$$|1\rangle = |\text{truth} = +1\rangle = |\text{true}\rangle = \frac{1}{2}|g\rangle - \frac{\sqrt{3}}{2}|m\rangle$$

$$|2\rangle = |\text{truth} = -1\rangle = |\text{false}\rangle = \frac{\sqrt{3}}{2}|g\rangle + \frac{1}{2}|m\rangle$$

Remember we already had the other basis sets:

$$|color = +1\rangle = |g\rangle = \frac{1}{\sqrt{2}}|h\rangle + \frac{1}{\sqrt{2}}|s\rangle$$

$$|color = -1\rangle = |m\rangle = \frac{1}{\sqrt{2}}|h\rangle - \frac{1}{\sqrt{2}}|s\rangle$$

$$|hardness = +1\rangle = |h\rangle = \frac{1}{\sqrt{2}}|g\rangle + \frac{1}{\sqrt{2}}|m\rangle$$

$$|hardness = -1\rangle = |s\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{1}{\sqrt{2}}|m\rangle$$

Now think of these states as representing a real physical system and think about probabilities.

We can make these statements:

**probability that an electron that is "up"  
will be measured to be "magenta" = 3/4**

**probability that an electron that is "false"  
will be measured to be "magenta" = 1/4**

**probability that an electron that is "soft"  
will be measured to be "magenta" = 1/2**

**probability that an electron that is "green"  
will be measured to be "hard" = 1/2**

and so on.

Let us now look at some details and some examples, that is, let us apply all of this stuff and expand our knowledge and capabilities as we proceed.

We begin with an example that includes only familiar classical properties like position, velocity, momentum, energy, etc, then return to explain the color-hardness experiments and finally in the later sections of the notes we investigate photon polarization experiments and Stern-Gerlach spin experiments.

We know from hundreds of years of experience, that the behavior of relatively big particles, with large masses (like rocks and shampoo bottles) is very well described by the classical mechanics of Newton. This says something about the quantum theory of particles, i.e., whatever the theory ends up saying about microworld particles it ought to predict that everyday particles, subject to everyday circumstances, will behave in the everyday, Newtonian way we all know and love.

Now the **position**  $\not\propto$  operator and the **momentum**  $\not\propto$  operator are incompatible (i.e., they are not simultaneously measurable - this is

the famous Heisenberg uncertainty principle as we shall show later). This means that

$$[\hat{x}, \hat{p}] \neq 0$$

In fact, the value is known to be

$$[\hat{x}, \hat{p}] = i\hbar$$

where  $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ Joule} - \text{sec}$ ,  $h = \text{Planck's constant}$  .

Note the appearance of the factor  $i$ .

Since  $\hat{x}$  is an operator representing a physical observable, a possible basis of the space is the set of eigenvectors of the  $\hat{x}$  operator.

Similarly for the eigenvectors of the  $\hat{p}$  operator.

They each form a possible basis because they are observables represented by Hermitian operators .

Because their corresponding operators do not commute, a state of definite momentum (an eigenvector of  $\hat{p}$ ) will be a **superposition** of states of definite position (eigenvectors of  $\hat{x}$ ) or, in other words, a state of definite momentum can only have probabilities of having definite  $x$ -values. Similarly, a state of definite position (an eigenvector of  $\hat{x}$ ) will be a superposition of states of definite momentum (eigenvectors of  $\hat{p}$ ).

The vector space when the  $\hat{x}$  eigenvectors are the basis is called the **coordinate representation**.

The vector space when the  $\hat{p}$  eigenvectors are the basis is called the **momentum representation**.

Now, the most important operator in quantum mechanics is the energy operator or **Hamiltonian**  $\hat{H}$ . It will determine the time evolution operator as we will see later.

The vector space when the  $\hat{H}$  eigenvectors are the basis is called the **energy representation**.

All are **equivalent** basis sets.

Our choice will depend on the questions we are asking or the measurements we are doing.

That choice is called going to the **HOME SPACE**.

Now the possible eigenvalues(allowed measured values) of  $\hat{x}$  and  $\hat{p}$  form a continuum extending from  $-\infty \rightarrow +\infty$  (called a continuous spectrum). The basis is infinite(non-denumerable) dimensional. In general, this present many difficult mathematical problems. For the systems we will be discussing, the infinite dimension cause no difficulties and so we can treat almost property as if the space had a finite dimension. The one exception will be normalization as we shall discuss.

The state  $|x\rangle$  corresponds to a system with a definite value of the position, namely,  $x$ . The state  $|p\rangle$  corresponds to a system with a definite value of the momentum, namely,  $p$ . The state  $|E\rangle$  corresponds to a system with a definite value of the energy, namely,  $E$ .

In general, the set of energy eigenvalues are both a **discrete** set over some range and a **continuous** set over a disjoint range.

The operators position  $\hat{x}$  and color  $\hat{C}$  are compatible and, therefore, we can simultaneously measure these observables.

This means that we can say that a magenta electron is located at  $x=7$  and it makes sense.

We write the state representing this case in the form

$$|color - value\rangle|x - value\rangle$$

i.e.,  $|m\rangle|7\rangle$  for a magenta electron located at  $x=7$ . (the formal name for this product is a **tensor-product**) When operators act on such states their operation takes the form

$$\hat{C}|m\rangle|7\rangle = -|m\rangle|7\rangle \text{ i.e., } \hat{C} \text{ only acts on the color space part}$$

$$\hat{x}|m\rangle|7\rangle = 7|m\rangle|7\rangle \text{ i.e., } \hat{x} \text{ only acts on the position space part}$$

Similarly statements can be made for position and hardness, momentum and color, momentum and hardness, energy and color, and energy and hardness. They all represent simultaneously measurable pairs.

As we said, the operator corresponding to the observable energy is called the Hamiltonian  $\hat{H}$  where

$$\hat{H}|E\rangle = E|E\rangle$$

As I mentioned, the energy basis is the most fundamental one in quantum theory. We will see why shortly.

The quantity

$$\psi_E(x) = \langle x | E \rangle \quad \text{for some state vector } |E\rangle$$

is called the energy **wave function**, that is, since it takes on different values for each value of  $x$ , it is a function of  $x$ .

It satisfies a famous equation that governs its time and space dependence - the Schrodinger equation. The Schrodinger equation in 1-dimension is an "ordinary differential equation" given by

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_E(x,t)}{dx^2} + V(x) \psi_E(x,t) = E \psi_E(x,t) = i\hbar \frac{d\psi_E(x,t)}{dt}$$

where  $V(x)$  is the potential energy of the system. We will derive and solve this equation later for simple systems.

The time evolution operator for simple systems is expressed in terms of the Hamiltonian  $\hat{H}$  by the relation  $\hat{U} = e^{-i\hat{H}t/\hbar}$ , which is an **operator function** of the operator  $\hat{H}$ .

As shown in earlier in these notes, in simple cases, functions of operators are easy to deal with. If

$$\hat{B}|b\rangle = b|b\rangle$$

i.e.,  $|b\rangle$  is an eigenstate of  $\hat{B}$ , then we have

$$f(\hat{B})|b\rangle = f(b)|b\rangle$$

for a function of the operator.

This implies that the energy eigenvectors have a very simple time dependence or time evolution, i.e.,

$$|E,t\rangle = \hat{T}|E,0\rangle = e^{-i\hat{H}t/\hbar}|E,0\rangle = e^{-iEt/\hbar}|E,0\rangle$$

where the complex exponential means  $e^{-iEt/\hbar} = \cos\frac{E}{\hbar}t - i\sin\frac{E}{\hbar}t$  as we showed earlier.

Using the eigenvectors and eigenvalues of the Hamiltonian, namely,

$$\hat{H}|E_n\rangle = E_n|E_n\rangle$$

as a basis We then have

$$|\psi(0)\rangle = \sum_n c_n |E_n\rangle$$

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar} |\psi(0)\rangle = e^{-i\hat{H}t/\hbar} \sum_n c_n |E_n\rangle = e^{-i\hat{H}t/\hbar} \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

$$|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle = \hat{U}(t) \sum_n c_n |E_n\rangle = \sum_n c_n \hat{U}(t) |E_n\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

Let us now explore the coordinate-space. Since we have continuous eigenvalues we must replace sums by integrals.

In the discrete spectrum case, we have, using a basis set  $\{|n\rangle\}$ , that

$$|\psi\rangle = \sum_n c_n |n\rangle \quad , \quad c_m = \langle m | \psi \rangle$$

$$|\psi\rangle = \sum_n \langle n | \psi \rangle |n\rangle = \sum_n |n\rangle \langle n | \psi \rangle = \left( \sum_n |n\rangle \langle n| \right) |\psi\rangle$$

$$\sum_n |n\rangle \langle n| = \hat{I} = \text{identity operator}$$

and

$$\hat{A} |a_n\rangle = a_n |a_n\rangle \quad , \quad |\psi\rangle = \sum_n c_n |a_n\rangle$$

$$\begin{aligned} \hat{A} |\psi\rangle &= \sum_n c_n \hat{A} |a_n\rangle = \sum_n c_n a_n |a_n\rangle = \sum_{n,m} c_n a_m |a_m\rangle \delta_{nm} \\ &= \sum_{n,m} c_n a_m |a_m\rangle \langle a_m | a_n \rangle = \sum_n c_n \left( \sum_m a_m |a_m\rangle \langle a_m| \right) |a_n\rangle \end{aligned}$$

so that

$$\hat{A} = \sum_m a_m |a_m\rangle \langle a_m|$$

which is the spectral representation of the operator  $\hat{A}$  in terms of its eigenvalues and eigenvectors.

Some of the changed relations look like

$$\hat{I} = \int_{-\infty}^{\infty} |x\rangle \langle x| dx \quad \text{instead of} \quad \hat{I} = \sum_n |n\rangle \langle n|$$

and

$$\int_{-\infty}^{\infty} g(x) |x\rangle \langle x| dx = g(x) \quad \text{instead of} \quad \sum_n a_m \langle m | n \rangle = a_n$$

Clearly, using our earlier discussion of the Dirac delta function, this result says that



$$\langle x|x\rangle = \delta(x-x)$$

Any arbitrary state can be written in terms of the basis states, i.e.,

$$|\psi\rangle = \mathcal{N}|\psi\rangle = \left( \int_{-\infty}^{\infty} |x\rangle \langle x| dx \right) |\psi\rangle = \int_{-\infty}^{\infty} \langle x|\psi\rangle |x\rangle dx = \int_{-\infty}^{\infty} \psi(x) |x\rangle dx$$

The probability (from Postulate #4) that we will find the system (particle) at  $x=q$  is given by

$$\begin{aligned} \text{prob}(x=q|\psi) = P(q) &= |\langle q|\psi\rangle|^2 = \left| \langle q \left| \int_{-\infty}^{\infty} \psi(x) |x\rangle dx \right. \right|^2 \\ &= \left| \int_{-\infty}^{\infty} \psi(x) \langle q|x\rangle dx \right|^2 = |\psi(q)|^2 = |\langle q|\psi\rangle|^2 \end{aligned}$$

or in general we say that

$$P(x) = |\langle x|\psi\rangle|^2 = |\psi(x)|^2$$

or the probabilities are given by the wave functions which will be solutions of the Schrodinger equation.

**Now we are beginning to get somewhere!**

The **general procedure** for figuring out the time dependence of a states and then answering the questions posed in experiments goes as follows (**this is tricky algebra but well worth spending time on**):

(1) Ask the question --- it usually takes the form

if we are in the state  $|\phi\rangle$  at time  $t=0$ , then what is the probability of being in the state  $|b_7\rangle$  at time  $t$

(2) Assume that  $|b_7\rangle$  is one of the eigenvectors of the operator  $\hat{B}$ .

(3) We solve for the energy eigenvectors of the system, i.e. we find the Hamiltonian (energy operator) and do the mathematics.

(4) We write  $|\phi\rangle$  in terms of the energy eigenvectors basis

$$|\phi\rangle = \sum_E c_E |E\rangle \quad \text{where} \quad c_E = \langle E|\phi\rangle$$

(5) Since the time dependence of the energy eigenstates is easy we can write down the time dependence of the state  $|\phi\rangle$  as

$$|\phi, t\rangle = \sum_E c_E |E, t\rangle = \sum_E c_E e^{-iEt/\hbar} |E\rangle$$

(6) We write  $|E\rangle$  in terms of the  $\mathcal{B}$  eigenvectors basis

$$|E\rangle = \sum_j d_j |b_j\rangle \quad \text{where } d_k = \langle b_k | E \rangle$$

(7) We write  $|\phi, t\rangle$  in terms of the  $\mathcal{B}$  eigenvectors basis

$$|\phi, t\rangle = \sum_E c_E e^{-iEt/\hbar} |E\rangle = \sum_E \langle E | \phi \rangle e^{-iEt/\hbar} \sum_j \langle b_j | E \rangle |b_j\rangle$$

(8) Then the probability of finding  $b_7$  is given by

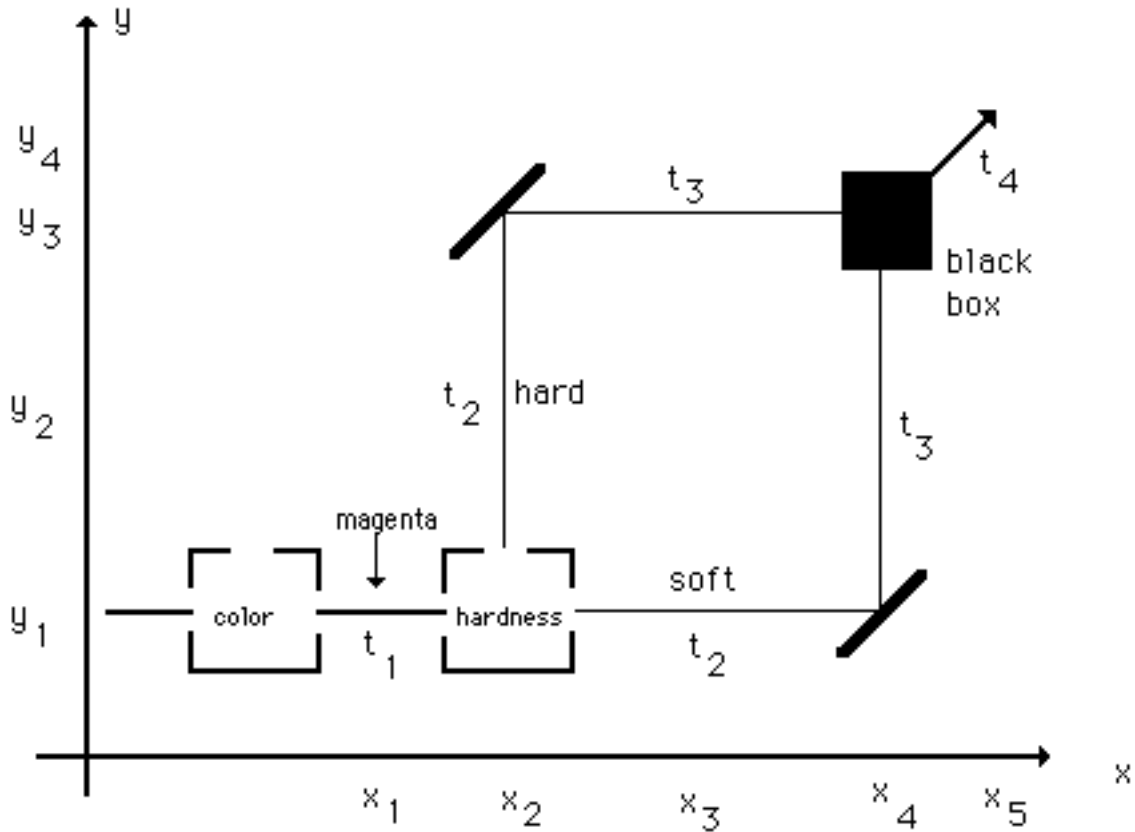
$$\begin{aligned} P &= \left| \langle b_7 | \phi, t \rangle \right|^2 = \left| \sum_E \langle E | \phi \rangle e^{-iEt/\hbar} \sum_j \langle b_j | E \rangle \langle b_7 | b_j \rangle \right|^2 \\ &= \left| \sum_j \langle b_7 | b_j \rangle \left( \sum_E \langle b_j | E \rangle \langle E | \phi \rangle e^{-iEt/\hbar} \right) \right|^2 \\ &= \left| \sum_E \langle b_7 | E \rangle \langle E | \phi \rangle e^{-iEt/\hbar} \right|^2 \end{aligned}$$

**This final result expresses the answer in terms of the initial and final states and the properties of the energy eigenvectors and energy eigenvalues, all of which are known.**

Thus, our formulation of quantum theory is capable of making the necessary predictions.

**Now back to our Two-Path Experiments:**

We now redo the **two-path color-hardness experiment** in more detail. See the figure below:



At time  $t_1$  when the particle is about to enter the apparatus, its state is (having just left a color box via the magenta aperture):

$$\begin{aligned}
 |color = magenta, x = x_1, y = y_1\rangle &= |m\rangle|x_1, y_1\rangle \\
 &= \left( \frac{1}{\sqrt{2}}|h\rangle - \frac{1}{\sqrt{2}}|s\rangle \right) |x_1, y_1\rangle = \frac{1}{\sqrt{2}}|h\rangle|x_1, y_1\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_1, y_1\rangle
 \end{aligned}$$

where we have changed to the hardness + position basis vectors because we are about to ask a hardness question (the hardness box comes next).

Here is how quantum theory says to calculate what happens next.

Let

$$\begin{aligned}
 \frac{1}{\sqrt{2}}|h\rangle|x_1, y_1\rangle &= |a\rangle \\
 \frac{1}{\sqrt{2}}|s\rangle|x_1, y_1\rangle &= |b\rangle
 \end{aligned}$$

so that

$$|m\rangle|x_1, y_1\rangle = |a\rangle - |b\rangle$$

Consider the following:

If the state at  $t_1$  were not the one above but, rather, just  $|a\rangle \rightarrow$  the "**hard**" part, and if the hardness box behaves properly, then the state at time  $t_2$  would be

$$|h\rangle|x_2,y_2\rangle$$

i.e., the electron would have just been diverted through the hard aperture and be on what we will call the "**hard path**".

If the state at  $t_1$  were just  $|b\rangle \rightarrow$  the "**soft**" part, and if the hardness box behaves properly, then the state at time  $t_2$  would be

$$|s\rangle|x_3,y_1\rangle$$

i.e., the electron would have just been diverted through the soft aperture and be on what we will call the "**soft path**".

However, as the state at  $t_1$  is in fact neither  $|a\rangle$  nor  $|b\rangle$ , but the superposition

$$|m\rangle|x_1,y_1\rangle = |a\rangle - |b\rangle$$

it follows that the state at  $t_2$  must be (if the hardness box behaves properly)

$$\frac{1}{\sqrt{2}}|h\rangle|x_2,y_2\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_3,y_1\rangle$$

This state involves **nonseparable** correlations between hardness/color and coordinate-space properties of the electron (the color and position properties were separated to start with - they had definite values, magenta and  $(x_1,y_1)$ ).

**No hardness or color properties of the electron in this state nor any of its coordinate-space properties (position, momentum, etc) has any definite value here.**

This is a superposition of states, where in one component the electron is travelling along the hard path and in the other component the electron is travelling along the soft path.

**This is the reason that earlier we had to make our earlier statements about not hard, not soft, not both and not neither. Those ideas are not only false, they are meaningless!**

This state is called an **entangled state**. It will get us very entangled later on!!

Continuing on..... the last state of the electron above (at  $t_2$ ) leads to the state of the electron at  $t_3$

$$\frac{1}{\sqrt{2}}|h\rangle|x_3,y_3\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_4,y_2\rangle$$

and then the state at time  $t_4$  is

$$\begin{aligned} & \frac{1}{\sqrt{2}}|h\rangle|x_5,y_4\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_5,y_4\rangle \\ &= \left( \frac{1}{\sqrt{2}}|h\rangle - \frac{1}{\sqrt{2}}|s\rangle \right) |x_5,y_4\rangle \\ &= |m\rangle|x_5,y_4\rangle \end{aligned}$$

We see that the color state and the coordinate-space state have **become separate** again. The position of the electron once again has a definite value and the color once again has a definite value.

So the fact that a hard electron fed into the total device will come out hard, and that a soft electron will come out soft (at the same point), together with our quantum assumptions, means that a magenta electron fed into the total device will come out (just as we found it did) magenta!

What if we were to stop the experiment in the middle by measuring the position of the electron at, say,  $t_3$ ?

Then the superposition would go away.

According to our postulates, a **collapse** would occur and the state just after the measurement would be either

$$|h\rangle|x_3,y_3\rangle \quad \text{or} \quad |s\rangle|x_4,y_2\rangle$$

each with probability 1/2 (in accordance with our probability assumption).

Then the state at  $t_4$  would be either

$$|h\rangle|x_5,y_4\rangle \quad \text{or} \quad |s\rangle|x_5,y_4\rangle \quad \text{respectively.}$$

Note that we collapsed the particle to "**hard**" by measuring its position in an entangled state. That is the way entangled or nonseparable states work - **collapsing one property collapses all the other properties.**

The state vector

$$\frac{1}{\sqrt{2}}|h\rangle|x_5,y_4\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_3,y_1\rangle$$

is nonseparable between hardness/color properties and coordinate-space properties. This means it is not associated with any definite values for hardness or color or position or momentum or anything like that.

Let me finally say something about how to build a "**total-of-nothing**" box.

If we are in a state  $|A\rangle$  and measure an observable  $B$ , then the probability of measuring a particular value of  $B$  is determined by doing the following:

[1] Write  $|A\rangle$  as a superposition of eigenvectors of  $B$  (the thing we want to measure)

$$|A\rangle = \sum_k a_k |b_k\rangle$$

where  $a_i = \langle b_i | A \rangle$  and  $|a_i|^2 = |\langle b_i | A \rangle|^2$  = the probability that the measurement will yield the value  $b_i$ .

Now consider the state  $-|A\rangle$ . If we measure  $B$  on this state, we will get all the same probabilities for every possible outcome, i.e.,

$$|\langle b_i | -|A \rangle|^2 = |-\langle b_i | A \rangle|^2$$

So if a vector  $|A\rangle$  changes to the vector  $-|A\rangle$  there are **NO OBSERVABLE EFFECTS - NOTHING HAPPENS** as far as quantum theory is concerned!!!. In fact, if a vector  $|A\rangle$  changes to the vector  $e^{i\alpha}|A\rangle$ , where  $|e^{i\alpha}|=1$ , there are **NO OBSERVABLE EFFECTS**.

So any box which changes the state of any incoming electron into -1 times that incoming state will be a "**total-of-nothing**" box, since it will not change any of the measurable values, i.e., any probabilities of the values of any of the observables of any electron which passes through it. Obviously, it will also not effect any electron which passes outside of it.

But the effects of such a box on an electron which is in a superposition of passing through it and outside of it may be quite a different matter.

Suppose the box is inserted in the soft path of the two-path device at  $(x_3, y_1)$ . Then, if the initial input electron were magenta, the state at  $t_2$  will be (as earlier)

$$\frac{1}{\sqrt{2}} |h\rangle |x_2, y_2\rangle - \frac{1}{\sqrt{2}} |s\rangle |x_3, y_1\rangle$$

and the state at  $t_3$  (after the "**passage**" through the box) will be not be

$$\frac{1}{\sqrt{2}}|h\rangle|x_3,y_3\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_4,y_2\rangle \quad (\text{as before})$$

but will be

$$\frac{1}{\sqrt{2}}|h\rangle|x_3,y_3\rangle + \frac{1}{\sqrt{2}}|s\rangle|x_4,y_2\rangle$$

where the sign of the second term has been changed by the "**total-of-nothing**" box in the soft path.

If we now follow this new state to  $t_4$  as before we will find

$$\begin{aligned} & \frac{1}{\sqrt{2}}|h\rangle|x_5,y_4\rangle + \frac{1}{\sqrt{2}}|s\rangle|x_5,y_4\rangle \\ &= \left( \frac{1}{\sqrt{2}}|h\rangle + \frac{1}{\sqrt{2}}|s\rangle \right) |x_5,y_4\rangle \\ &= |g\rangle|x_5,y_4\rangle \end{aligned}$$

instead of

$$\begin{aligned} & \frac{1}{\sqrt{2}}|h\rangle|x_5,y_4\rangle - \frac{1}{\sqrt{2}}|s\rangle|x_5,y_4\rangle \\ &= \left( \frac{1}{\sqrt{2}}|h\rangle - \frac{1}{\sqrt{2}}|s\rangle \right) |x_5,y_4\rangle \\ &= |m\rangle|x_5,y_4\rangle \end{aligned}$$

So the "**total-of-nothing**" box has changed the color of all the electrons from magenta to green (as we had said earlier) even though it has no measurable effect on the electrons that "passed through it"!!!!.

That is the way quantum mechanics works!!