Bessel Function Identities

$$J_0'(x) = -J_1(x)$$

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$

$$J_n'(x) = \frac{1}{2}\left(J_{n-1}(x) - J_{n+1}(x)\right)$$

$$xJ_n'(x) = xJ_{n-1}(x) - nJ_n(x)$$

$$xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$$

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$$

$$\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$$

$$\int xJ_0(x)\,dx = xJ_1(x)$$

$$\int x^2 J_0(x)\,dx = x^2 J_1(x) + xJ_0(x) - \int J_0(x)\,dx$$

$$\int x^m J_0(x)\,dx = x^m J_1(x) + (m-1)x^{m-1} J_0(x) - (m-1)^2 \int x^{m-2} J_0(x)\,dx$$

$$\int \frac{J_0(x)}{x^2}\,dx = J_1(x) - \frac{J_0(x)}{x} - \int J_0(x)\,dx$$

$$\int \frac{J_0(x)}{x^m} dx = \frac{J_1(x)}{(m-1)^2 x^{m-2}} - \frac{J_0(x)}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2} \int \frac{J_0(x)}{x^{m-2}} dx$$

$$\int J_1(x)\,dx = -J_0(x)$$

$$\int xJ_1(x)\,dx = -xJ_0(x) + \int J_0(x)\,dx$$

$$\int x^m J_1(x)\,dx = -x^m J_0(x) + m\int x^{m-1} J_0(x)\,dx$$

$$\int \frac{J_1(x)}{x} dx = -J_1(x) + \int J_0(x)\,dx$$

$$\int \frac{J_1(x)}{x^m} dx = -\frac{J_1(x)}{mx^{m-1}} + \frac{1}{m} \int \frac{J_0(x)}{x^{m-1}} dx$$

$$\int x^n J_{n-1}(x)\,dx = x^n J_n(x)$$

$$\int x^{-n} J_{n+1}(x)\,dx = -x^{-n} J_n(x)$$

$$\int x^m J_n(x)\,dx = -x^m J_{n-1}(x) + (m+n-1)\int x^{m-1} J_{n-1}(x)\,dx$$

$$\int xJ_n(\alpha x)J_n(\beta x)\,dx = \frac{x\left[\alpha J_n(\beta x)J_n{}'(\alpha x) - \beta J_n(\alpha x)J_n{}'(\beta x)\right]}{\beta^2 - \alpha^2}$$

$$\int xJ_n^2(\alpha x)\,dx = \frac{x^2}{2}\left[J_n'(\alpha x)\right]^2 + \frac{x^2}{2}\left(1-\frac{n^2}{\alpha^2 x^2}\right)\left[J_n(\alpha x)\right]^2$$

$$\int_0^\infty e^{-ax}J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}$$

$$\int_0^\infty e^{-ax}J_n(bx)dx = \frac{\left(\sqrt{a^2+b^2}-a\right)^n}{b^n\sqrt{a^2+b^2}} \quad , \quad n>-1$$

$$\int_0^\infty \cos(ax)J_0(bx)dx = \begin{cases} \dfrac{1}{\sqrt{a^2+b^2}} & , \quad a>b \\ 0 & , \quad a<b \end{cases}$$

$$\int_0^\infty J_n(bx)dx = \frac{1}{b} \quad , \quad n>-1$$

$$\int_0^\infty \frac{J_n(bx)}{x}dx = \frac{1}{n} \quad , \quad n=1,2,3,......$$

$$\int_0^\infty e^{-ax}J_0(b\sqrt{x})dx = \frac{e^{-b^2/4a}}{a}$$

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \frac{\alpha J_n(\beta)J_n'(\alpha) - \beta J_n(\alpha)J_n'(\beta)}{\beta^2 - \alpha^2}$$

$$\int_0^1 xJ_n^2(\alpha x)dx = \frac{1}{2}\left[J_n'(\alpha)\right]^2 + \frac{1}{2}\left(1 - \frac{n^2}{\alpha^2}\right)\left[J_n(\alpha)\right]^2$$

$$J_0(x) = \frac{1}{\pi}\int_0^\pi \cos(x\sin\theta)\,d\theta$$

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)\,d\theta \quad , \quad n = \text{integer}$$

$$J_n(x) = \frac{x^n}{2^n\sqrt{\pi}\,\Gamma(n+1/2)}\int_0^\pi \cos(x\sin\theta)\cos^{2n}(\theta)\,d\theta \quad , \quad n > -1/2$$

$$\cos(x\sin\theta) = J_0(x) + 2J_2(x)\cos(2\theta) + 2J_4(x)\cos(4\theta) + \ldots\ldots..$$

$$\sin(x\sin\theta) = 2J_1(x)\sin(\theta) + 2J_3(x)\cos(3\theta) + 2J_5(x)\cos(5\theta) + \ldots\ldots..$$

$$J_n(x+y) = \sum_{k=-\infty}^{\infty} J_k(x)J_{n-k}(y)$$

$$1 = J_0(x) + 2J_2(x) + ... + 2J_{2n}(x) + ...$$

$$x = 2\left(J_1(x) + 3J_3(x) + 5J_5(x) + ... + (2n+1)J_{2n+1}(x) + ...\right)$$

$$x^2 = 2\left(4J_2(x) + 16J_4(x) + 36J_6(x) + ... + (2n)^2 J_{2n}(x) + ...\right)$$

$$\sin(x) = 2\left(J_1(x) - J_3(x) + J_5(x) - ....\right)$$

$$\cos(x) = J_0(x) - 2J_2(x) + 2J_4(x) - ....$$