An Interference Experiment with Photons

Direct laser beam at a half-silvered mirror.

For **intense** light beams,

such mirrors reflect half of light and allow half to pass straight through.

When intensity of laser beam high,

two beams seen emerging from mirror,

each having 1/2 intensity of incoming beam.

Arrangement is called **beam-splitter**.

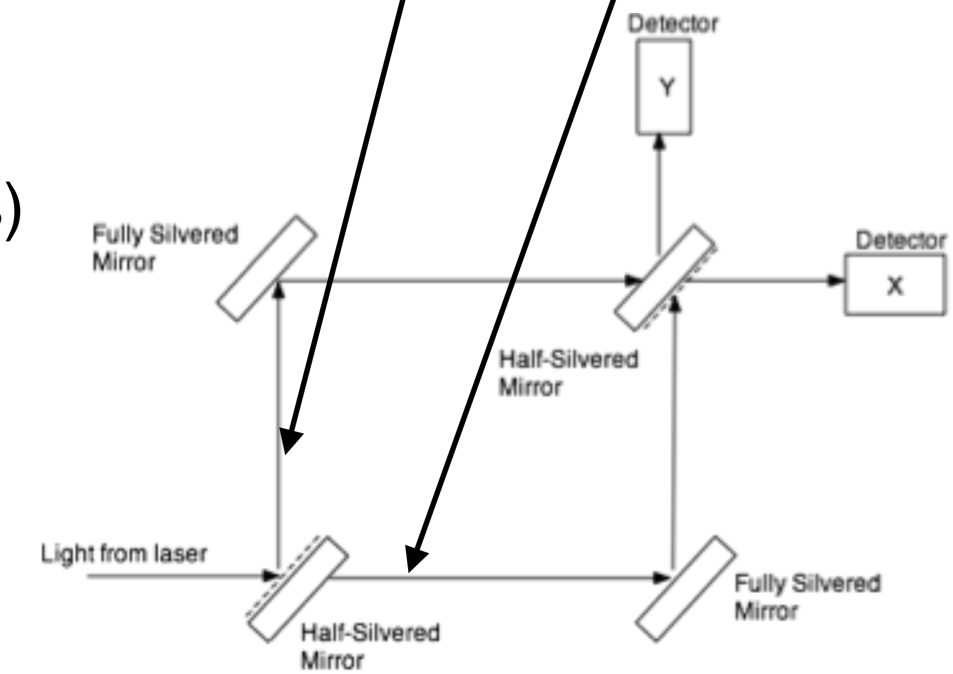
If turn intensity of laser down(photons emerge with time gaps)

-> only one photon at any given time,

and use pair of C(harged)C(oupled)D(device) detectors

to detect reflected and transmitted beams,

something very interesting happens.



For each photon that leaves laser,

one photon is detected either at transmission CCD (on transmitted path)

or at reflection CCD (on reflected path).

Photons are not split in some odd manner

so that half photon goes one way at mirror and half other way. See figure.

Instead, seems to be 50:50 chance(probability)

that photon transmitted or reflected by half-silvered mirror.

No measurable difference between photons as they approach mirror,

i.e., no property seems to determine which way they will go (sound familiar).

—> Fundamental point that will come up repeatedly in context of quantum theory.

Next step is to remove detectors and replace them with two mirrors (fully silvered)

that divert two beams(change their directions by 90°)

to second half-silvered mirror as in figure.

At this point,

same thing happens,

with 1/2 of light arriving at each mirror

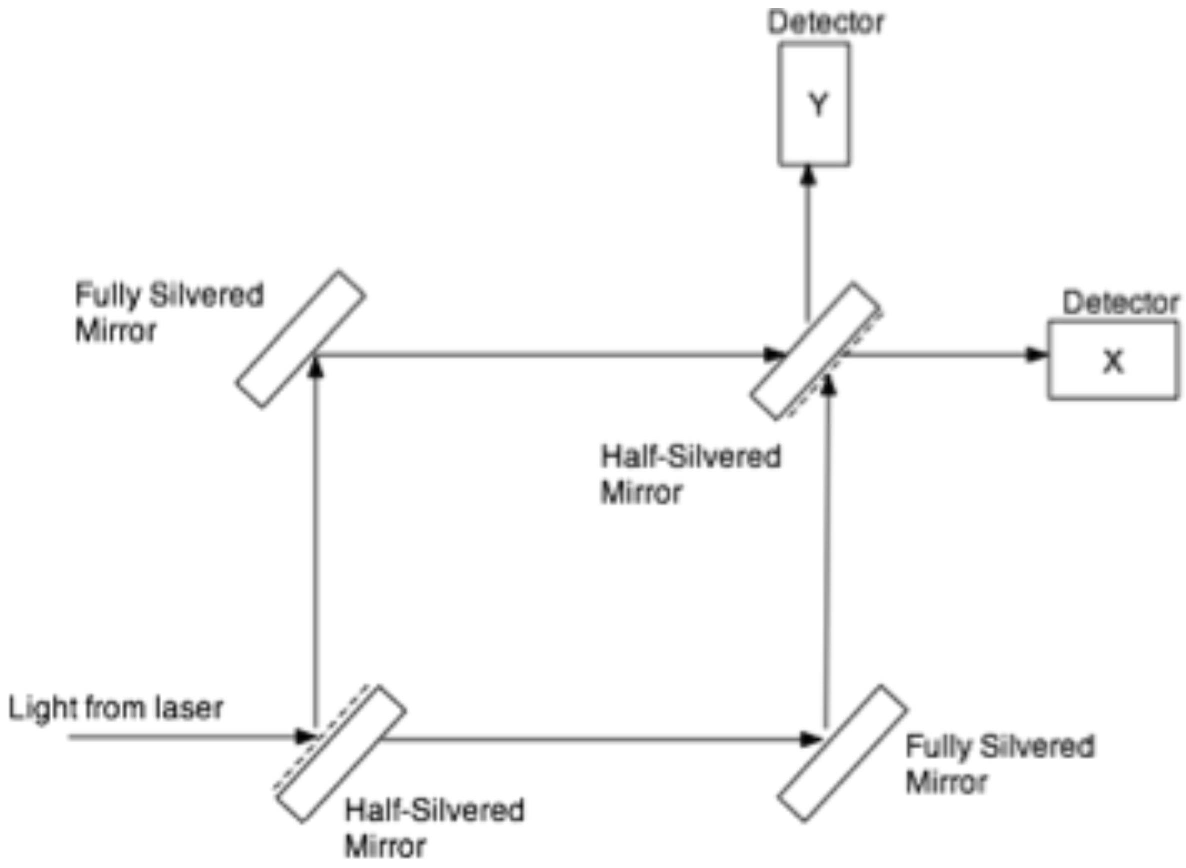
passing straight through

and other half being reflected.

Two new beams emerge

and eventually travel to pair of detectors

placed at X and Y. See figure



Beam heading to detector X

is combination of light that was reflected by 1st half-silvered mirror (travelled top path),

then transmitted by 2nd half-silvered mirror,

with light that transmitted by 1st half-silvered mirror(along bottom path)

and reflected by 2nd.

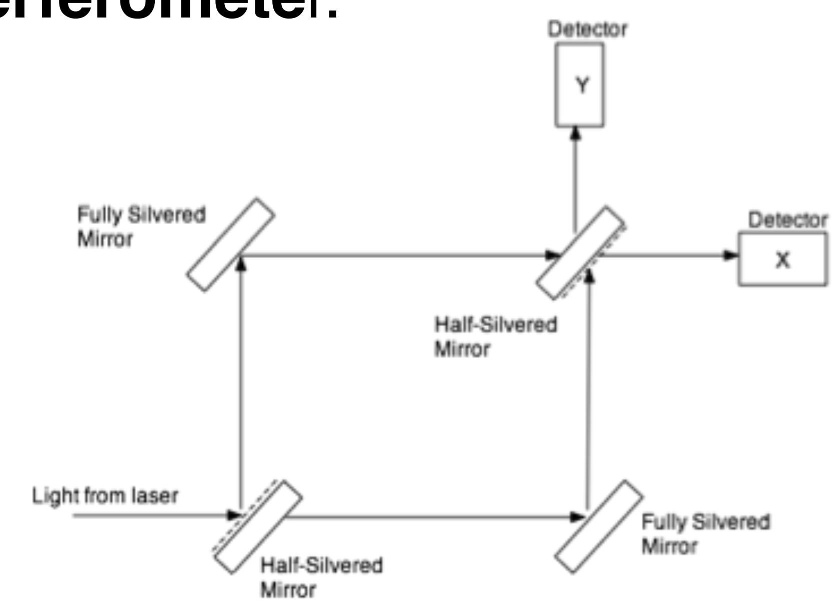
Detector Y collects light that is similar mixed combination.

Arrangement of mirrors and detectors called **Mach-Zehnder interferometer**.

Once set up, easy to confirm that

intensity of light reaching each detector depends critically

on **distances travelled** by light along top and bottom paths.



If equipment finely adjusted

so paths exactly same length,

detector Y records no light

while detector X records all of intensity (all photons)entering experiment.

Without critical adjustment,

X and Y collect light in varying amounts:

more light at X -> less reaches Y (and vice versa).

Using classical physics methods these effects can be completely explained by saying that light is a **wave when the intensity is high**.

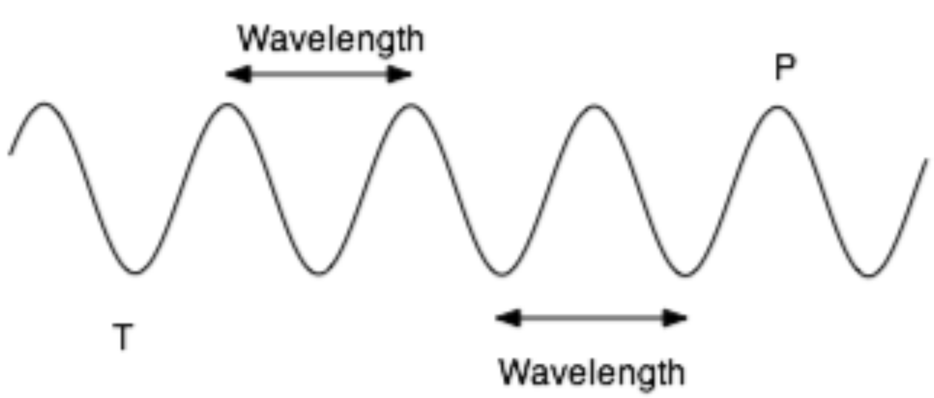
Interference as Wave Effect (reminder)

Consider ripples crossing surface of lake.

Ripples consist of places where water level is higher than normal (peaks)
and places where dropped below normal (troughs).

Wavelength of ripple is distance between successive peaks(P),
same as distance between successive troughs(T).

Frequency of wave is rate at which complete cycles (peak to trough to peak again)
pass fixed point, and period is time taken for one cycle.



Light(E/M fields) is more complicated than a water wave.

Peaks and troughs of light wave are not physical distances (height of water wave)
but are variations in strength of fields(E and B).

Mathematically, however, they are the **same** phenomena.

Thus, light waves are very sensitive measures of distance.

In interference experiment with interferometer,

divide distance travelled by light wave on route to detector into sections,

each having length equal to wavelength of wave.

Distance probably not whole number of wavelengths.

Furthermore, two different possible routes through experiment

have to be precisely same length to be precisely same number of wavelengths long.

If distances not precisely same,

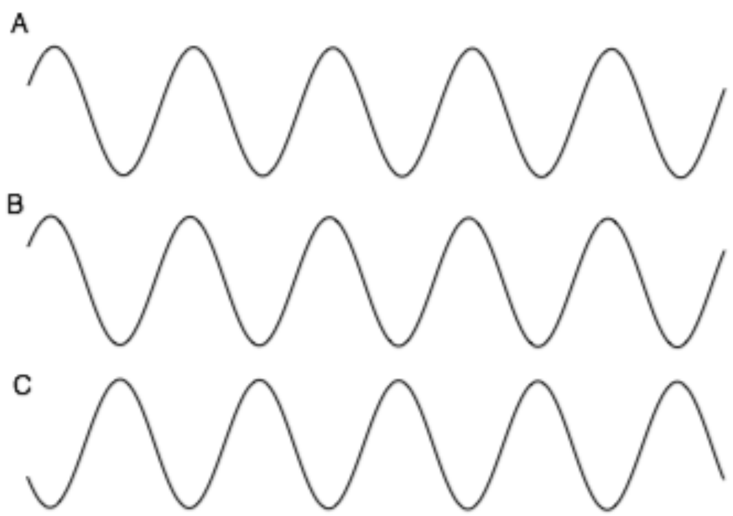
light traveling along each route consequently

have gone through different number of complete waves when gets to the detector.

As light in two beams has common source at first half-silvered mirror,

two beams will set off on different routes in phase

(i.e., in step - simplest definition of phase - both at same point on wave) with each other.



Waves labelled A and B in phase(peak to peak),

waves B and C exactly out of phase (peak to trough).

Above ->> set off peak for peak - so were in phase.

By time they get to detector

two beams may no longer be in phase (different distances travelled).

One could be arriving at peak, and other at trough (B and C).

If this happens, then waves will cancel each out —> no energy entering detector

called **destructive interference**

If still in phase would add up

called **constructive interference**).

Exact cancellation only happens if waves meet precisely peak to trough

not possible for any extended length of time

due to small variations in distance (mirrors shaking slightly)

and fluctuations in laser output.

Detailed analysis of interference experiment

also takes into account what happens to light at various mirrors

-> also influence phase of waves.

When light reflects off mirror,

reflected wave out of phase with incoming wave by half wavelength.

Using λ = wavelength, wave has undergone $\lambda/2$ phase shift (shift by 1/2 wavelength)

on refection.

Slightly different with half-silvered mirror,

surface that reflects from either side mounted on thin block of glass.

Dashed line in figures indicates reflecting surface.

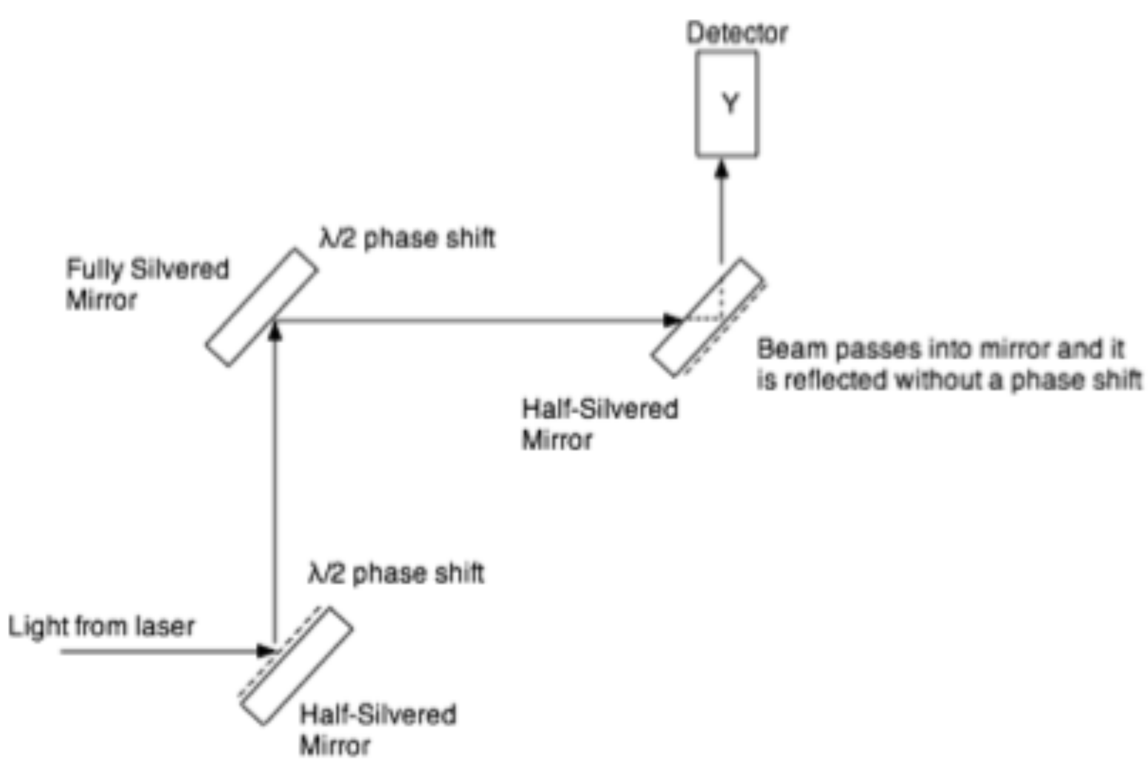
If reflection takes place off surface before light enters glass block,

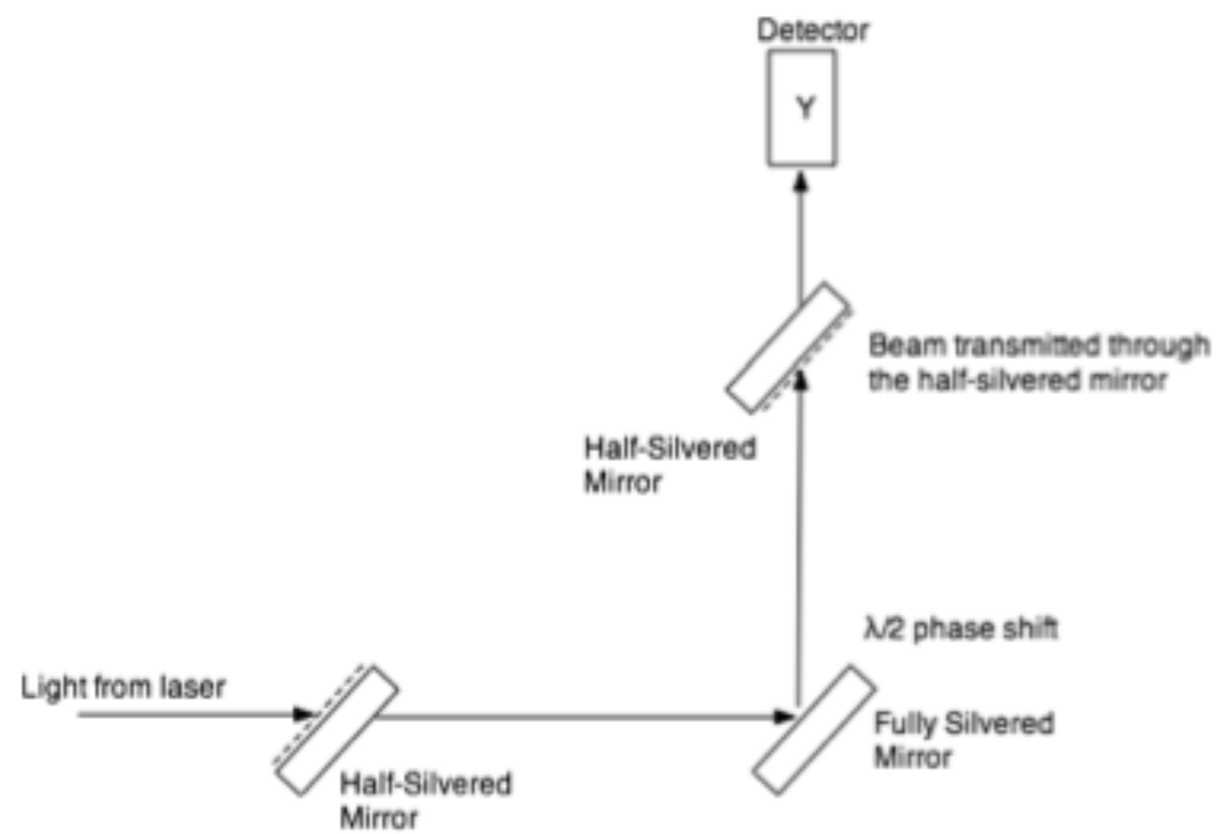
then ordinary λ/2 phase shift takes place.

However, any light that passes through block

before reaching reflecting surface not phase shifted on reflection.

See two figures below.





Two light beams reaching detector Y will be (overall) $\lambda/2$ out of phase.

Consequently, waves B and C and will completely cancel (if travelled equal distances).

If carry out same analysis for detector X,

i.e., chart progress of waves through interferometer to detector X,

find arrive in phase provided travelled equal distances.

Most experimental setups,

paths through interferometer not exactly equal

so waves not exactly in/out of phase.

Consequently, some light reaches both X and Y.

If equipment allowed movement of fully silvered mirrors,

so relative path lengths were changed,

then variation in brightness of light in X and Y

could be studied as mirror moved - as relative paths changed.

Modern version of Mach-Zehnder interferometer with Photons

Suppose we are able to turn down laser intensity

so that light beam made up of single photons entering apparatus at any time.

Assume have done so —> only one photon in experiment at any time.

Also have control over average rate.

Expect photons arriving at half-silvered mirror

to have 50:50 chance of going through/reflecting off.

Another possibility is

two reduced energy photons emerge from mirror, in each direction.

Can determine experimentally what happens:

place photon detectors just after mirror in path of each possible beam.

Simple experiment produces interesting result.

Half the time photon is reflected, and half the time is transmitted;

never get two photons coming out at same time.

However, no inherent difference

between those photons that get through and those that reflect.

No pattern to sequence,

except that after long time half reflect and half get through.

Sound familiar!

Effect is common in quantum physics.

Some aspects of nature's behavior lie beyond ability to predict (e.g., which way photon will go).

Question - does this reflect fundamentally random aspect to nature,

or is something more subtle going on that have not discovered yet?

Having established that photon reaching

1st half-silvered mirror in Mach-Zehnder interferometer

will either reflect and travel top path through device,

or transmit and follow bottom path,

now turn attention to what happens at detector end of device.

First find, between them, detectors pick all photons that enter experiment.

Number of photons arriving at either detector in given time depends on two path lengths,

i.e., if exactly equal then no photons ever arrive at Y.

If paths not exactly equal, then find that detection rate at each detector

reflects intensity of interference pattern

that would be observed when intensity is turned up.

What do we mean by that?

Let's imagine that had arranged for path lengths

such that 70% of total light intensity entering experiment arrives at X and 30% at Y.

No double photon firings.

Experiment done under well-controlled conditions

and no doubt that photon arrival rate directly reflects an interference pattern.

Doesn't sound like a problem, but there is.

If photon is small particle of light,

then how can different path lengths have any effect on one single photon?

Confirmed that photons randomly choose reflection or transmission at half-silvered mirror.

After that, surely proceed along one path or other to detector.

Hard to imagine single photon going along both paths at same time

remember rejected by experimental results

(detectors only registered one photon at a time).

Now a wave can do this.

Can spread out throughout experiment (ripples on lake)

so that parts of wave travel along each path at same time

(i.e., wave energy divides between paths).

When two parts of wave combine at far side of experiment,

information about both paths is being compared,

which leads to interference pattern.

A single photon must surely have information about only one path,

so how can single photon experiments produce interference patterns?

Flaw in arguments.

The flaw is extremely subtle

and leads to primary issue physicists face when dealing with quantum world.

We confirmed that photons divert at half-silvered mirror by placing detectors in two paths.

However, doing this eliminated any chance of picking up interference pattern.

If detectors have stopped photons, then **have not** travelled paths.

In principle, experiment does not tell anything about what happens when no detectors present.

Common sense to assume that photons do same thing with or without detectors,

but as already seen,

interference pattern for photons is not a matter of common sense.

In addition, color/hardness experiments

say that important whether or not detectors are present!

Investigate this further.

Place one photon detector after half-silvered mirror - in path of reflected beam.

If detect photon there, then would not get one at far side of experiment.

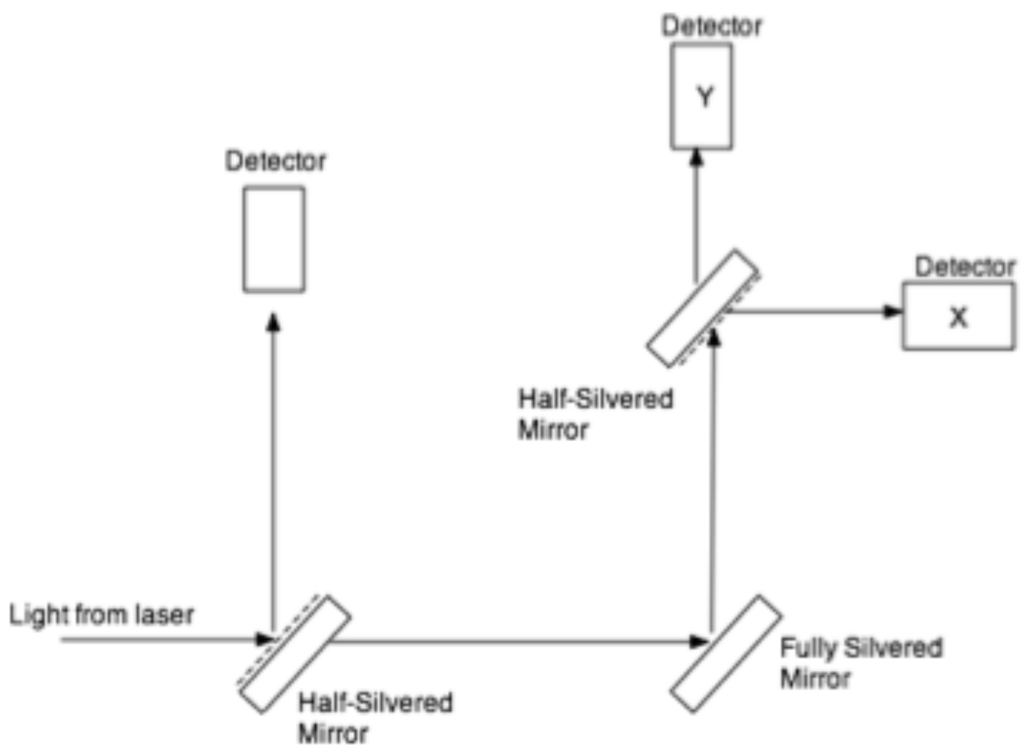
On other hand, if do not pick one up at detector

then it has passed through mirror rather than reflecting and should see it at far end.

Experiment easily done and confirms

that for every photon leaving laser pick one up either at far end

or in reflected beam as below.



Find for transmitted photons

half of arrive at Y and other half at X,

no matter what length of path is.

In other words, no interference takes place.

Removing detector on reflected path

(and replacing corresponding half-silvered

mirror) opens up that route

to far side of experiment again.

At same time

it removes any direct knowledge that might have

about behavior of photons at the half-silvered mirror.

Observe that doing this restores interference pattern!

Remember what happened in the color/hardness experiments.

Summarizing logic so we can expose what is happening and find flaw.

(1) Rate of photons arriving at far side of experiment related to intensity of bright beam.

(2) Moving mirror with bright beam maps out interference pattern in detectors.

(3) Reducing intensity of beam does not affect interference pattern - instead arrival rate of photons now depends on position of mirror

(4) If set up experiment so that can tell which path taken by photon (directly or indirectly), then interference pattern is destroyed.

(5) If unable to tell paths of photons, then there is interference pattern, implies photons arriving have information about both routes through experiment.

(6) Opening up top path (removing detector) can reduce number of photons arriving at Y.

In fact, if path lengths are same, opening up top path means never get any photons at Y.

All experimental results equivalent to Young-type two-slit interference experiment

and the two-path color-hardness experiment discussed earlier.

Changing experiment to a Delayed Choice configuration makes result even stronger!

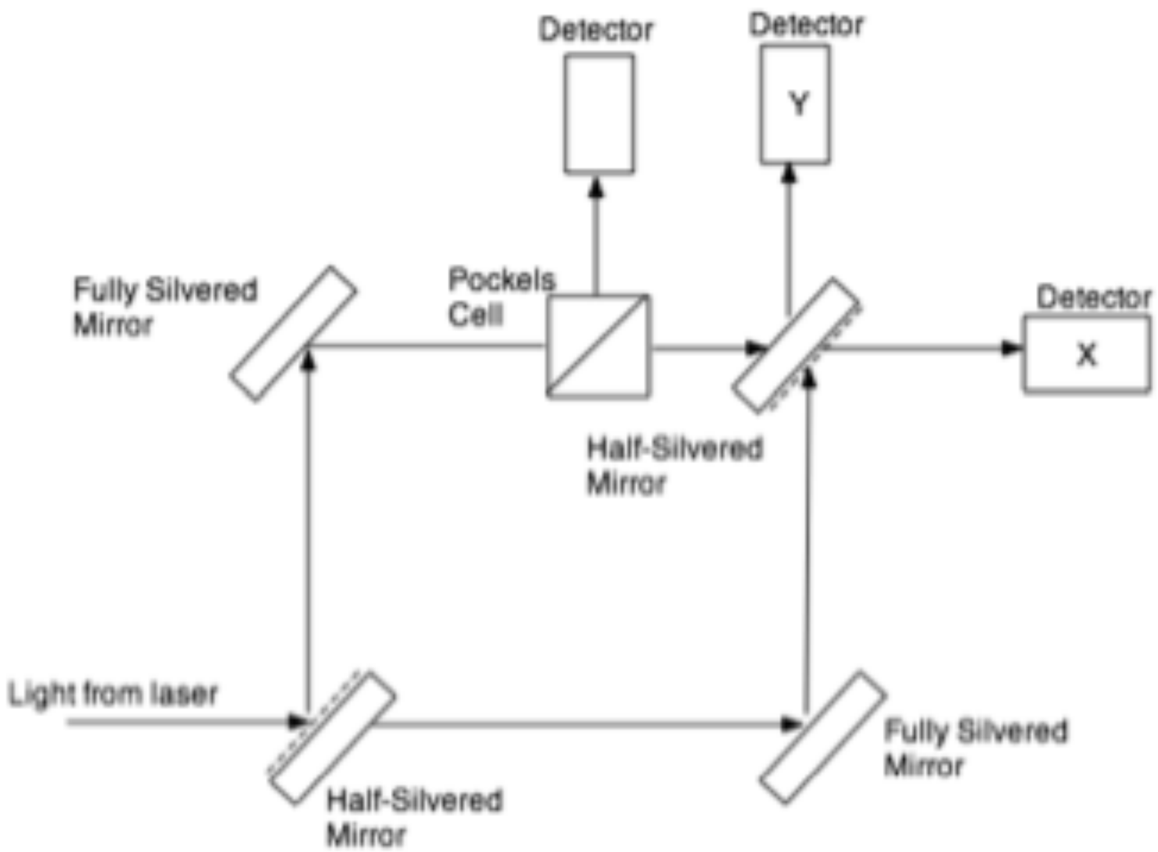
Let us see how.

Introduce Pockels cell(PC) (can divert photons extremely fast) on one route. Explain.

Try to **find out what photons doing while in interferometer.**

Consider setup where PC set to divert photons(to extra detector).

Photon leaves laser and arrives at first half-silvered mirror.



If reflected,

then PC will divert it so don't see at X or Y.

However, if photon transmitted

by first half-silvered mirror,

misses PC, and turns up at either X or Y.

Either case

no interference pattern

(have gotten which-path information).

If set PC to pass photons,

then changes what happens and get interference pattern.

For extreme case of equal path lengths, no photons ever arrive at Y.

Assume two path lengths exactly same. So have:

(1) If PC set to transmit, then get no photons at Y and all at X.

(2) If PC set to divert, then half of photons detected have equal chance of either X or Y.

Result should make us stop and think.

If photon takes lower route with PC set to divert,

then can get to X or Y.

If takes lower route with PC set to pass,

then photon can never arrive at Y.

But if takes lower route doesn't go anywhere near PC,

so how can setting of that device affect things?

Is this further hint that somehow or other photon travels both routes at same time?

Again this should sound familiar!

Now experimentalists get devious.

Set PC to divert photons, but while photon in flight,

switch cell over to transmit setting.

Ability to do this very rapidly means can make change

after photon has interacted with first half-silvered mirror.

No magic in doing this. If know when photon left laser,

can estimate how long takes to get to half-silvered mirror.

Provided we switch PC after this time,

but before photon had time to reach detectors X and Y,

then can perform experiment as described.

If setting of PC has(someway) influenced photon,

then original setting should have determined that photon takes one path or other

and certainly not both at once.

Now think we have changed setting after decision made by photon

[**NOTE:** Of course, word decision not appropriate.

Photons do not make decisions.

Clearly, hard not to be anthropomorphic when describing experiments].

In fact, can trigger PC in random manner.

Record setting of PC and match arrival of photons at one detector or another.

Can then run experiment for many photons

and record arrival at different detector positions.

After experiment run for while,

can analyze data.

Have some photons arriving at Pockels detector

and some at far end of experiment.

Latter group sorted out into those that arrived

when PC set to divert,

and those that made it when PC set to transmit.

Remarkably, when data separated,

photons that arrived at far side with PC set to transmit show interference pattern.

Other photons that arrived with PC set to divert

(obviously committed to other path and so missed it) show no interference pattern.

In every case PC set to divert photons and switched after photon left first mirror.

With PC set to divert, photons follow one route or other.

Then switched PC, destroying ability to know which path photons travelled,

and producing interference pattern.

Hard to believe that changing setting of PC can have influence

that seems to **travel backward in (retrocausality)** to when photon reaches first mirror.

Last statement, made using ordinary words,

not mathematics(nothing strong happening), may be total nonsense of course!

From both photon experiments and color/hardness experiments

have seen that quantum physics is contextual theory

-> adequate description of behavior of quantum object (light/electrons)

requires an understanding of whole (the context) experimental setup.

Quantum behavior depends on **context**!

Experimental results (answers to questions) **depend** on what questions we are asking!

Another Interference Experiment

Clearly, get into difficulty with experiments

when piece together understanding of whole

by looking at component parts on own.

When everything together, things behave differently. **whole > sum of parts!!**

Results of one experiment not accurate guide to another.

If this is case,

then conclude that photons always take only one route,

as indicated in experiments that look for route followed.

However, know that as soon as do not have ability

to tell path of photons

they take both routes at once

or something that is equivalent to that,

or maybe nothing we can imagine with our macro world minds/words!

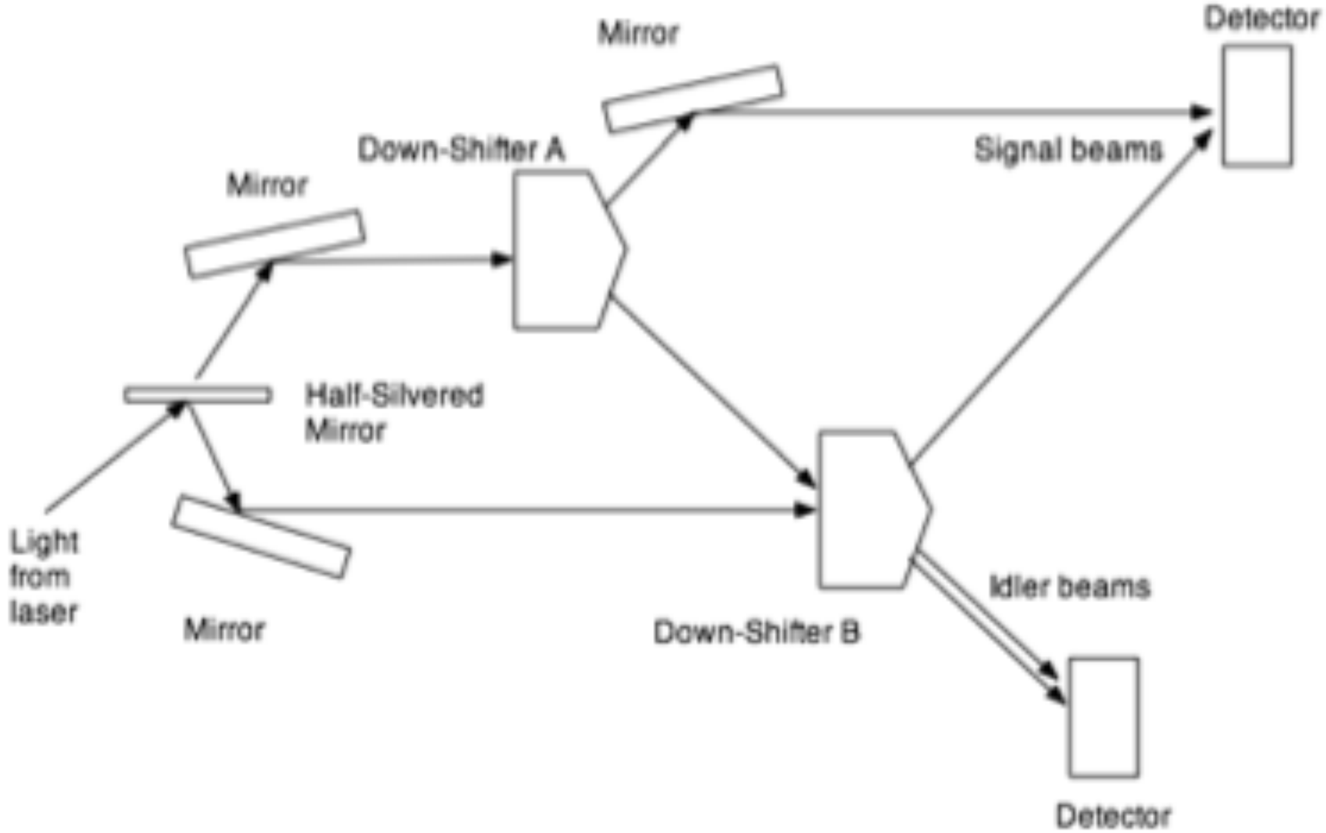
Another experiment pushed notion further

by showing interference pattern can be destroyed

without direct influence on photons creating it.

Experiment uses crystal known as down-shifter.

Device absorbs photons to produce 2 new photons, each with half the energy.



Laser light is sent onto 1/2-silvered mirror

and 2 beams separately

directed into down-shifters.

Each down-shifter produces

signal beam and idler beam,

difference between two beams

nothing more than **subsequent** way used.

2 signal beams directed to detector

produce interference pattern (different path lengths)(same as Mach-Zehnder).

Idler beam from down-shifter A mixed with that from down-shifter B

and both beams arrive at second detector.

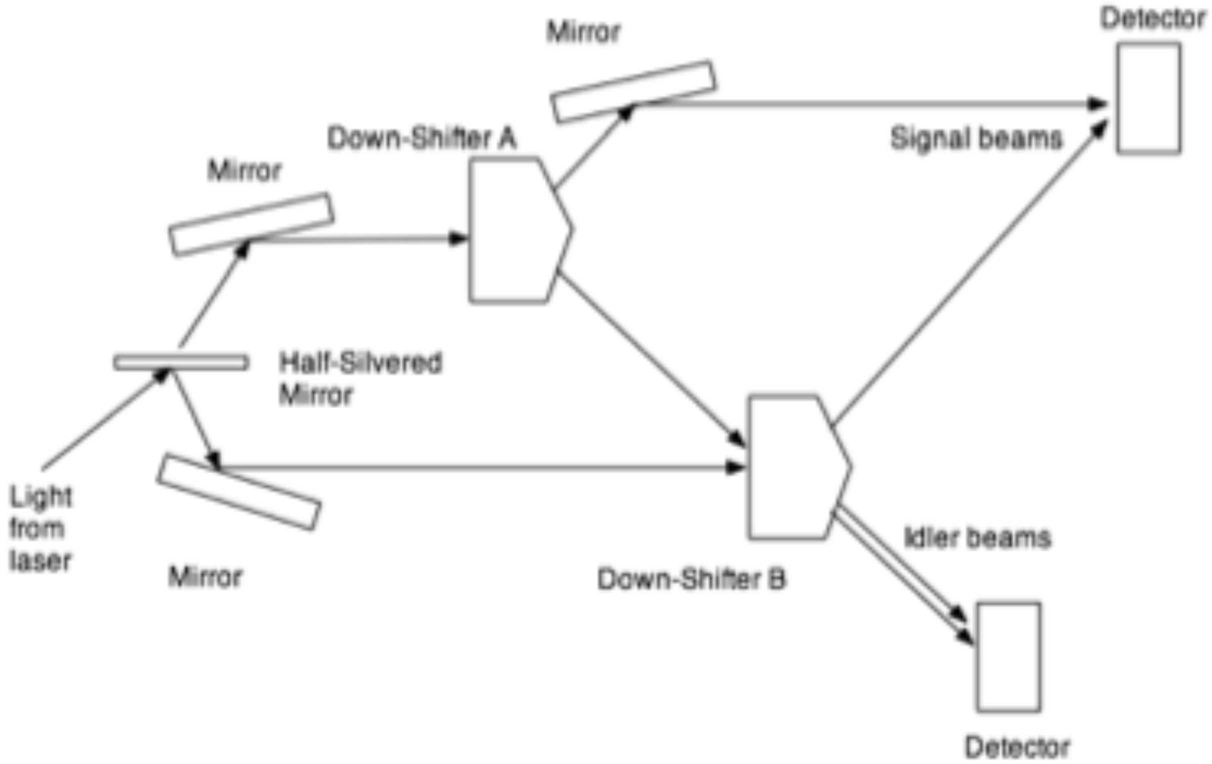
Upshot is that every time photon leaves laser,

photon of half energy arrives at each detector.

Fact that interference pattern emerges

-> in some manner,

each photon appears to have travelled along both signal beam paths.



Say photon arrives at half-silvered mirror

and goes on top path (only that path).

Photon arrives at down-shifter A

and produces 2 further photons,

one ends up at signal detector

and other at idler.

No interference pattern

since no information carried to signal detector about other route.

Same true if photon took lower route through experiment.

Only way to get interference pattern at signal detector

is for information to arrive from both routes,

have to be two signal beams, one from each down-shifter.

If true,

then down-shifters have to be activated

by something arriving at each one —->

makes it appear that photon from laser went both ways at half-silvered mirror.

Presence of two signal beams

doesn't imply that two photons are arriving at signal detector

there is only one at a time arriving.

Most bizarre feature of experiment

is way in which interference pattern can be destroyed

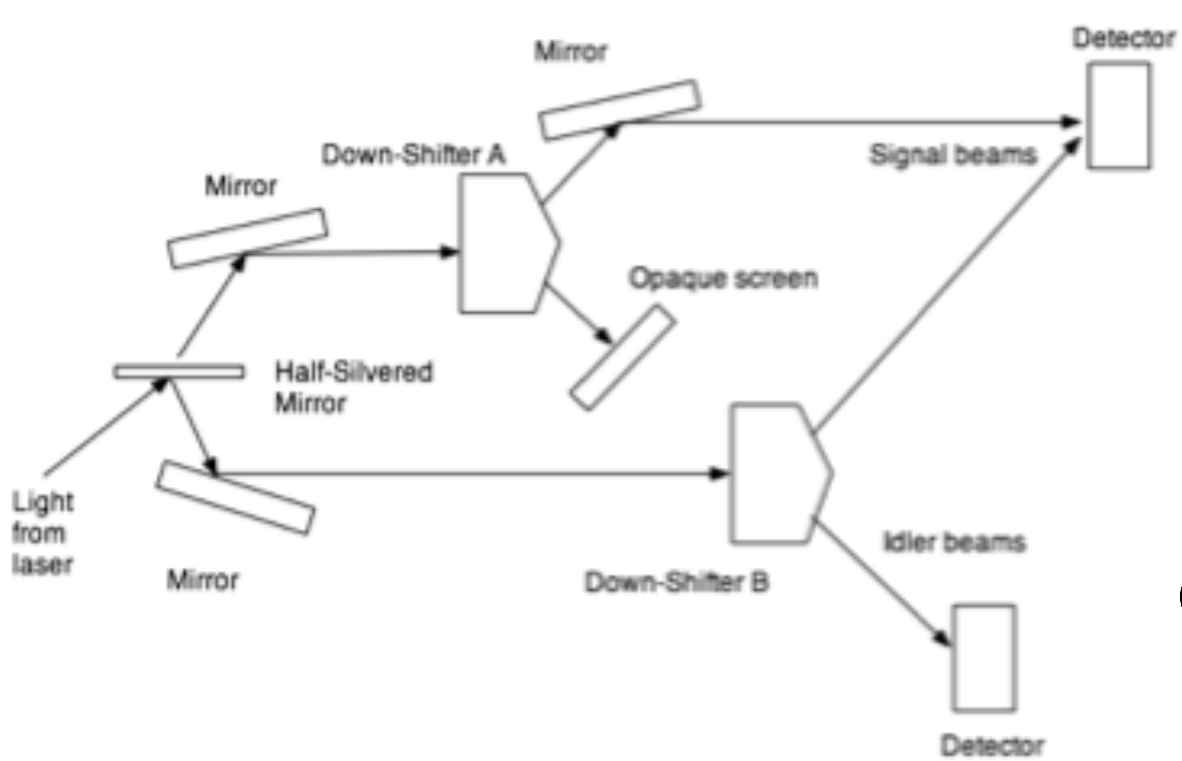
if have ability to tell path of the photons, even if don't choose to use this information.

Threat of doing this, or rather fact that

experiment(context) is set up to allow possibility, is enough to destroy interference.

Dramatically confirmed by blocking one of idler signals, say from down-shifter A

Logic here remarkable.



Whenever photon picked up

at idler detector we know must have come

from down-shifter B.

Means that photon from half-silvered mirror

must have hit down-shifter to be converted

into photon that arrives at idler detector.

Can further deduce that other photon

from down-shifter B travelled to signal

detector and therefore is photon detected there.

Tracing argument further back,

photon that definitely hits down-shifter B

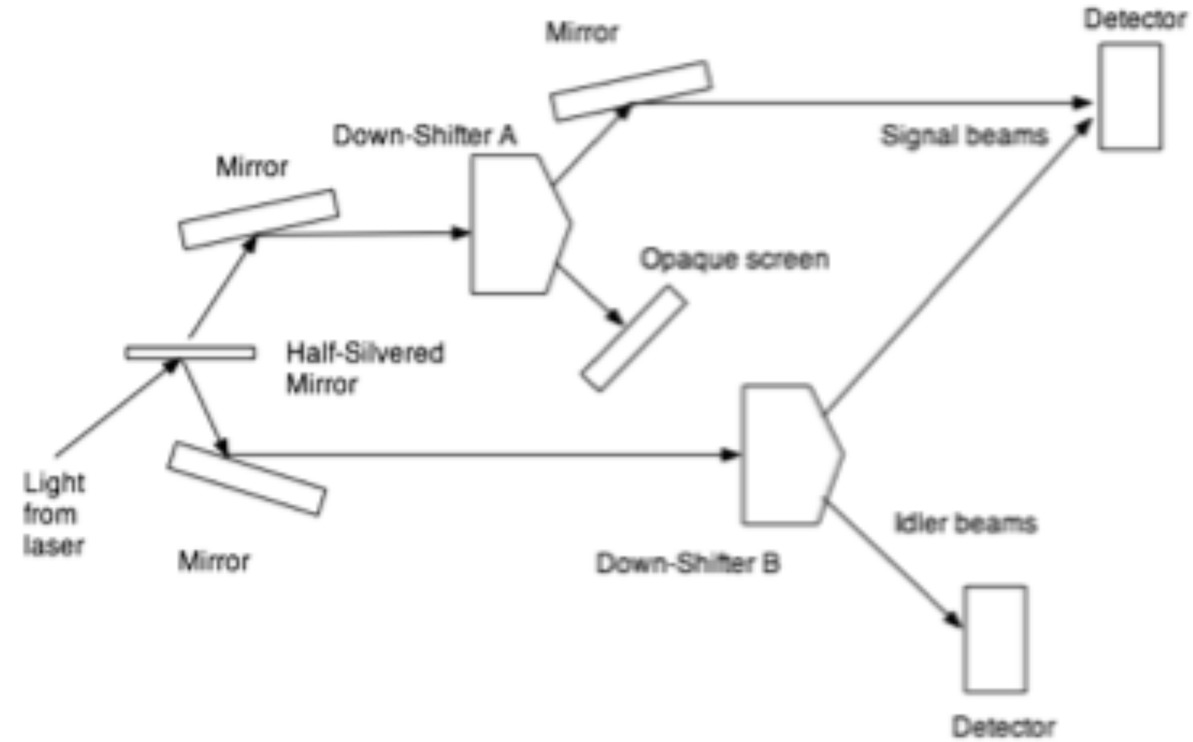
must have come from half-silvered mirror.

There is no ambiguity about route that this

photon takes from mirror.

Nothing goes along top route; nothing produced

so interference pattern disappears.



As long as idler route from down-shifter A open,

have no way of telling which shifter photon came from.

Ambiguity sufficient to guarantee interference pattern at signal detector.

If don’t know that photon at idler detector came from B (or A),

then don’t know about signal photon either.

Under those circumstances, can’t say which route photon took at half-silvered mirror,

so takes both routes or “something” like that.

Seems that behavior of photon determined by **context** of experiment as whole.

Know no photons coming from down-shifter A,

why does it matter that idler route from A is blocked?

How is information conveyed back to half-silvered mirror

so as to determine what happens there?

words FAIL us, MATH works!!

Quantum Erasers (Complementarity and Entanglement)

In earlier discussions, we saw that closing the locality loophole involved switching between different analyzer orientations while emitted photons were still in flight.

The choice between the nature of the measurement was therefore delayed with respect to the transitions that originally created the photons.

Is it possible to make this delayed choice between measuring devices of a more fundamental nature?

For example, in discussion of double-slit measurements, found that if allow sufficient number of photons individually to pass through slits, one at a time, interference pattern will be built up.

It **seems** that photon has knowledge of the paths through passes through both slits.

As noted earlier, a skeptical physicist who places a detector over one of slits to show that photon passes through one or other does indeed prove their point - photon detected, or not detected, at one slit.

But then the interference pattern can no longer be observed.

Advocates of local hidden variable theories could argue that the photon is somehow affected by way we choose to set up our measuring device.

It thus adopts a certain set of physical characteristics (owing to existence of hidden variables) if apparatus is set up to show particle-like behavior, and adopts different set of characteristics if apparatus is set up to show wave interference.

However, if we design an apparatus that allows us to choose between these totally different kinds of measuring device, we could delay our choice until photon was (according to local hidden variable theory) **committed** to showing one type of behavior.

Suppose that photon cannot change its mind **after** it has passed through slits, when it discovers which kind of measurement is being made (whatever last sentence given in "words" may actually mean!!)

Ultimate Delayed Choice or Quantum Eraser Experiment

QM says systems can change behavior depending on measurements made on them or in response to decision that has not yet been made.

One part of an entangled pair can affect properties of its partner instantaneously, no matter where in universe partner happens to be.

A so-called quantum eraser experiment has now been done...it dramatizes several aspects of quantum strangeness at once.

Experiments dramatically show non-local effects, i.e., ability of experiment in one place to influence outcome of another regardless of time or distance – but without transmitting any signals.

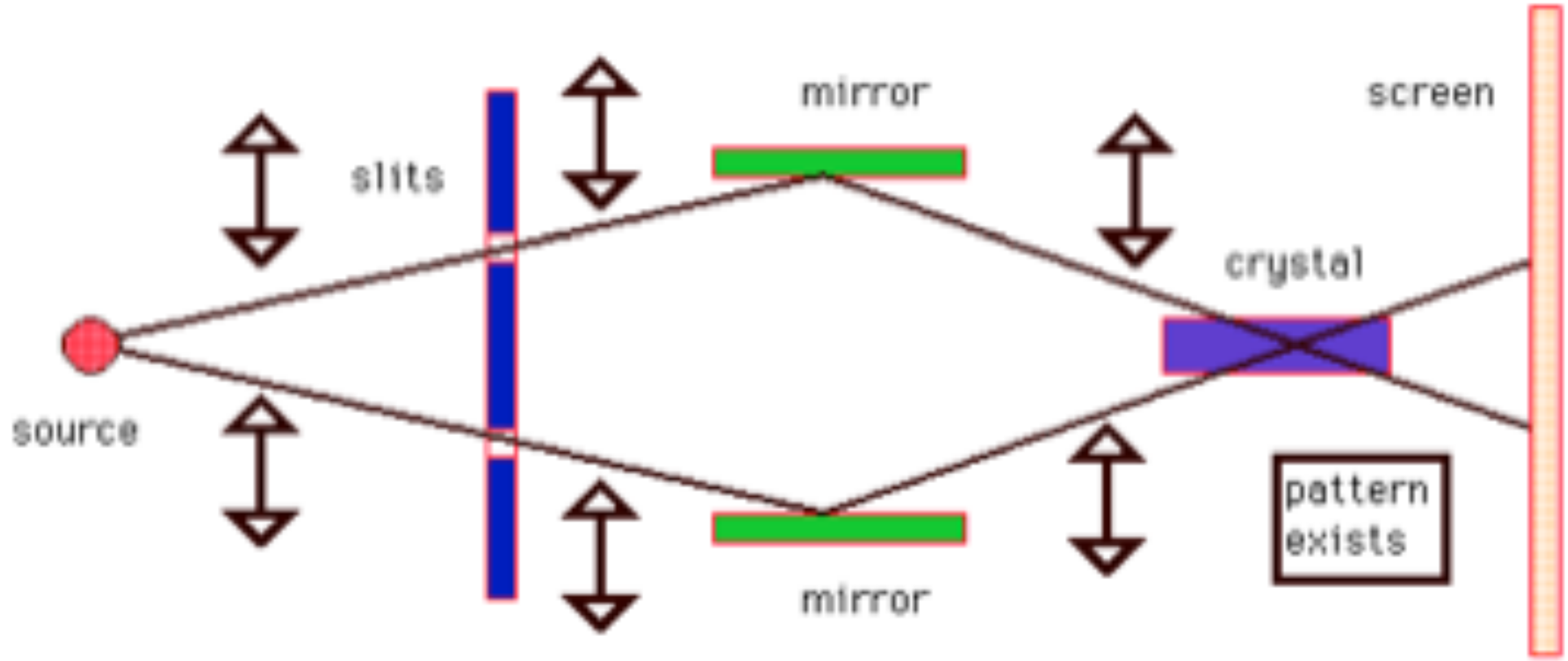
The idea behind the quantum eraser is to make paths(like those in a 2-slit system) distinguishable, which eliminates any interference effects, but then erase the which-path information just before light reaches screen where we actually observe the interference pattern.

QM predicts that interference pattern should then reappear and it does.

A quantum mystery of following sort.

A photon approaching slits will need to know whether or not there is an eraser further down path(in its future), so that it can decide whether to pass through slits as a superposition of all possibilities (paths are indistinguishable) and produce an interference pattern later on screen or that should behave as if paths are distinguishable and produce no interference pattern later on screen!!

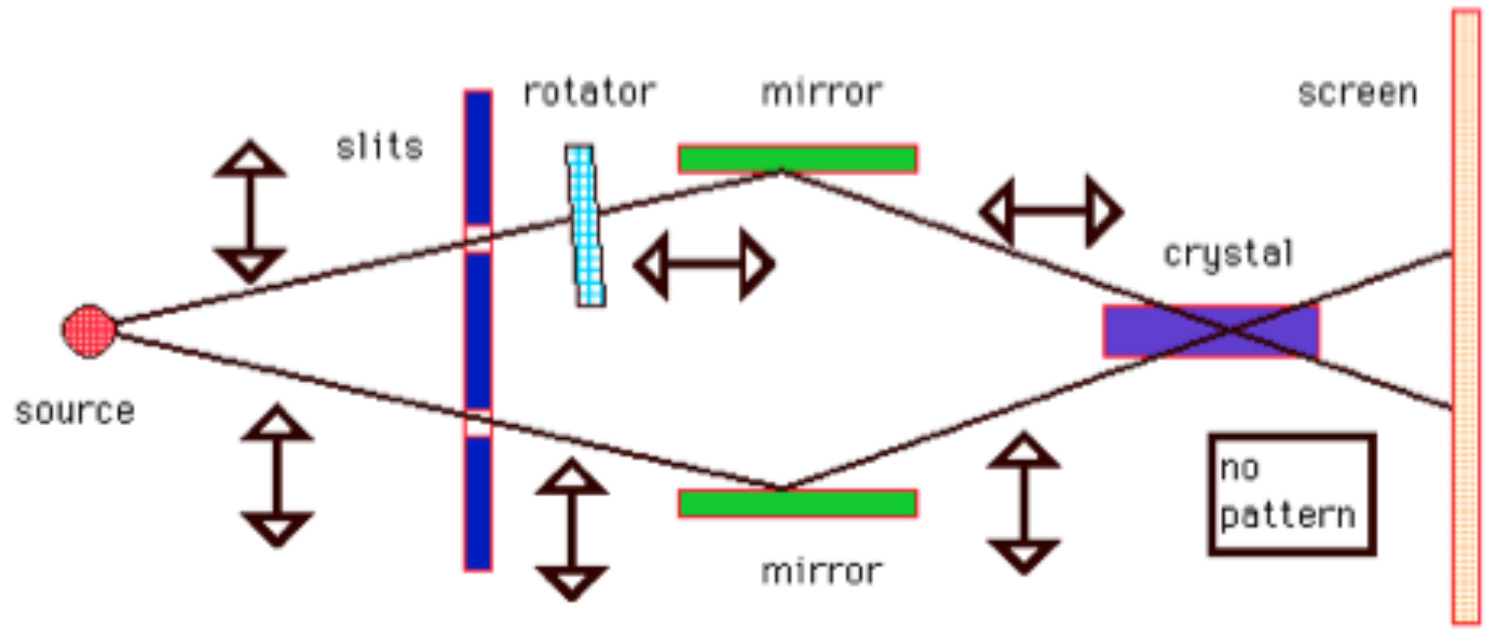
An early eraser experiment can be visualized as the two-slit experiment as shown in diagram below:



Photons passing through double slit are all vertically polarized.

They can get to recombination crystal by two paths as shown, which remakes beam that produces an interference pattern on screen, i.e., if paths are indistinguishable, then have superposition of all possible paths(2 paths in this case) and get an interference pattern.

Now insert a polarization rotator on one path only

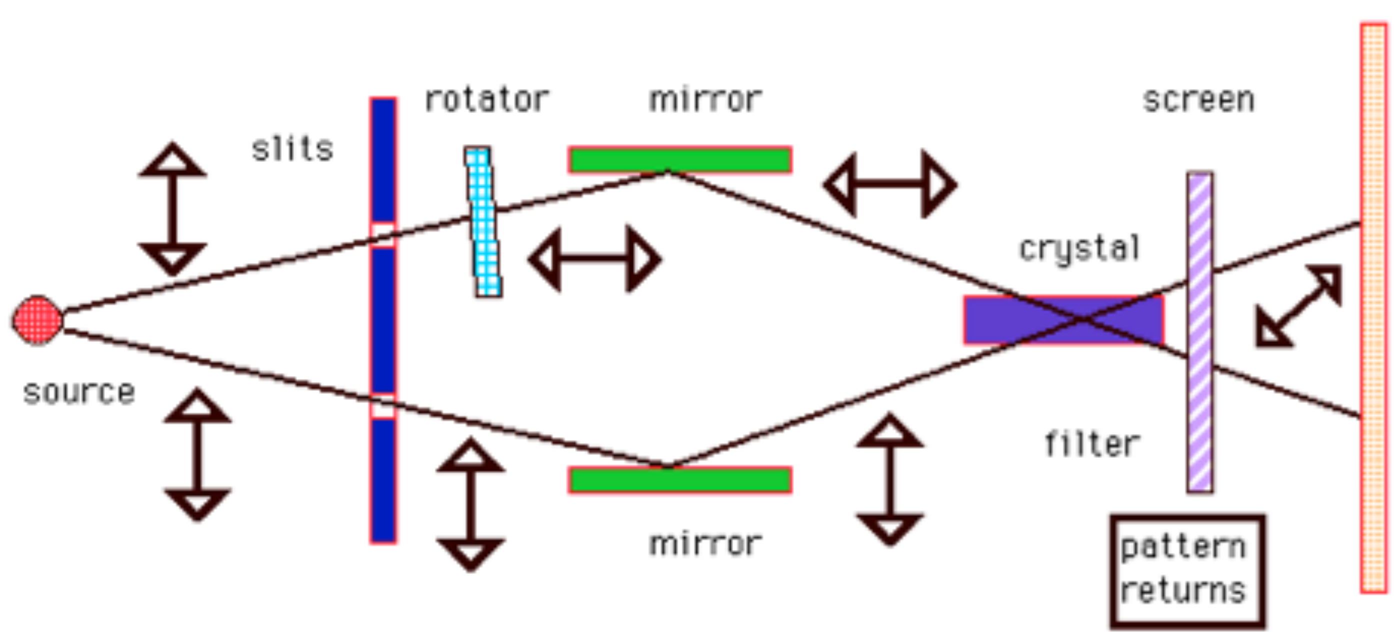


and rotate the polarization of photons to horizontal in that path.

Since the paths now produce distinguishable photons, we get particle like behavior and the interference should disappear.

It does!

Now add the quantum eraser after recombination crystal —> a polaroid at 45°.



The polaroid filter produces equal numbers of photons with both vertical and horizontal polarization (with respect to the new direction).

Half of new vertically polarized photons come from horizontal polarized photons on top path and half of new vertically polarized photons come from vertical polarized photons on bottom path (similarly for new horizontally polarized photons).

It is impossible to tell whether photon was vertically or horizontally polarized before the eraser.

Thus, the two paths are once again indistinguishable.

If the interference pattern reappears, then the photon approaching slits somehow needs to know whether or not there is an eraser down line so can decide whether to pass through slits as superposition and produce interference effects or as mixture and produce no interference.

The pattern reappears immediately thus confirming the quantum mechanical prediction.

The filter erases the which-path information caused by the rotator.

A truly astounding result.

What does this say about the classical idea that it is the two-slit system that is the real cause of the interference pattern?

Worry:

Experimenters, knowing the fundamental importance of these results, wanted to leave no possible source of controversy intact.

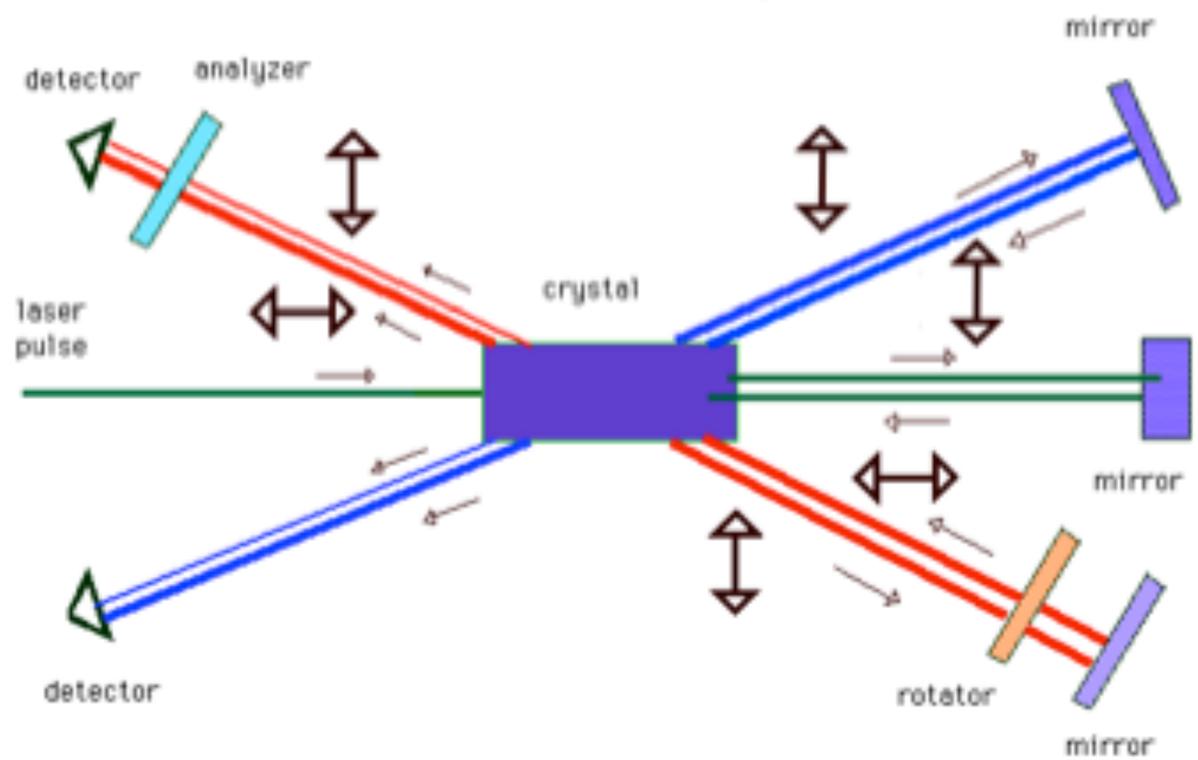
In the above version of experiment, there is a potential problem because the which-path information is carried by same photons that interfere, making experiment difficult to interpret.

New version of experiment...

Here which-path information is not carried by what one would naively call interfering photon.

Instead, carried by 2nd photon and along way it also demonstrates the striking non-local effects of QM.

Experiment looks like:



High intensity laser photons are sent into a parametric down-conversion non-linear crystal such as lithium iodate.

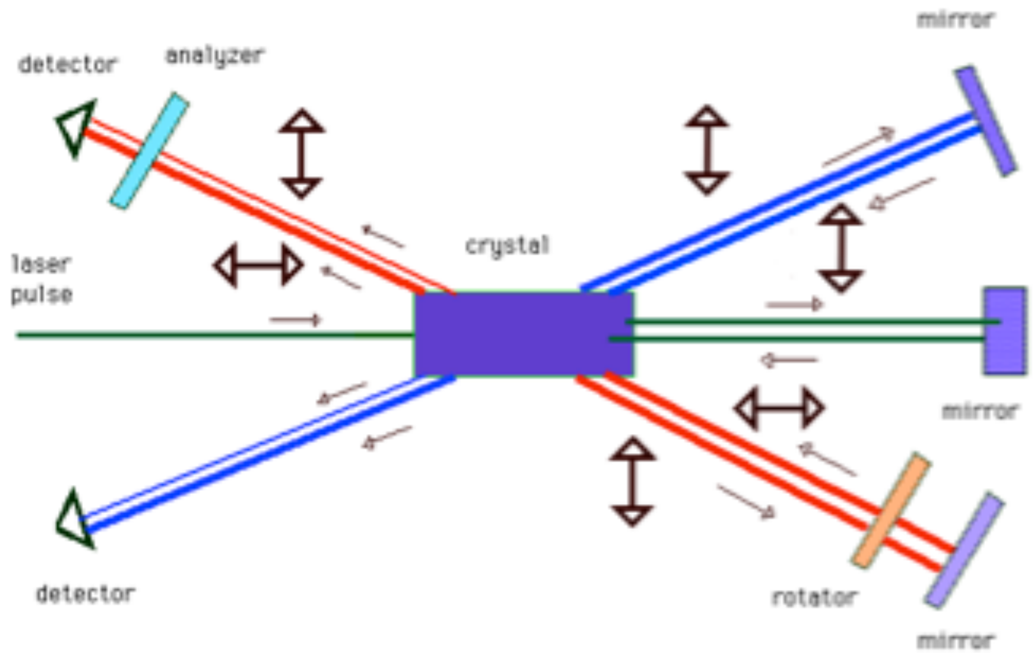
The crystal converts some incoming photons(green) into pairs of identical photons with lower energy and vertical polarization moving at angle to original direction.

The photons are produced as entangled partners.

Measurement on one photon automatically tells us about other with no direct measurement on 2nd photon necessary.

Twin beams exit the crystal at angle to the original path.

These are the thick red and blue lines



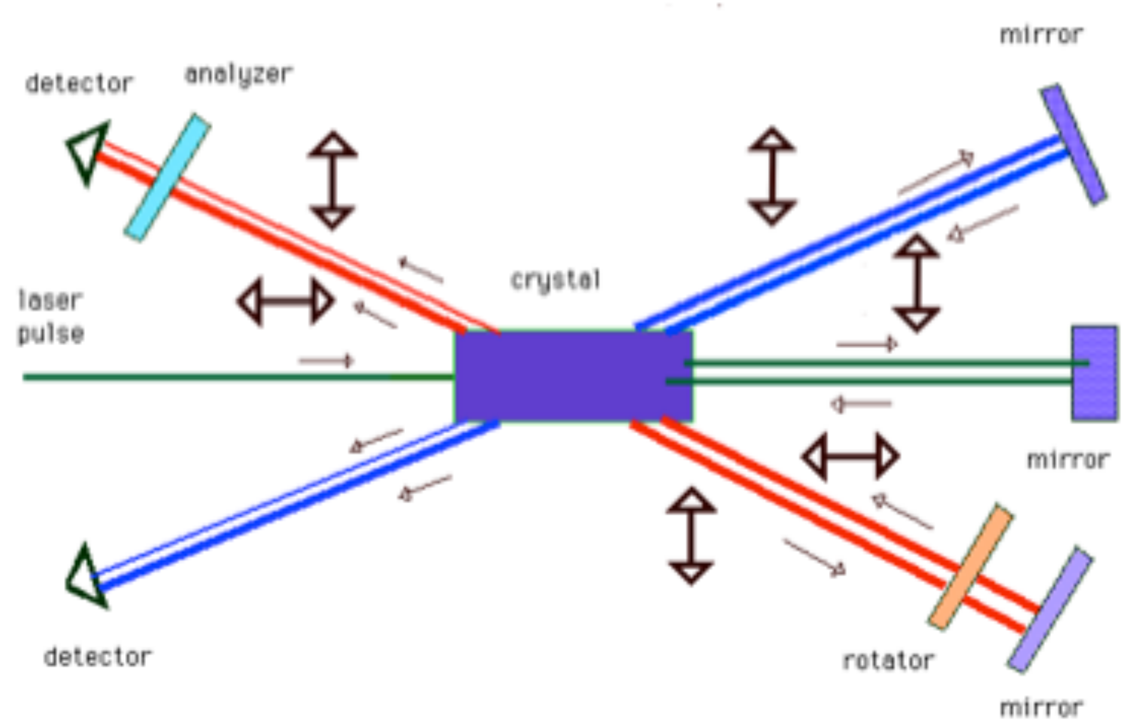
They are reflected back towards the crystal by mirrors and pass straight through it to detectors (the second-pass intensity is now too weak to cause down-conversions).

These are thick red and blue lines.

However, not all laser light gets converted on 1st pass.

Some goes straight through crystal to another mirror(green) and is reflected back into the crystal(still high intensity) where the crystal creates more photon pairs which then follow same path as other beams to the detectors.

These are thin red and blue lines.

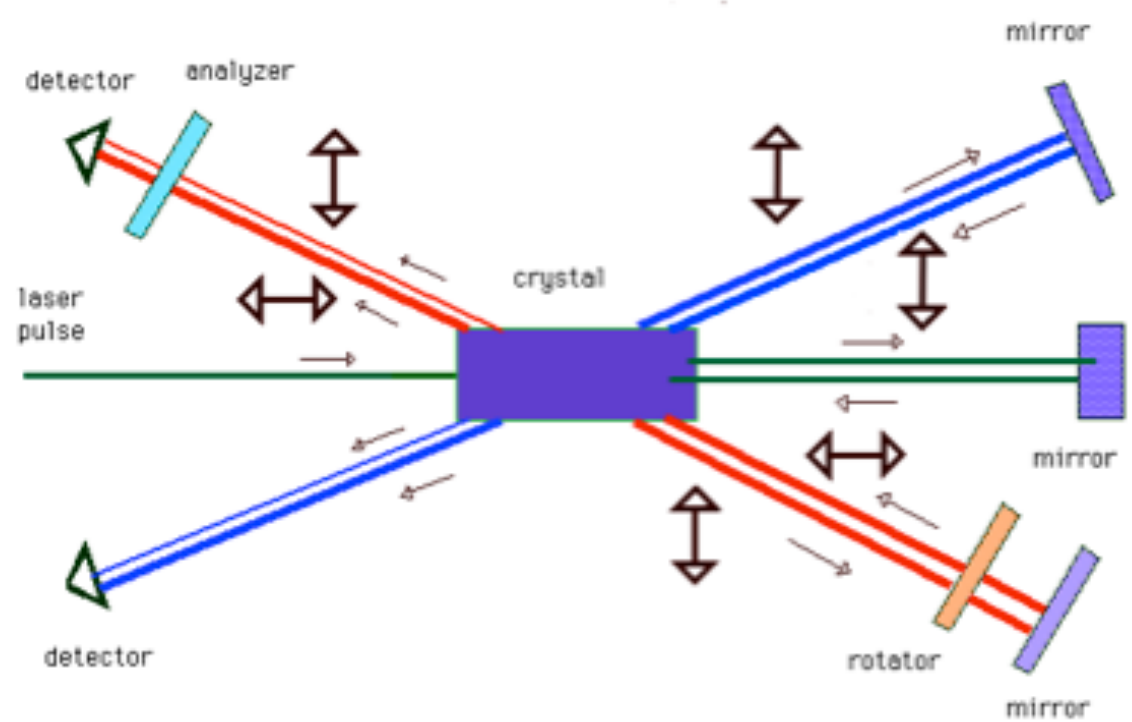


As a result, have two different beams heading towards each detector each having follow different paths.

Each pair of beams corresponds to a separate double slit experiment.

If there is no way to distinguish photons created on first pass through crystal from those created on 2nd pass(and there is not), both detectors should have interference patterns and they do!!

Now make one returning beam in one leg distinguishable from other by inserting polarization rotator into red path(as shown below) converting vertical to horizontal polarization in that leg.



The interference pattern in top detector vanishes instantly as it should since the two (interfering) beams now have distinguishable paths.

Now, however, also find that interference pattern disappears in bottom detector!!!!!!! Why?

We have done nothing to disturb these beams so that beams in the bottom detector still correspond to indistinguishable paths and photons!!!!

Remember, however, that photons are created in entangled pairs, so when red-path photons become labeled with which path info, the same info becomes available to blue-path photons, no matter where they are!!!

This is non-locality at work.

To erase which-path info, we now add a 45° polaroid in red path, just in front of detector.

The interference pattern immediately reappears in the top detector as it should (same as previous experiment).

It seemed however, that pattern did not reappear in bottom detector.

One might imagine this is so because erasing red-path photon information does not erase any information from blue path.

In addition, if it did reappear immediately, then we would be able to send signals faster than light with such an eraser.

However, as in EPR(we will see) experiment we find, if we bring the two data sets back together and compare them, the pattern had, in fact, been restored along both paths (need both data sets) to see **correlations** among individual photons.

It seems like the correlations are entangled and not the state vectors!!

We will have more to say about this later when we talk about measurement.

Alternatively, one can do coincidence measurement, which only looks at those photons counted in each detector simultaneously and one can see interference pattern return directly, that is, which way and interference effects are being recorded for single photons.

Thus, inserting or removing which-path information transforms behavior of light throughout entire system simultaneously demonstrating amazing quantum eraser and dramatic non-local behavior of QM.

This experiment makes it clear that there is direct relationship between tests of complementarity and tests of quantum non-locality.

Interference effects are a direct manifestation of non-local behavior.

These effects can be encoded in the mathematical structure of quantum entanglement - in this case, the entanglement of the states is responsible for the interference with the state used to detect the "which way" information.

These states cannot be disentangled without forcing the system to reveal one type of behavior or other.

They cannot be disentangled to reveal both types of behavior simultaneously.

Though still a subject for debate, consensus is building that complementarity - and hence non-locality and entanglement - is the mechanism for mutual exclusivity in the dual wave-particle nature of quantum objects - what Richard Feynman described as central mystery at heart of quantum mechanics.

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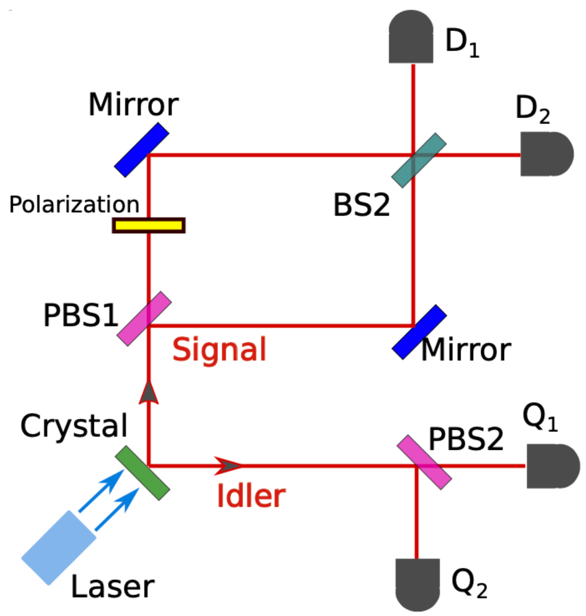
The Vienna Delayed-Choice Quantum Eraser Leaves No Choice

https://arxiv.org/pdf/2010.00049.pdf



Experimental Setup

Abstract

A realizable delayed-choice quantum eraser, using a modified Mach-Zehnder (MZ) interferometer and polarization entangled photons, is theoretically analyzed here. The signal photon goes through a modified MZ interferometer, and the polarization of the idler photon provides path information for the signal photon. The setup is very similar to the delayed-choice quantum eraser experimentally studied by the Vienna group. It is demonstrated that in the class of quantum erasers with discrete output states, the delayed mode leaves no choice for the experimenter. The which-way information is always erased, and every detected signal photon fixes the polarization state of the idler, and thus gives information on precisely how the signal photon traversed the two paths. The analysis shows that the Vienna delayed-choice quantum eraser is the first experimental demonstration of the fact that the delayed mode leaves no choice for the experimenter, and the which-way information is always erased.

I leave this for you to look at if interested. You will find you can understand all the Dirac quantum mathematics in the papers and should be able to follow it.