Stable Facts, Relative Facts

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FACTS IN QUANTUM THEORY

The common textbook presentation of quantum theory assumes the existence of a classical world.

A measurement involves an interaction between the classical world and a quantum system.

The measurement produces a definite result, for instance a dot on a screen.

The result is a fact by itself, but also establishes a fact about a quantum system.

For instance, a certain measurement resulting in a definite record establishes that at some time the z-component of the spin of an electron is $L_z = \hbar/2$

This is a fact.

Quantum probabilities are probabilities for facts, given other facts.

The facts are therefore entries of which the probability amplitudes are function.

In particular, facts are used as conditionals for computing probabilities of other facts.

For instance, if the spin of the electron mentioned above is then immediately measured in a direction at an angle θ from the z-axis, the probability to find the value $L_\theta = \hbar/2$ (a fact), given the fact that $L_z = \hbar/2$ is $P(L_\theta = \hbar/2 \mid L_z = \hbar/2) = \cos^2(\theta/2)$.

Hence, facts are what quantum mechanics is about.

Facts ascertained in a conventional measurement are *stable* in the following sense.

If we know that one of N mutually exclusive facts a$_i$ (i = 1...N) has happened, the probability P(b) for a fact b to happen is given by

$$P(b) = \sum_i P(b|a_i)P(a_i), \qquad (1)$$

where P(a$_i$) is the probability that a$_i$ has happened and P(b|a$_i$) is the probability for b given a$_i$.

We take equation (1) as a characterization of stable facts.

This textbook presentation of quantum mechanics is incomplete because it assumes the existence of a classical world.

An exactly classical world might exist because current quantum theory has limited validity—for instance it is violated by physical collapse mechanisms, or cannot be extended to systems with an infinite number of degrees of freedom, or else.

But the universal success of the theory and all current empirical evidence strongly suggest that real physical objects are 'classical', meaning they do not display quantum properties, only approximatively.

There are no *exactly* classical objects, strictly speaking, as everything we interact with is made of atoms and photons, which obey quantum theory.

In formulating the fundamental theory of nature, the use of effective concepts valid only within an approximation is unconvincing.

Therefore the attempts to interpret quantum theory as a universal theory, such as Many Worlds, Hidden Variables, and others do not rely on postulating classical objects.

A possibility to interpret quantum theory as a universal theory neither postulating classical objects, nor postulating unobservable worlds, unobservable variables, or unobserved physics, is Relational Quantum Mechanics (RQM). RQM bases the interpretation of the theory on a larger ensemble of facts, of which stable facts are only a subset.

These are called *relative* facts.

Relative facts

Relative facts are defined to happen whenever a physical system interacts with another physical system.

Consider two systems S and F.

If an interaction affects F in a manner that depends on the value of a certain variable L_S of S, then the value of L_S is a fact relative to F.

This is true by definition irrespectively of whether F is a classical system or not.

That is, whenever the two systems interact, the value of the variable L_S becomes a fact relative to F.

The interaction with F is the context in which L_S takes a specific value; we call the system F, in this role, a 'context'.

The interaction with the context determines the fact that a certain variable, L_S , has a certain value.

Stable facts are only a subset of relative facts, as there are many relative facts that are not stable facts.

Quantum theory provides probabilities that relate relative facts, but these satisfy (1) only if b and the a_i are facts relative to the same system.

That is, if we label facts with the system they refer to, writing $a^{(F)}$ for a fact relative to the system F, it is always true that

$$P(b^{(\mathcal{F})}) = \sum_i P(b^{(\mathcal{F})}|a_i^{(\mathcal{F})})P(a_i^{(\mathcal{F})}), \qquad (2)$$

while it is in general false that

$$P(b^{(\mathcal{W})}) = \sum_i P(b^{(\mathcal{W})}|a_i^{(\mathcal{F})})P(a_i^{(\mathcal{F})}) \qquad (3)$$

if W ≠ F.

When (3) holds, we say that the facts $a^{(F)}$ are stable with respect to W.

The failure of (3) is easily understood in terms of the standard language of quantum theory.

If F is sufficiently isolated it may be possible to maintain the quantum coherence of the coupled system S−F formed by S and F together.

The interaction entangles the two systems and interference effects between different values of the variable L_S can later be detected in the measurements by an observer W.

The probabilities for facts of the S−F system relative to W, indeed, can be computed from an entangled state of the form

$$|\psi\rangle = \sum_i c_i |Fa_i\rangle \otimes |\psi_i\rangle, \qquad (5)$$

where a_i are values of L_S and Fa_i are values of F's 'pointer variable' L_F.

Probabilities computed from this state feature interference terms, and this violates (3) because what sums is amplitudes, not probabilities.

The value of L_S, therefore, is not a stable fact.

Hence facts relative to a system F cannot in general be taken as conditionals for computing probabilities of facts relative to a different system W.

Equation (1) holds only if b and a_i are facts relative to the same system, but fails in general if used for facts relative to different systems.

The notation S for 'system', F for 'Friend' and W for 'Wigner' is meant to to evoke the famous Wigner's friend ideal experiment.

There is no assumption, however, about the system F being quantum or classical, microscopic or macroscopic.

Relative facts play a central role in the Relational Interpretation of Quantum Mechanics (RQM).

We shall discuss this role in detail later on.

First, however, we ask the following questions: what exactly characterizes a stable fact, among the relative facts?

What gives rise to stable facts?

Decoherence

Since stability is a characteristic feature of the classical world (whose facts invariably satisfy (1)), answering the questions above amounts to explaining, in terms of relative facts, what it takes for a system to be classical.

Various characterizations of a classical or semiclassical situation can be found in the literature - large quantum numbers, semiclassical wave packets or coherent states, macroscopic systems, large or infinite number of degrees of freedom...

All these features play a role in characterizing classical systems in specific situations.

But the key phenomenon that makes facts stable is decoherence - the suppression of quantum interference that happens when some information becomes inaccessible.

Consider two systems F and E (E for 'Environment'), and a variable L_F of the system F.

Let F_{ai} be the eigenvalues of L_F. A generic state of the coupled system F–E can be written in the form

$$|\psi\rangle = \sum_i c_i |Fa_i\rangle \otimes |\psi_i\rangle, \qquad (5)$$

where $|\psi_i\rangle$ are normalized states of E.

Define
$$\epsilon = \max_{i,j} |\langle \psi_i|\psi_j\rangle|^2. \qquad (6)$$

Now, suppose that: (a) ϵ is vanishing or very small and (b) a system W does not interact with E.

Then the probability P(b) of any possible fact relative to W resulting from an interaction between F and W can be computed from the density matrix obtained tracing over E, that is

$$\rho = \text{Tr}_{\mathcal{E}} |\psi\rangle\langle\psi| = \sum_i |c_i|^2 |Fa_i\rangle\langle Fa_i| + O(\epsilon). \qquad (7)$$

By posing $P(F_{ai}^{(E)}) = |c_i|^2$, we can then write i

$$P(b^{(\mathcal{W})}) = \sum_i P(b^{(\mathcal{W})}|Fa_i^{(\mathcal{E})})P(Fa_i^{(\mathcal{E})}) + O(\epsilon). \qquad (8)$$

Thus, probabilities for facts b relative to W calculated in terms of the possible values of L_F satisfy (3) up to a small deviation.

Hence the fact $L_F = a_i$ relative to E is stable with respect to W to the extent to which one ignores effects of order ϵ.

In the limit $\epsilon \to 0$, the variable L_F of the system F is exactly stable with respect to W.

The extensive theoretical work on decoherence has shown that decoherence is practically unavoidable and extremely effective as soon as large numbers of degrees of freedom are involved.

The variables of F that decohere, namely the variables for which ϵ becomes small, are determined by the actual physical interactions between F and E, they are those that commute with the interaction Hamiltonian.

The decoherence time, namely the time needed for ϵ to become so small that interference effects become undetectable by given observational methods, can be computed and is typically extremely short for macroscopic variables of macroscopic objects.

All this is well understood.

It is important for what follows to emphasize two subtle aspects of decoherence.

First, decoherence is not an absolute phenomenon, but a relative one:

it depends on how the third system W interacts with the combined system F–E.

Indeed, assumption (b) above is just as crucial as assumption (a) in deriving (8). Another system W' interacting with F − E differently might be able to detect interference effects.

Second, decoherence implies that an event regarding two systems F and E is stable with respect to at *third* system W.

That is, the variable L_F is stable relative to W even if the latter has not interacted with it, so there is no fact relative to W (yet).

This allows us to say that with respect to W the 'state of the system F has collapsed into the state $|F_{ai}\rangle$ state with probability $P(F_{ai}^{(E)}) = |c_i|^2$, even though W has not interacted F.

These observations show that decoherence does not imply that there is a perfectly classical world of absolute facts, but it does explain why (and when) we can reason in terms of stable, hence classical, facts.

The ubiquity of decoherence makes very many facts largely stable with respect to us.

Measurements

If two systems S and F interact and their respective variables L_S and L_F get entangled, and if L_F is stable with respect to W, it follows immediately from the definitions that the stability of L_F with respect to W extends to L_S as well.

This is precisely what happens in a typical quantum measurement of a variable L_S in a laboratory. Thinking of S, F and W as, respectively, the system being measured, the apparatus and the experimenter, we can separate the measurement in three stages:

1. An interaction between the system and the apparatus entangles L_S with a pointer variable L_F of the apparatus.

2. L_F gets correlated with a large number of microscopic variables (forming E) that are inaccessible to observer W.

3. The observer W interacts with the pointer variable L_F to learn about L_S.

Let's trace this same story in terms of relative facts:

1. A relative fact is established between S an F.

2. A relative fact is established between F and E. Since W does not interact with E, this established the previous fact for W.

3. The value of L_F becomes a fact for W and , since it is correlated withe L_S, the value of the variable L_S also becomes a fact with respect to W.

Already at stage 2, the observer might say that L_S 'has been measured' and apply (3), since the interaction with the inaccessible degrees of freedom greatly suppresses interference terms.

In other words: stability allows W to 'de-label' facts when they are facts relative to F.

In the mathematical formalism, W can assume that 'the wave function has collapsed'.

The value of L_S, still, does not become a fact relative to the observer—and cannot be known by the observer—until she actually interacts with a variable correlated with it.

It is the way that W, F and E couple to each other that make F a measuring apparatus for W.

The stability of F with respect to W extends to all other variables that interact with S, hence W applying quantum mechanics, might say that F causes S to collapse.

However another system W' that couples differently to these systems might still be able to detect interference effects.

In summary, we can distinguish two notions of facts that play a role in quantum theory: relative facts and stable facts.

Quantum theory allows us to talk about relative facts and compute probabilities for them.

Equation (2) holds but (3) does not.

The violation of (3) is quantum interference.

Stable facts are a subset of the relative facts.

They satisfy (3).

A relative fact about a system F is stable with respect to a system W if W has no access to a system E which is sufficiently entangled with F.

But stability is only approximate (in principle, no fact is exactly stable for any finite ϵ) and relative (depends on how the 'observer' system couples to the system and the environment).

FACTS AND REALITY

Up to now, we have given definitions of relative and stable facts, and studied their properties.

In this section we discuss the roles of relative and stable facts for the interpretation of quantum theory, namely for the relation between the formalism and the reality it describes.

The link between the theory the world

Let us compare advantages and difficulties of interpreting either stable or relative facts as the link between theory and reality.

Stable facts are taken as the link between the formalism and the world in textbook interpretations of quantum theory.

They are the conventional 'measurement outcomes' in a macroscopic laboratory.

They are similar to the facts of classical mechanics because in the world described by classical mechanics all facts (variables having certain values at certain times) are exactly stable: the (epistemic) probabilities for them to happen are *always* exactly consistent with (1).

In quantum mechanics, facts stable with respect to us are ubiquitous because of the ubiquity of decoherence.

There are however two difficulties in taking stable facts as the basis of the quantum ontology.

First, stability is relational.

Facts are stable only relative to a system that does not have sufficiently precise interactions with an environment system.

Therefore one does not avoid relationalism by restricting to stable facts.

Second, more seriously, stability is generically approximate only.

The system and environment are still in a superposition with respect to a third system.

These are serious difficulties if we want to take stable facts as the only primary elements of reality.

How stable does a fact need to be before it is real?

And with respect to which systems does it have to be stable, in order to be real?

Any answer to these questions is bound to be as unsatisfactory as the textbook interpretation that requires a classical world.

The alternative is to embrace the contextuality of the theory in full, and base its ontology on all relative facts.

Relative facts form the basis of a realist interpretation in Relational Quantum Mechanics (RQM).

The fundamental contextuality that characterizes quantum theory is interpreted in RQM as the discovery that facts about a system are always defined relative to another system, with which the first system interacts.

In the early history of quantum theory it was recognized that every measurement involves an interaction, and it was said that variables take values only upon measurement.

RQM notices that every interaction is in a sense a measurement, in that it results in the value of a variable to become a fact.

These facts are not absolute, they belong to a context; and there is no 'special context': any system can be a context for any other system.

The quantum state ('the wave function') does not have an ontic interpretation in RQM.

The state is not a 'thing', nor a condition of a system.

Rather, it is what a physicist uses to calculate probabilities for relative facts between physical systems to happen, given the relevant information she has.

Unlike other epistemic interpretations of quantum theory, the ontology of RQM is realist in the sense that it is not about agents, beliefs, observers, or experiences: it is about real facts of the world and relative probabilities of their occurrence.

The ontology is relational, in the sense that it is based on facts labelled by physical contexts.

Relative facts, therefore, provide a relational but realist interpretation to quantum theory which does not need to refer to complex agents.

Conclusions and final comments

The insight of Relational Quantum Mechanics (RQM) is that recognizing the relative nature of facts offers a straightforward solution to the measurement problem.

The measurement problem is the apparent incompatibility between two postulates: the 'projection' and the 'linear evolution' postulate.

Both postulates can be correct: they refer to facts relative to different systems.

Say that S interacts with F, so that a fact relative to F is established.

Then the projection postulate is used to update the state of S with respect to F, while the unitary evolution postulate is used to update the state of $S - F$ with respect to a third system W.

In a slogan: 'Wigner's facts are not necessarily his Friend's facts'. See Lecture_8c.

This by no means implies that when Wigner and his friend compare notes they find contradictions.

Interactions between S and F do have influence on the facts relative to W.

Indeed, after an interaction, S and F are entangled relative to W, meaning that in interacting with the two systems, W will find the two correlated.

Therefore Wigner will always agree with his Friend about the value of L_S once he too interacts with them.

In this sense, relative facts correspond to real events, they have universal empirical consequences.

Still, accepting the relativity of all facts is a strong conceptual step

It amounts giving up the absolute nature of facts, namely the existence of an absolute 'macroreality' and of 'observer-independent facts' in the language used in discussions of Bell's inequalities.

Such a macroreality only emerges approximately, relative to systems for which decoherence is sufficiently strong.

Decoherence has always played a peculiar role in the discussions on the measurement problem.

On the one hand, it is simply a true physical phenomenon, obviously relevant for shedding light on quantum measurement.

On the other hand, there is consensus that decoherence alone is not a solution of the measurement problem, because it does not suffice to provide a link between theory and reality.

Decoherence needs an ontology.

Relative facts provide such a general ontology, which is well defined with or without decoherence.

Decoherence clarifies why a large class of relative facts become stable with respect to us and form the stable classical world in which we live.

The violation of (1) when used for facts relative to different systems sheds also some light on the underpinnings of quantum logic.

The violation of (1), indeed, has been interpreted as a violation of classical logic, as it can be written as

$$P(b \text{ and } (a_1 \text{ or } a_2)) \neq P((b \text{ and } a_1) \text{ or } (b \text{ and } a_2)), \quad (9)$$

in contradiction with the classical logic theorem

$$b \text{ and } (a_1 \text{ or } a_2) = (b \text{ and } a_1) \text{ or } (b \text{ and } a_2). \quad (10)$$

The apparent violation of logic is understood in RQM as a result of forgetting that facts are relative: labelled by a context, as Bohr has repeatedly pointed out.

Facts relative to a context cannot be used, in general, to compute probabilities of facts related to other contexts because what is a fact in a certain context is not necessarily a fact in other contexts.

As a final remark, observe that if the quantum state has no ontic interpretation, the only meaning of 'being in a quantum superposition' is that interference effects are to be expected.

To say 'Friend is in a quantum superposition' does not mean anything more than saying that Wigner would be mistaken in using (3).

It has no implications on how Friend would 'feel' in being in a superposition.

Friend sees a definite result of his measurement, a fact, and this does not prevent Wigner from having the chance to see an interference effect in his facts.

Wigner's friend does not stop being an observer simply because Wigner has a chance to detect interference effects in his facts.

Schrodinger's cat has no reason to feel 'superimposed'.