Physics 50 **Assignment #8** **Spring 2010**

Readings: Riley, Hobson and Bence - Chapter 15
Boccio - 011_ODEs, 012_ODEs and 013_ODEs

Problems:

15.04 An ODE
15.07 An ODE
15.09 Two ODEs
15.10 ODEs and Laplace Transform
15.24 Variation of Parameters
15.33 An ODE

Even # Answers:

15.04 - General solution is: $f(t) = (A+Bt)e^{-3t} + \frac{1}{4}e^{-t}$

15.10 - (a) $f(t) = 2e^{-3t} - e^{-2t}$

(b) $f(t) = e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$

15.24 - (a) $y(x) = Ae^{x} + Be^{-x} - \frac{n!}{2}\sum_{m=0}^{n}\frac{x^m}{m!}\left[1+(-1)^{n+m}\right]$

(b) $y(x) = \left(A + Bx + \frac{1}{3}x^3\right)e^{x}$

EP-1 For the ODE

$$x^2\frac{d^2y}{dx^2} - x(1-x)\frac{dy}{dx} + (1-x)y = 0$$

one solution is $y_1(x) = x$. Find the **Wronskian** and use it to find a **second** solution.

Answer: $y_1(x) = x \quad , \quad y_2(x) = x\ln(x) + x\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{nn!}$

EP-2 For the ODE with **constant** coefficients

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = x^2 + 2\sin x$$

find the **homogeneous** solutions and then use the **variation of parameters** method to find the **particular** solution

Answer: $y(x) = Ae^{-5x} + Be^{-x} + \frac{1}{4}\left(\frac{8}{13}\sin x - \frac{12}{13}\cos x + \frac{4}{5}x^2 - \frac{48}{25}x + \frac{248}{125}\right)$

EP-3 Solve the ODE

$$u'' + 7u' + 10u = \cos 2x \quad , \quad u(0) = u'(0) = 0$$

Answer: $u(x) = \frac{9}{348}\cos 2x - \frac{29}{348}e^{-2x} + \frac{21}{348}\sin 2x + \frac{20}{348}e^{-5x}$

EP-4 Solve the ODE

$$x^2u'' - 2xu' - 4u = x\cos x$$

Answer: $u(x) = Ax^4 + \frac{B}{x} - \frac{1}{5x}\cos x - \frac{1}{5}\sin x + \frac{x^4}{5}\int \frac{\cos x}{x^4}dx$