FIRST YEAR PHYSICS

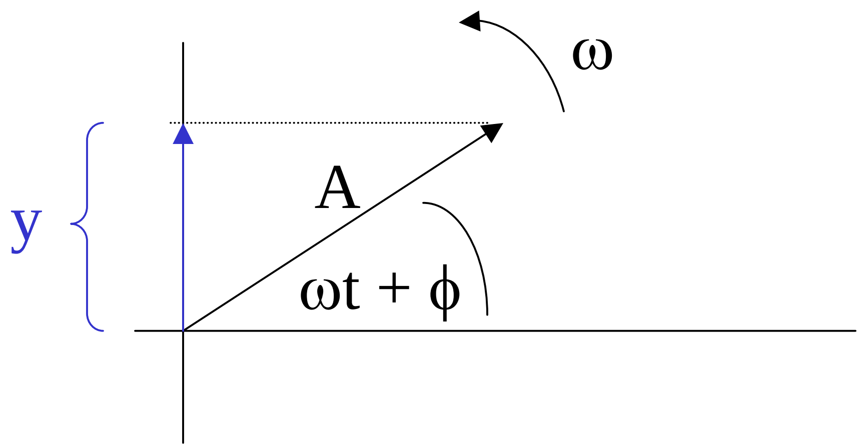
Unit 4:
Light II

Contents

PHASORS

In the previous unit we talked a lot about adding waves having different amplitudes and phases. We shall now discuss a simple graphical method that can be used for any harmonic wave

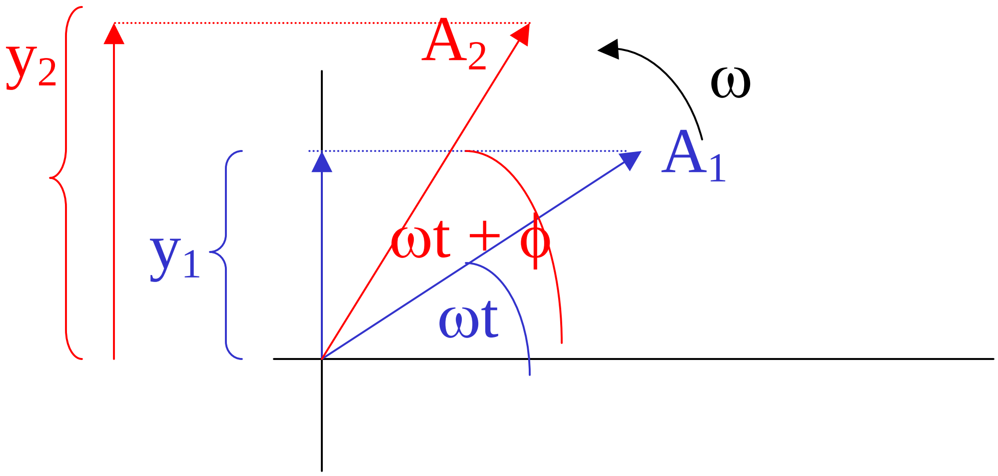
- Assume a sinusoidal wave $y = A \sin(kx - \omega t + \phi)$
- This arbitrary wave can be represented graphically as the 'y' component of a rotating vector that we call "phasor"



- If we have two waves and we would like find the resultant of these two waves,

$$y_1 = A_1 \sin(kx - \omega t) \text{ and } y_2 = A_2 \sin(kx - \omega t + \phi)$$

- Using analytical methods this can be messy, especially when we have lot of waves.
- Using phasors, this is pretty simple: we add the two y components:

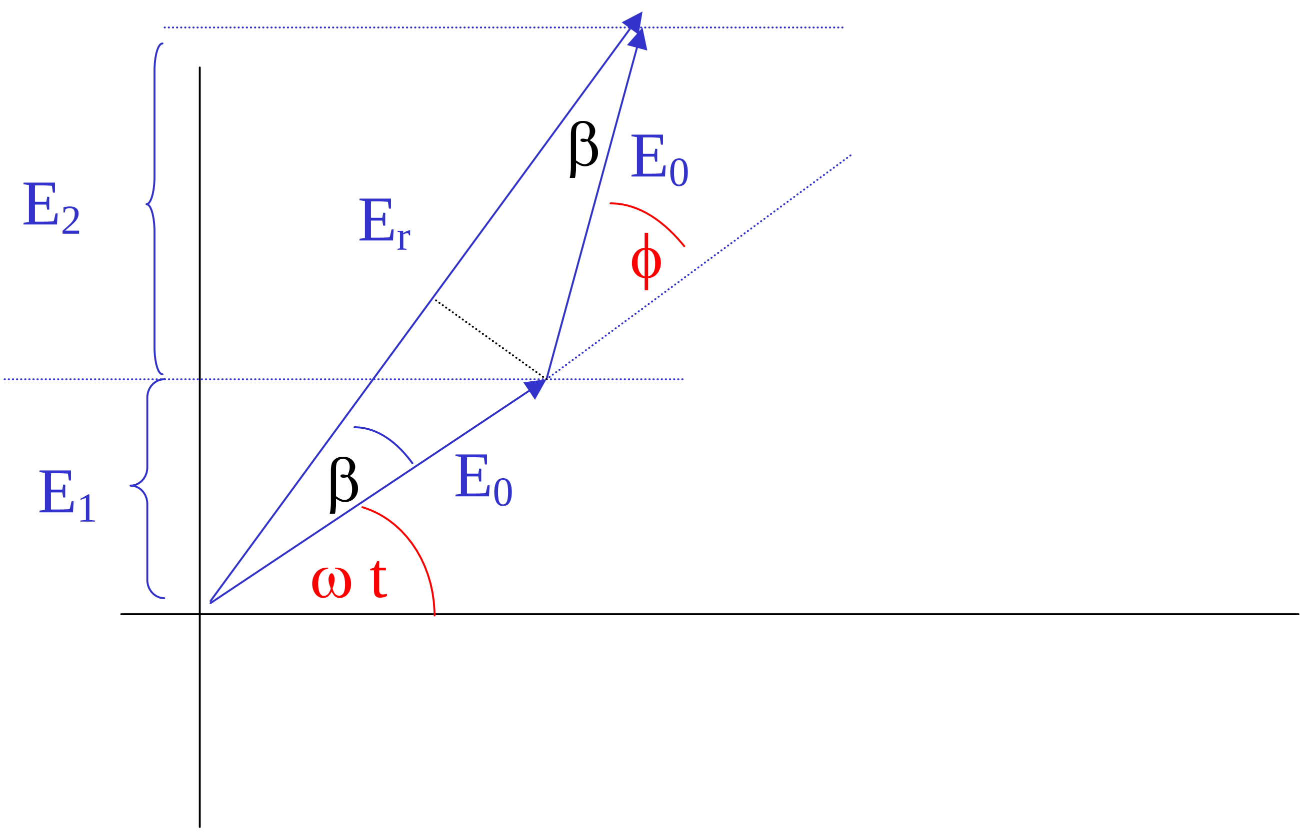


As an example of using phasors, let's revisit the question of the intensity distribution of light on a screen coming from two slits (ie. Young's double slit experiment).

- Two waves having identical amplitudes, E_0, arrive at a point P. There will be a phase difference of (see section on Young's experiment):

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

- We first calculate the resultant electric field amplitude, E_r. The intensity is simply proportional to the square of the amplitude
- We use standard trigonometry:



- For the resultant wave, the amplitude is E_r and the phase shift relative to the original wave is β
- From the diagram we see that ϕ is an exterior angle of the equilateral triangle, therefore $\beta = ½\phi$
- Using simple trig we can write $E_r = 2(E_0 \cos\beta) = 2E_0\cos(½\phi)$
- We square this to get the intensity:

$$I = [2E_0\cos(½\phi)]^2 = 4I_0\cos^2(½\phi) = 4I_0\cos^2\left(\frac{\pi}{\lambda} d \sin\theta\right)$$

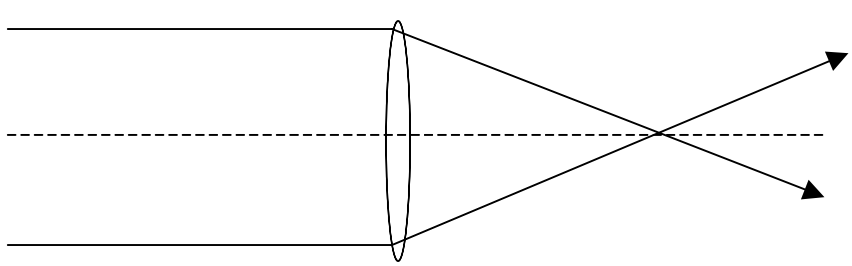
- This is exactly what we found in the previous unit.
- Phasor are a simple graphical method to calculate the sum (or difference) of any number of waves.

Now use the phasor method to calculate the intensity distribution from three slits.

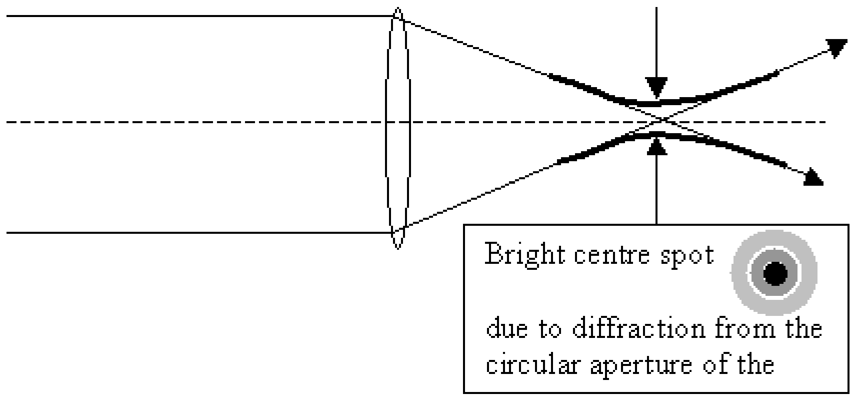
RESOLUTION OF OPTICAL INSTRUMENTS

The wave nature of light has fundamental consequences on the workings of optical instruments (including the human eye).

- According to geometrical optics, the image of an object which is infinitely far is a point in the focal plane of the objective (focusing) lens.



- But looking at this same situation from the point of view light waves, light coming from infinitely far should be considered as (plane) waves that will diffract as they pass through aperture of the objective lens. That is, the objective lens is a circular aperture and we have to consider diffraction effects caused by this aperture:



- Therefore the focal point is not really a ‘point’ at all. It is a focal ‘spot’ or focal blur whose size is determined by diffraction from the lens:
  - You remember, the angular spread of light caused by an aperture of ‘d’ diameter is given by

$$\theta = \sin^{-1}(1.22\ \lambda\ /\ d)$$

  - For small angles $\sin\theta \approx \theta$, thus

$$\theta \approx (1.22\ \lambda\ /\ d)$$

  - If we use a lens of diameter ‘d’ and focal length ‘f’, the diameter ‘D’ of the smallest possible ‘image’ will be

$$D = f\theta = (1.22\ f\lambda\ /\ d)$$

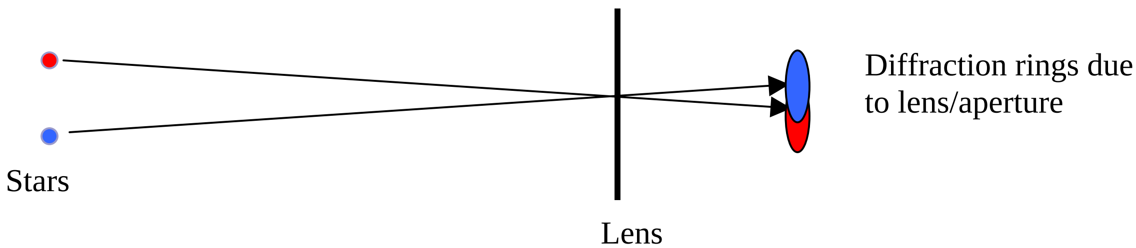
Note:

  - to reduce the spot size, we need to:
    - increase the diameter (d) of the lens/mirror
    - reduce the wavelength (λ). This is (one of) the reasons why
      - Telescopes have large mirrors,
      - Photolithography uses short wavelengths
      - Electron microscopes give higher resolution than optical microscopes (the wavelengths involved in electron microscopy are much shorter than that of visible light)
  - In practice, the actual image of a distant point will be somewhat larger than that given above. This is caused by various lens ‘aberrations’ or defects. However, the important point is that even if you eliminate all the lens defects, the diffraction pattern given above will remain! This is an inherent property of the wave nature of light.

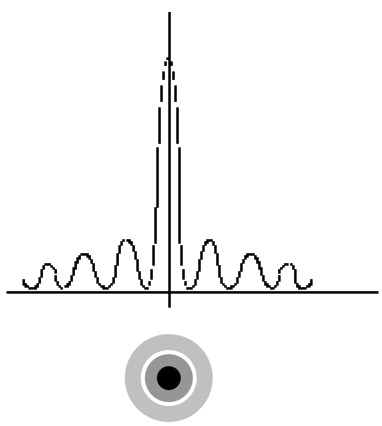
Rayleigh’s criterion

Assume we are looking at two stars through a telescope. The diameter of the telescope mirror is ‘d’, and the optical detector attached to the telescope only ‘sees’ one wavelength, λ. We ask the following question: How close can these two stars be to able to resolve them? Or in other words, can we define some rule that helps us decide if the image we are looking at is made of two smaller images or just one.

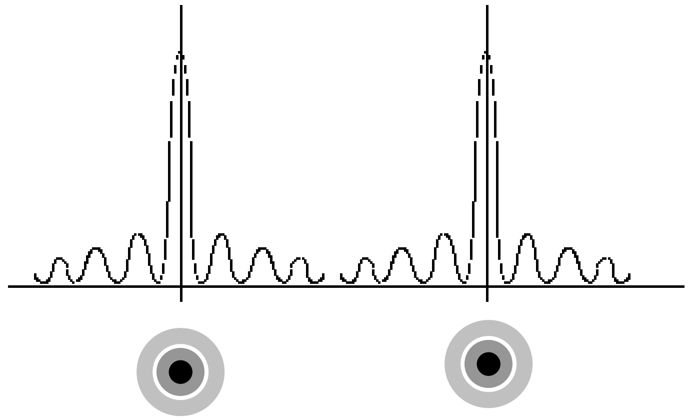
The answer can be found by looking at the diffraction patterns generated by the two objects (eg stars):



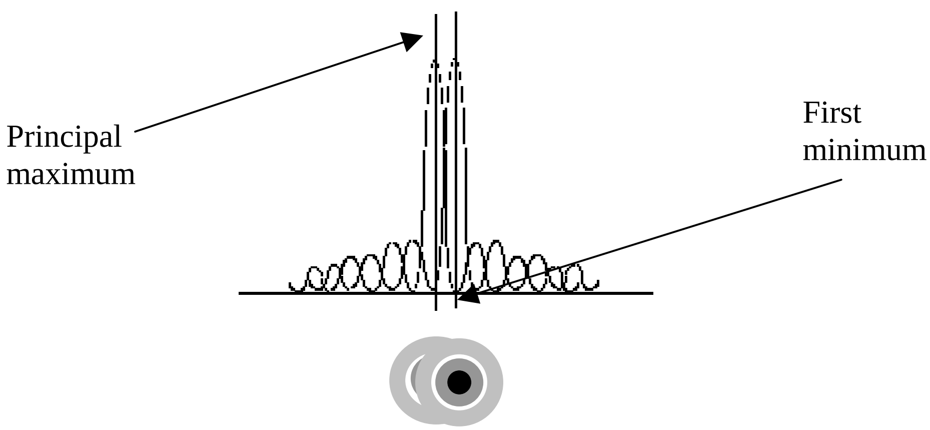
The diffraction pattern of each star looks something like this:



When the two stars are well separated this is what we would measure:



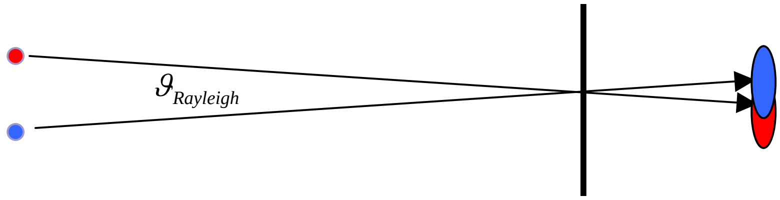
If we are looking at stars that are closer together, their images also get closer and their diffraction patterns overlap. It becomes difficult to decide whether we are looking at two or just one star. The limit of resolvability occurs when the principal maximum of one of the diffraction patterns coincides with the first minimum of the other diffraction pattern. We call this condition **Rayleigh's criterion**. The diffraction patterns at Rayleigh's limit would look something like this:



When Rayleigh's condition is met, the angular separation between the two objects (eg. stars) is given by the difference between the principle maximum and the first minimum of the diffraction pattern of a circular aperture, which is given by

$$\vartheta_{Rayleigh} = \sin^{-1}(1.22\ \lambda\ /\ d\ )$$

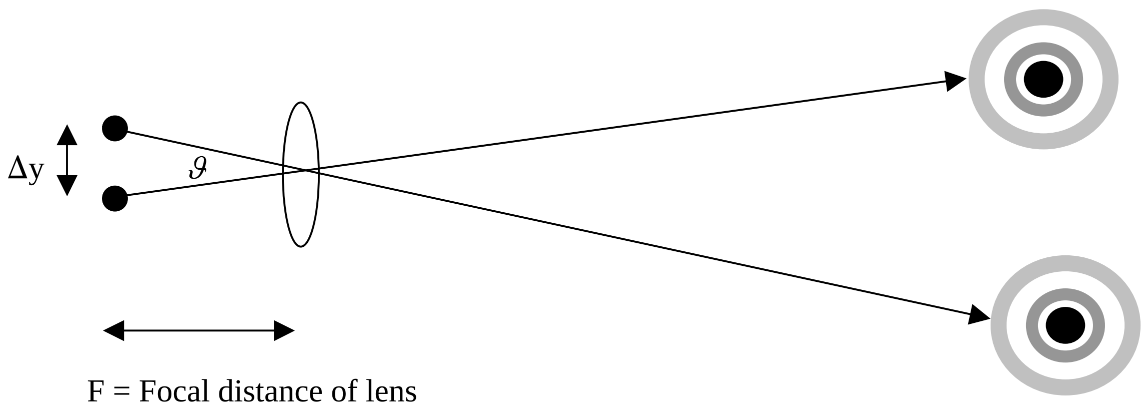
That means that the angular separation of the two stars has to be at least $\vartheta_{Rayleigh}$ otherwise the stars cannot be resolved.



Rayleigh's criterion is used for other areas of optics (physics) not only astronomy, (eg. cameras, microscopes, eyes, diffraction gratings, etc)

Let's look at microscopes: the question again is "What is the closest two objects can be and still be resolved with a microscope?"

- In the case of the microscope the objects are very close to the focal point of the lens:
- Let's denote the distance between these two objects by Δy



- To be able to resolve the two points, the angular separation between these two objects has to be greater than the angle specified by Rayleigh's criterion:

$$\vartheta > \vartheta_{Rayleigh} = \sin^{-1}(1.22\ \lambda\ /\ d) = 1.22\ \lambda\ /\ d$$

  where 'd' is the diameter of the lens.

- From the diagram we see that angular separation between the two points is given by

$$\vartheta = \Delta y / F$$

- Therefore, the minimum angle between the two objects is:

$$\vartheta_{min} = \Delta y_{min} / F = 1.22\ \lambda\ /\ d$$

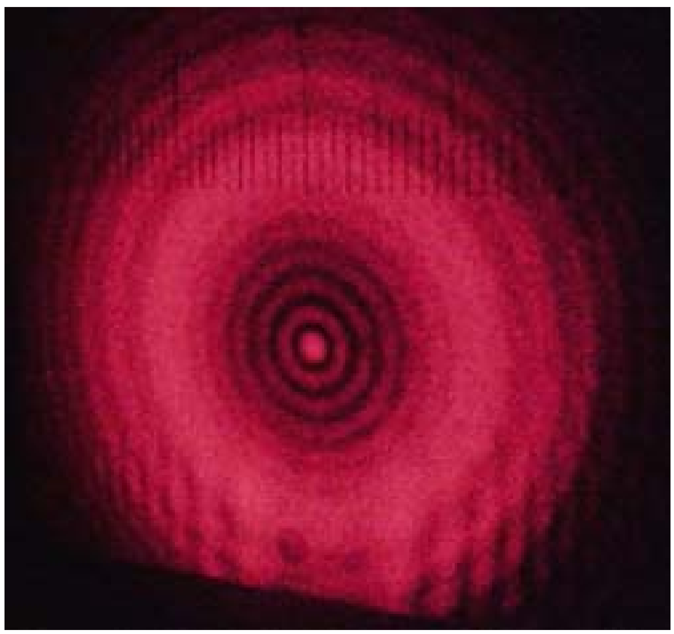
- The shortest focal length cannot be shorter than the radius of the lens, thus

$$\Delta y_{min} \geq F(1.22\ \lambda / d) = (d/2)(1.22\ \lambda\ /\ d) \approx \lambda / 2$$

This is an important result: the smallest distance we can resolve is approximately equal to the wavelength of light! Or in other words, we cannot focus any wave to spot with dimensions < λ! No matter how many lenses, mirrors, magnifications, etc you try, you cannot beat diffraction.

Comments:

- Diffraction also occurs for opaque 'apertures'. That is, we will see diffraction around opaque slits (eg. wires), or around opaque circular apertures (eg. a ball bearing or something similar). In the figure below is the diffraction pattern from a small ball bearing. Note the bright spot in the centre! (from http://dustbunny.physics.indiana.edu/~dzierba/P360n/KPAD/Exps/Poisson/poisson.html) (local copy)



For diffraction of a wire, see for example: http://www.physics.montana.edu/demonstrations/video/optics/demos/thinwirediffraction.html (local copy)

- The type of diffraction we described above assumed that the 'screen' (detector, eye, etc) is far from the object compared to the wavelength of light. For example the slit width was 0.1 mm, the distance from the slit 10m, the wavelength, 0.001mm . This type of diffraction is called Fraunhofer diffraction.
- When the screen is closer to the object, the diffraction pattern changes. Diffraction in this range is called Fresnel diffraction. (We'll study Fresnel diffraction in higher years.)

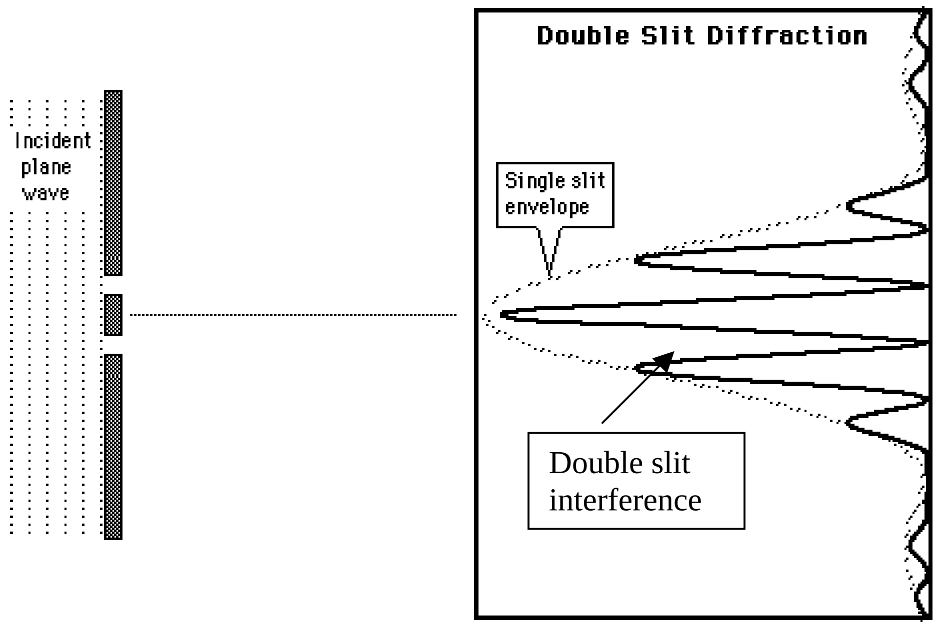
MORE ON DIFFRACTION

Multiple slits:

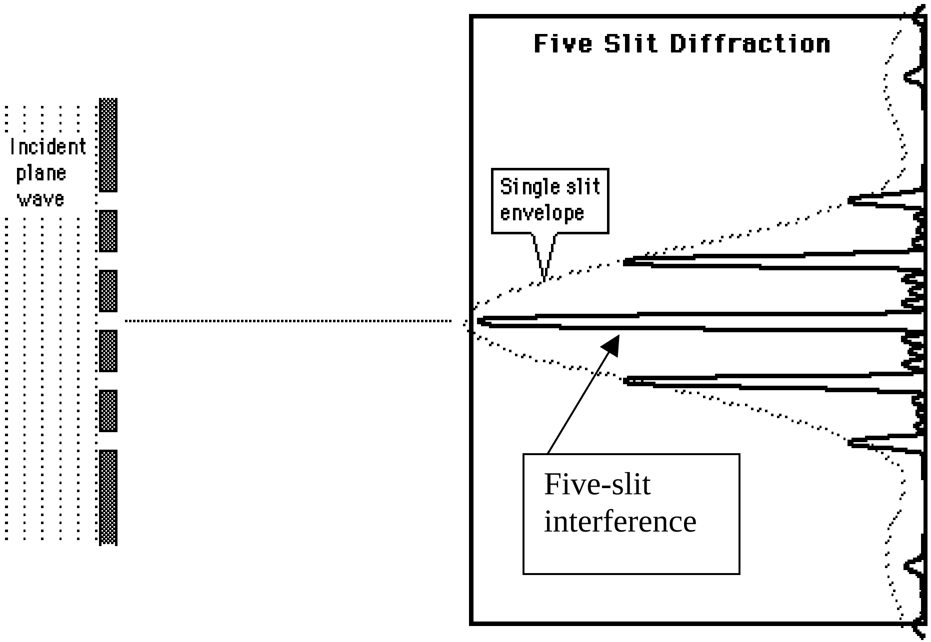
Remember two-slit interference? We found that the product of the single-slit diffraction pattern and the double-slit interference pattern gives the intensity pattern:

$$I(\theta) = I_0 (\sin \beta / \beta)^2 (\cos^2\alpha)$$

where $\beta = \pi a \sin\theta / \lambda$ and $\alpha = \pi d \sin\theta / \lambda$



(*) How would the diffraction/interference pattern of many slits look? For example 5 slits (N=5)?

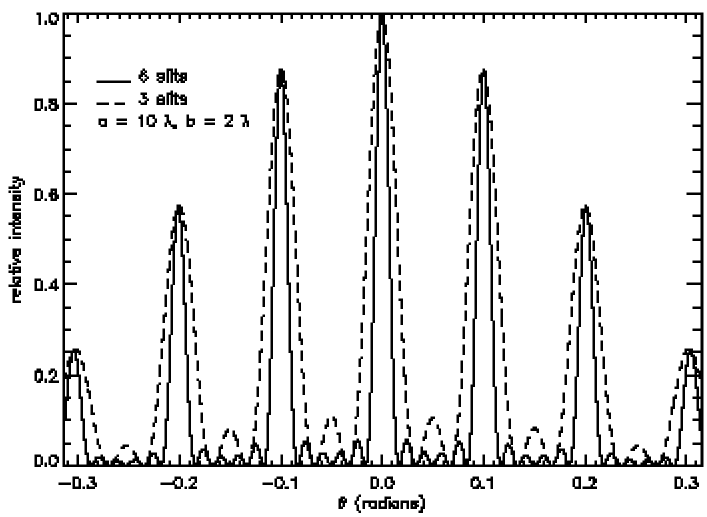


- Maxima occur when: $d\sin\theta_m = m\lambda$ (same as 2 slits)
- Minima occur when $d\sin\theta_p = p\lambda/N$ (different from 2 slits)

  where p = 1,2, .. but $p \neq 5, 10$, ..

So the more slits, the more minima between the principal maxima. On the figure below, we compare the 3 and 6 slit pattern. Note two important points: the maxima become narrower as we increase the number of slits, and the number of minima increases:

(from http://www.dur.ac.uk/r.g.bower/OpticsI/optlec/img132.gif)



- In general, for N slits:

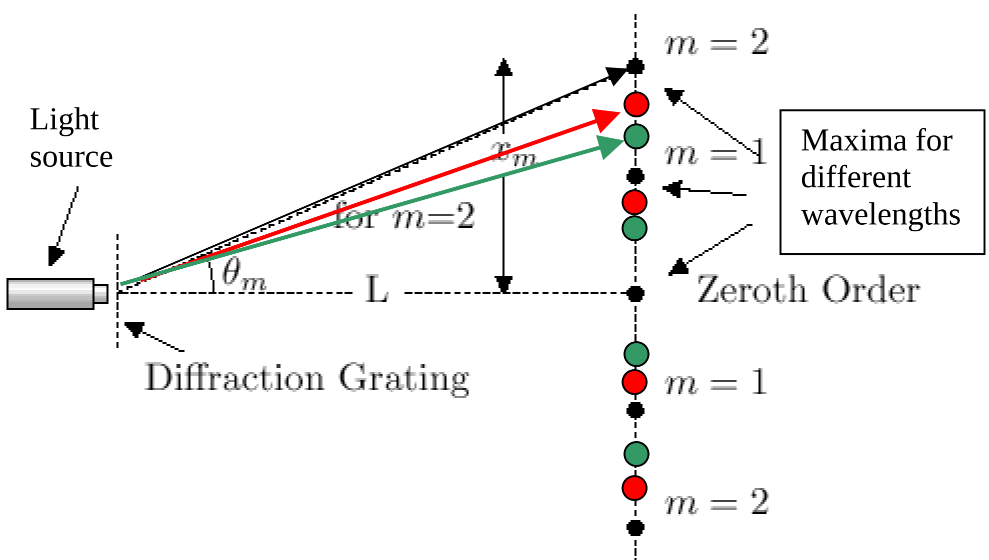
$$\text{Maxima: } d\sin\theta_m = m\lambda$$

$$\text{Minima: } d\sin\theta_p = p\lambda/N \text{ where } p= 1,2,3 \ldots \text{ but } p\neq N$$

See for example, http://www.physics.nwu.edu/ugrad/vpl/optics/diffraction.html (local copy)

Note:

- If there are N slits, there are N minima between two main maxima
- As N is increased the maxima become sharper
- If N is very large (eg. N = 10,000 or more), interference fringes become sharp interference 'lines'
- Different wavelengths will diffracted by different angles
- If we shine white light on such a multiple slit device, each 'line' corresponds to a wavelength, λ
- A device with very large large number of slits (10^4 or 10^5) is called a **diffraction grating**
- Diffraction gratings are used to separate light into its components (similar to a glass prism)



Diffraction gratings

Question: How wide is a ‘line’ produced by a grating for a given λ?

Answer:

$$\sin\theta_m = m\lambda/d$$

$$\sin\theta_{m+1} = (m+1)\lambda/d$$

The angular separation between different order is therefore given by:

$$\Delta\theta_m = \theta_m - \theta_{m+1} = \lambda /d \quad \text{(assuming that } \sin\theta \approx \theta)$$

Since there are N minima between two maxima, the width of each line representing a given wavelength, λ is given by:

$$\Delta\theta_\lambda = \Delta\theta_m /N$$

which can be written as:

$$\Delta\theta_\lambda = \lambda /Nd$$

Note: *Nd* is the width of the grating. The wider the grating the narrower the line width.

Now that we know the angular width of a ‘line’, let’s calculate the angular separation between two wavelengths, λ_1, and λ_2, where $\lambda_2 = \lambda_1 + \Delta\lambda$

- We know that

$$\sin\theta_1 = m\lambda_1/d$$

$$\sin\theta_2 = m\lambda_2/d$$

therefore the angular separation between these two wavelengths is

$$\Delta\theta = \theta_1 - \theta_2 \approx m\Delta\lambda /d$$

- But we can only resolve λ_1 and λ_2 if their angular separation ($\Delta\theta$) is larger than their angular width ($\Delta\theta_\lambda$):

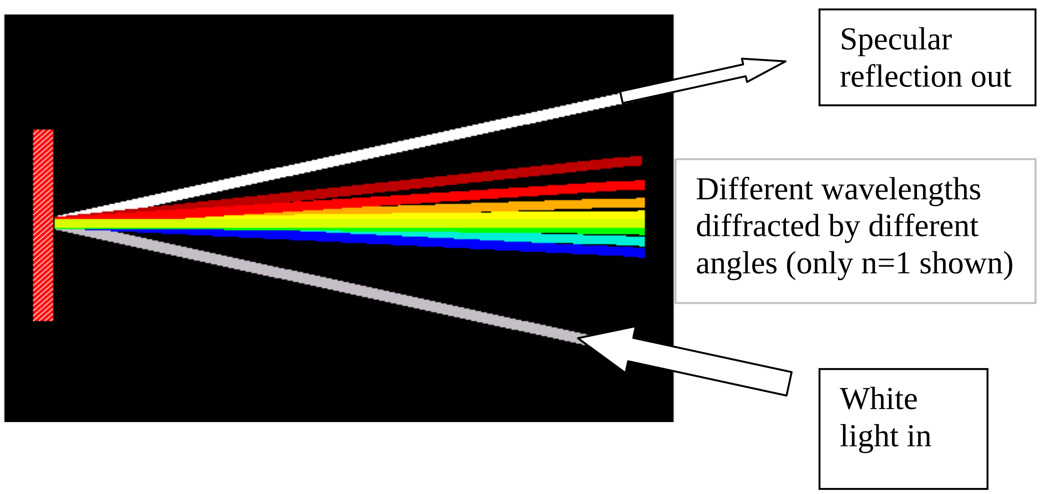
$$\Delta\theta \geq \Delta\theta_\lambda$$

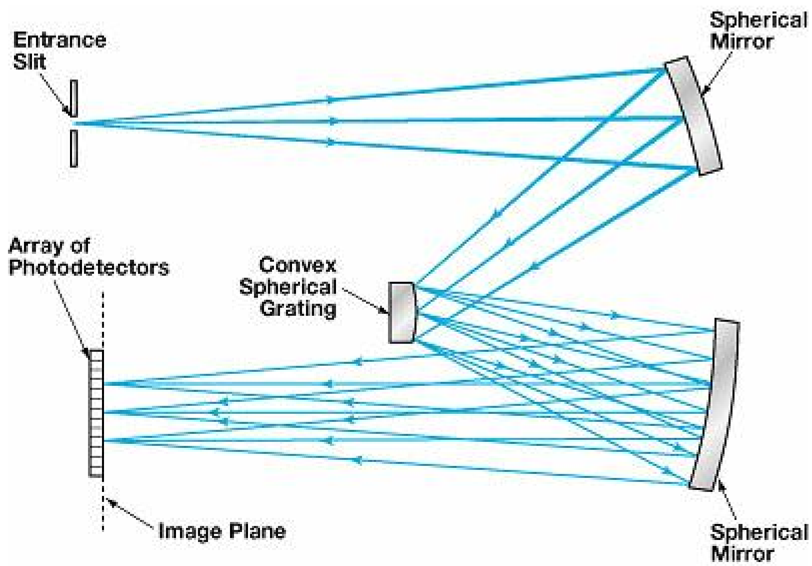
$$\therefore\ m\Delta\lambda\ /d \geq \lambda\ /Nd$$

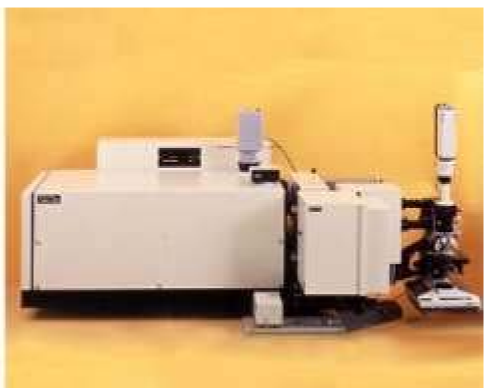
- Therefore the smallest possible $\Delta\lambda$ that can be resolved with a grating having N slits (grooves) is:

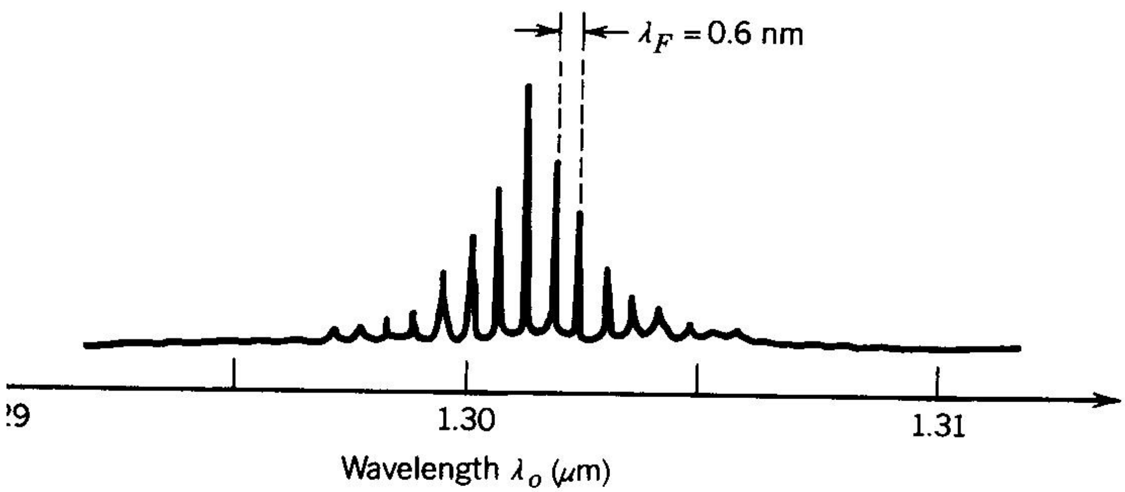
$$\lambda/\Delta\lambda = Nm$$

- R = $\lambda/\Delta\lambda$ is called the **resolving power** of the grating.
  - Gratings are used in monochromators, spectrometers. These are instruments that are used to study the composition of light.
  - Gratings can either work in transmission or reflection




Entrance Slit
Spherical Mirror
Array of Photodetectors
Convex Spherical Grating
Image Plane
Spherical Mirror




$\lambda_F = 0.6$ nm
1.30
1.31
Wavelength λ_o (μm)

X-RAY DIFFRACTION

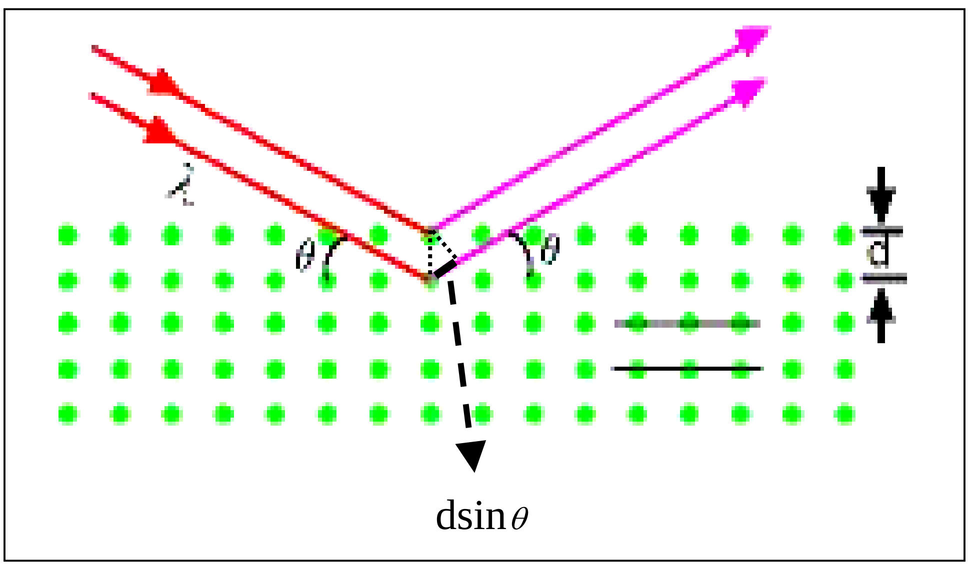
- X-rays are short wavelength E.M. waves: $\lambda \approx 0.1\text{nm} = 10^{-10}\text{m}$
- Can interference or diffraction be observed with such short wavelengths?
- Let's try to use a standard optical diffraction grating at these wavelengths.
  - Typical optical diffraction grating: d = 3μm
  - λ=0.1nm,
  - From the grating equation (assuming m = 1):

$$\theta = \sin^{-1}(\lambda/d) \approx 0.001^0$$

  - $\theta=0.001^0$ basically means that the X-rays are not diffracted enough to be measurable. We need a better grating.
- To observe diffraction we need to have the spacing between the slits to be $d \approx \lambda$
- How can we make a diffraction grating such that d = 0.1nm?
- We rely on nature: crystals are 3 dimensional gratings such that d $\approx 10^{-10}$ m

Where does the diffraction occur?

- In crystals X-rays get scattered by the atoms/molecules
- In some directions scattered x-rays interfere constructively
- Condition for constructive interference:



http://www.mrl.ucsb.edu/mrl/centralfacilities/xray/xray-basics/Xray-basics.html#x1
(local copy)

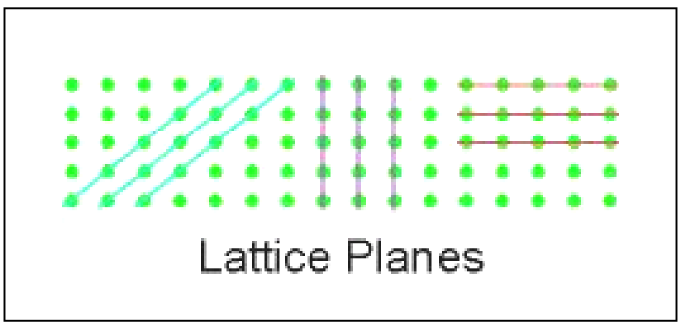
When

$$2d\sin\theta = m\lambda$$

we get constructive interference.   This equation is called Bragg's law.

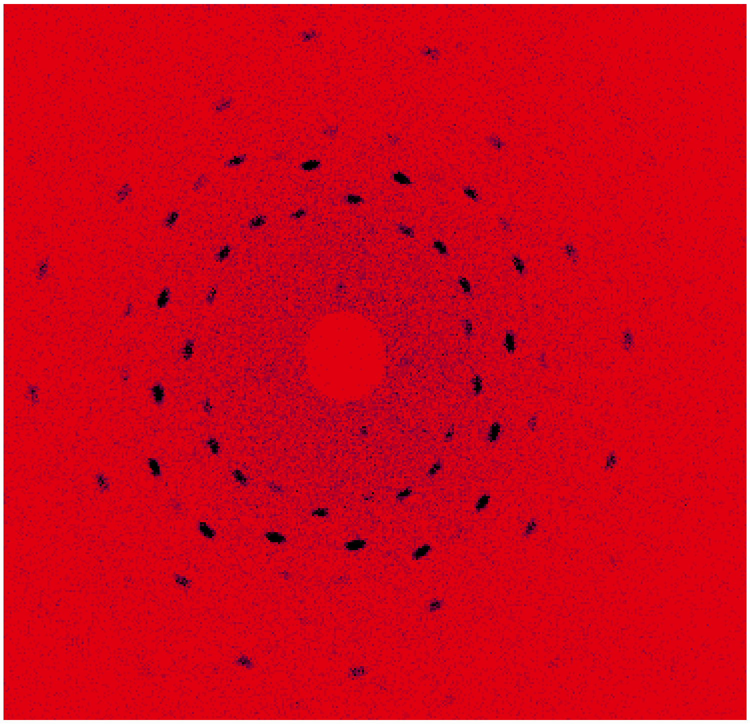
Note:

- '*d*' is the spacing between atomic planes, not the spacing between atoms. It is called the inter-planar or lattice spacing
  - *d* depends on the crystallographic structure
  - Rotating the crystal, the beam will see different '*d*'
  - By measuring *d* we learn about the crystal structure
  - The symmetry of the crystal structure will be mirrored by the symmetry of the diffraction pattern



- θ is the angle between incoming beam and the atomic planes, and not between the incoming beam and the normal to the planes (as is expected in optics)
- x-ray diffraction is mainly used:
  - to study crystal structure
  - nowadays an important use is the study the structure of large bio-molecules
  - The use of X-rays as an instrument for the systematic study of the way in which crystals are built was due to the William and Lawrence Bragg (father and son). They were recognized by the award of the Nobel Prize jointly in 1915.
  - An example of a x-ray diffraction from a silicon crystal is shown below (from http://images.google.com.au/imgres?imgurl=www.digiray.com/diffraction/Silicon0556.gif&imgrefurl=http://www.digiray.com/diffraction/&h=358&w=371&prev=/images%3Fq%3Dx-

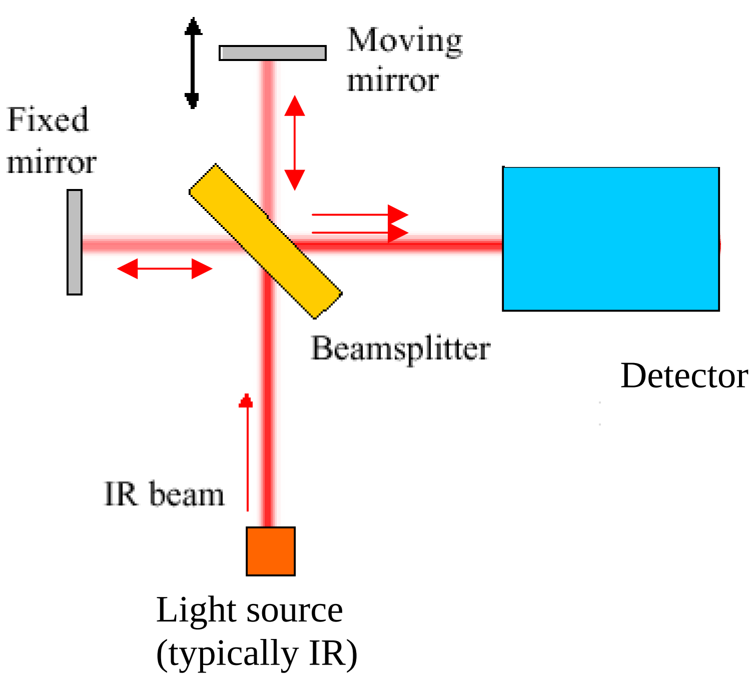
ray%2Bdiffraction%26start%3D20%26svnum%3D10%26hl%3Den%26lr%3D%26ie%3DUTF-8%26oe%3DUTF-8%26sa%3DN)



For a nice review of x-ray diffraction, see for example:
http://www.mrl.ucsb.edu/mrl/centralfacilities/xray/xray-basics/Xray-basics.html#x1
(local copy)

Michelson's Interferometer

- Historical importance:
  - The Michelson interferometer is associated with experiments that lead to special relativity. These experiments by Michelson and Morley provided evidence against the existence of an absolute frame of reference (based on ether). (see for example, http://scienceworld.wolfram.com/physics/Michelson-MorleyExperiment.html) (local copy)
  - But Michelson's pioneering contributions to interferometry are much broader than just one experiment. He was awarded the Nobel Prize in 1907 for his many discoveries in optics.
  - The Michelson interferometer has become an indispensable tool in many scientific applications such as high-resolution spectroscopy, atomic length standards, and in practical applications, where displacements as small as a fraction of the wavelength of visible light must be measured.
  - We shall review the instrument that Michelson and Morley used.
- Interferometers are often used to measure very small distances but an interferogram can also be used to provide spectral information (see for example, http://scienceworld.wolfram.com/physics/FourierTransformSpectrometer.html) (local copy)
- In the Michelson interferometer, light travels through 2 paths as shown in the diagram below



See for example, http://www.3dimagery.com/michelsn.html (local copy)

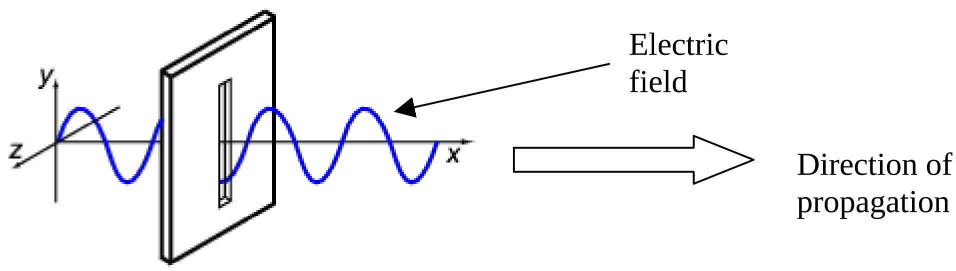
- Differences in the paths: $\Delta L = 2(D_1 - D_2)$ gives rise to phase difference between the waves:

$$\Phi = (2\pi / \lambda)\Delta L$$

- Depending on the phase shift between the two beams travelling different paths, the interference between the waves can be constructive or destructive
- By moving one of the mirrors, the path difference changes, and so do the interference fringes from constructive to destructive or vice versa
  - By moving the mirror by ¼λ we go from a bright spot (constructive interference) to a dark spot (destructive interference)
  - By counting the number of dark - bright switches (fringes) we can measure distances in terms of λ

POLARIZATION

- Light (EM radiation) is a transverse wave
- One of the properties of a transverse wave is that it can be polarised.
- This means that all the oscillations of the wave are in the same plane.
- Light waves are electromagnetic waves, made up of electric and magnetic fields that oscillating perpendicular to each other.
- When we talk about the oscillations of a light wave, we will be describing the oscillating electric field. (For clarity, the magnetic fields will not be shown. This is the normal practice when describing electromagnetic waves.)



- The plane containing the $\underline{E}$ and $\underline{k}$ vectors (k => direction of propagation) is called the **plane of vibration**
- Since the electric (magnetic) fields are vector quantities they can be written as:

$$\underline{E}(x,t) = \underline{E}_z(x,t) + \underline{E}_y(x,t)$$

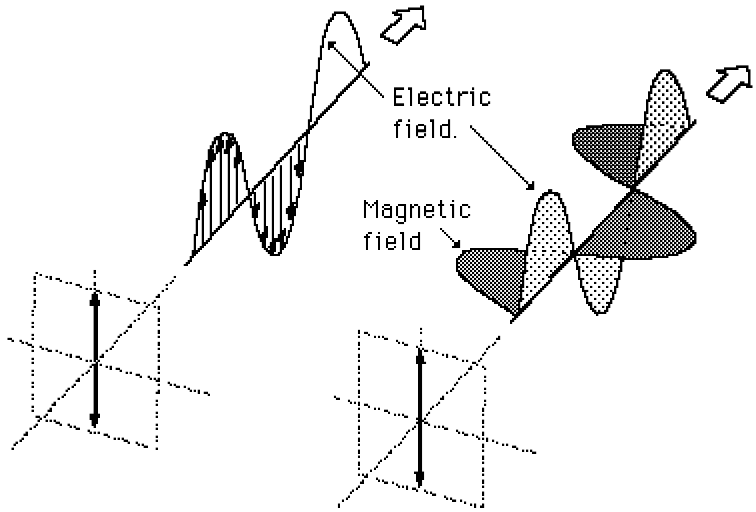
where

$$\underline{E}_z(x,t) = \underline{\mathbf{k}}E_{oz}\cos(kx-\omega t)$$

$$\underline{E}_y(x,t) = \underline{\mathbf{j}}E_{oy}\cos(kx-\omega t+\Phi)$$

where (**i,j,k**) are the unit verctors

- When $\Phi = 0$, $\underline{E}_z(x,t)$ and $\underline{E}_y(x,t)$ are in phase, and the wave (light) is said to be **linearly polarised:**



(from http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html#c2) (local copy)

- When $\Phi \neq 0$, light is not linearly polarised. In the special case of $\Phi = \pi/2$, and $E_{oz} = E_{oy}$ (= E_o) light is **circularly polarised:**

$$\underline{E}_z(x,t) = \underline{\mathbf{k}}E_o\cos(kx-\omega t)$$

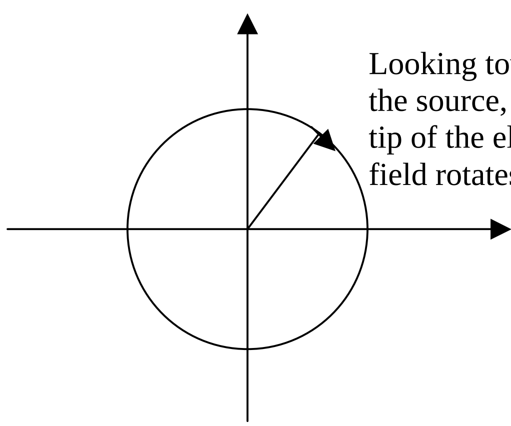
$$\underline{E}_y(x,t) = \underline{\mathbf{j}}E_o\cos(kx-\omega t+\pi/2) = \underline{\mathbf{j}}E_o\sin(kx-\omega t)$$

$$\underline{E}(x,t) = \underline{E}_z(x,t) + \underline{E}_y(x,t)$$

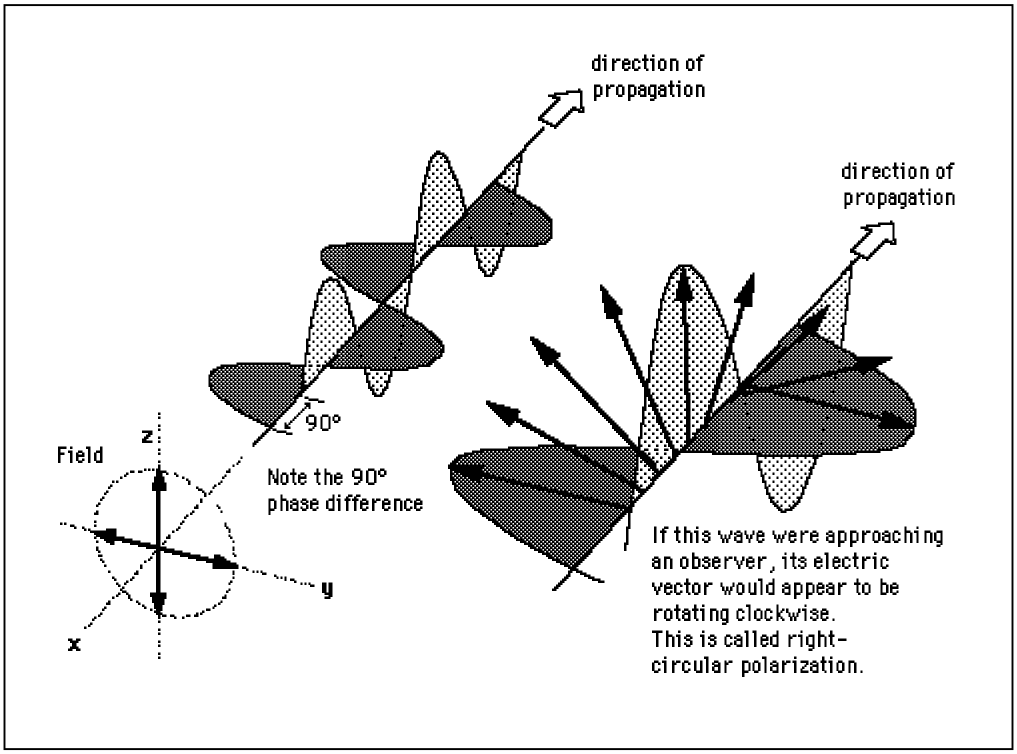
$$\underline{E}(x,t) = \underline{\mathbf{k}}E_o\cos(kx-\omega t) + \underline{\mathbf{j}}E_o\sin(kx-\omega t)$$

$$E(x,t) = E_o(\mathbf{k}\cos(kx-\omega t) + \underline{\mathbf{j}}\sin(kx-\omega t))$$

- When we look at the beam head on, we'll see the tip of the E field rotate in a circle. For example,
  - when $(kx-\omega t) = 0$, $E(x,t)$ points in the z-direction:
  - at some other time, when $(kx-\omega t) = \pi/2$ then $E(x,t)$ points in the y-direction:
- Therefore the tip of the electric field vector goes around in a circle! Hence **circularly polarised** light.



Looking towards the source, the tip of the electric field rotates



from http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html#c3 (local copy)

- The vector can rotate clock-wise or anti clock-wise
  - Right-circularly polarised light and left c.p:

$$\text{R.C.P.}\quad \underline{E}_{+}(x,t) = \underline{\mathbf{k}}E_{oz}\cos(kx-\omega t) + \underline{\mathbf{j}}E_{oy}\sin(kx-\omega t)$$

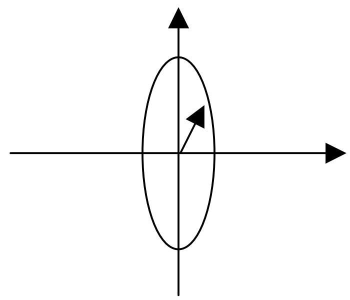
$$\text{L.C.P.}\quad \underline{E}_{-}(x,t) = \underline{\mathbf{k}}E_{oz}\cos(kx-\omega t) - \underline{\mathbf{j}}E_{oy}\sin(kx-\omega t)$$

- When we add a RCP and a LCP we get:

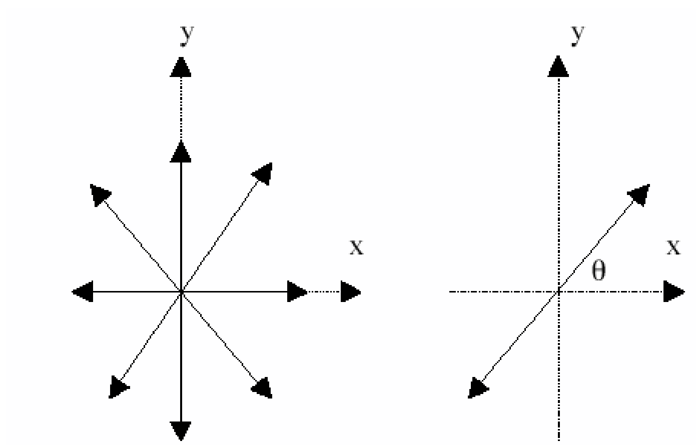
$$\underline{E}_{+}(x,t) + \underline{E}_{-}(x,t) = 2\underline{\mathbf{k}}E_{oz}\cos(kx-\omega t) \Leftarrow \textbf{linearly polarised light}$$

This was the case when $\Phi = \pi/2$, and $E_{oz} = E_{oy}$ (= E_o). What happens if $\Phi = \pi/2$, but $E_{oz} \neq E_{oy}$?

- When $\Phi = \pi/2$, but $E_{oz} \neq E_{oy}$ light is called elliptically polarised. The tip of the electric field writes an ellipse:



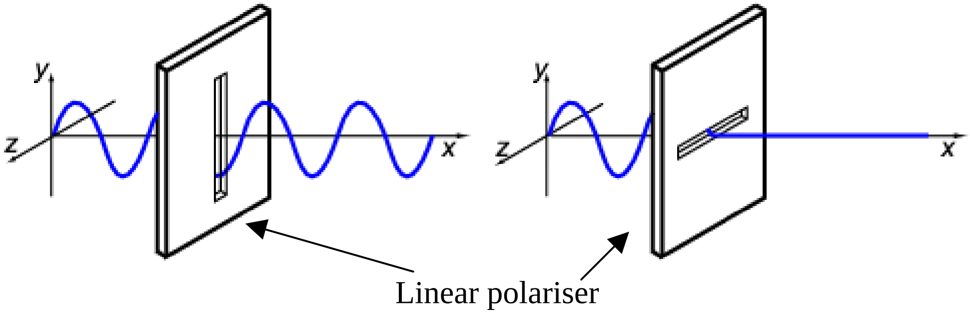
- When light is composed of a rapidly varying succession of different polarisations, we talk about **unpolarised** light. Sunlight is unpolarised. (NB. unpolarised light is not circularly polarised light):



unpolarised light linearly polarised light

Polarisers

How can we 'make' light oscillate in a certain direction? Polarisers do that.

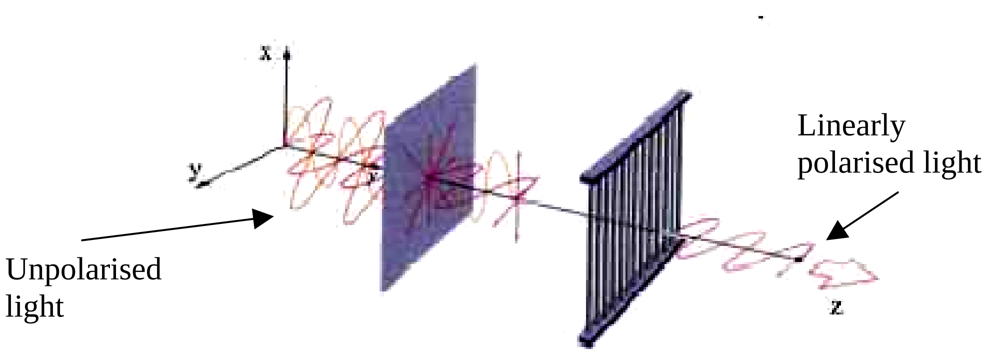


- ‘Instruments’ that produce well defined polarisation out of unpolarised light are called polarisers
- Polarisers <=> analysers

Linear polarisers

Many types. We described some here:

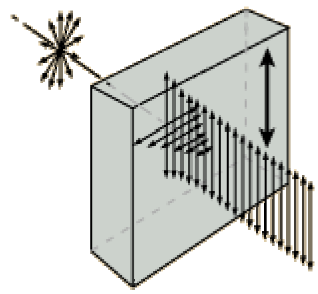
- **Wire-grid type polarisers**: light oscillating along the wire is absorbed, light oscillating perpendicular to the wires is not absorbed => transmitted light linearly polarised.



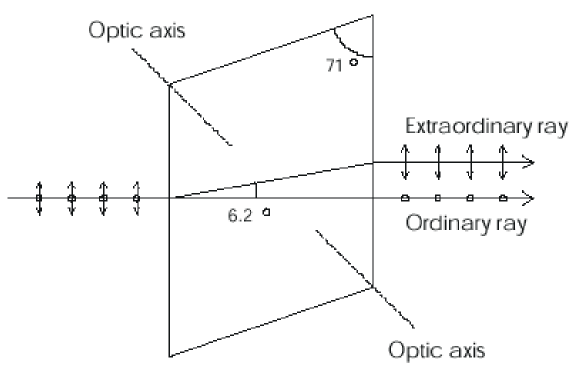
A Wire Grid Polarizer (Hecht figure 25.8)

- OK for microwaves, IR or in general longer wavelengths
- polaroid: similar to wire-grid but ‘wires’ are molecules. OK for shorter wavelengths, eg. visible radiation (Invented in 1928 by Land, P/G student)

- **Dichroic crystals**: selective absorption of one direction of polarisation. Some crystalline materials absorb more light in one polarisation than in the other, so as light propagates through the material it becomes more and more polarised. This anisotropy in absorption is called dichroism. There are several naturally occurring dichroic materials. (The polaroid discussed above also polarises by selective absorption.)

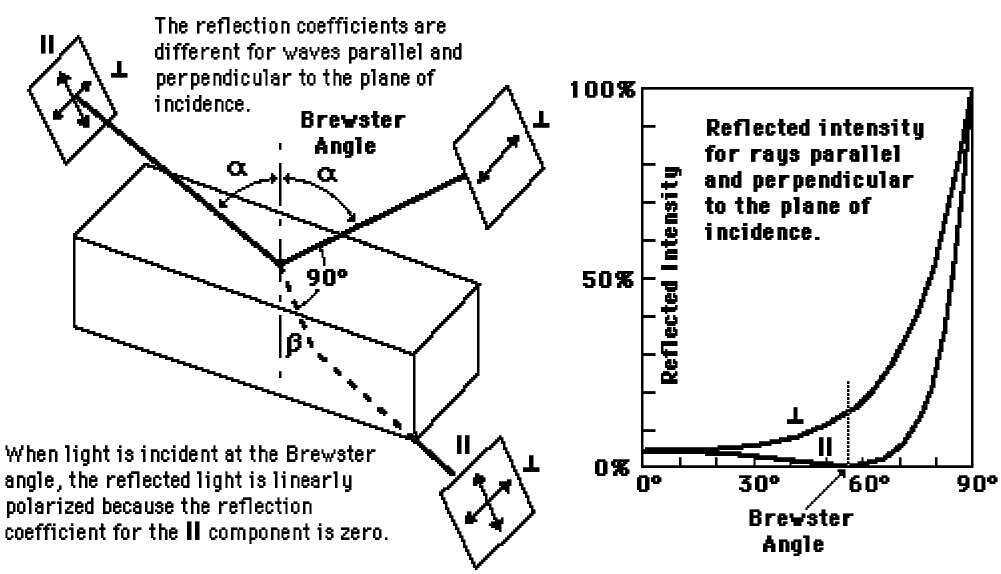


- **Birefringence:** in some crystals the refractive index depends on the direction of polarisation of light. So there are two refractive indexes depending on the polarisation: n_o (ordinary) and n_e (extraordinary). Since the r.i. are different, so is refraction: light will bend differently depending on the polarisation. These materials are anisotropic. In a birefringent material, light is split into two perpendicular polarizations with each being refracted slightly differently. Because they are refracted differently, the polarizations emerge in slightly different directions. Calcite is birefringent.



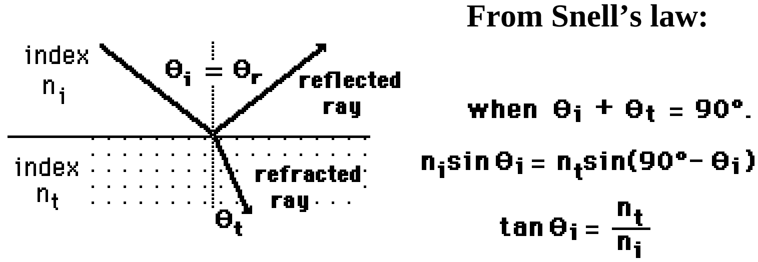
(from http://www.city.ac.uk/optics/optics/EX12.pdf) (local copy)

- **Polarisation by reflection:** when unpolarised light is reflected from a dielectric, the reflected light is found to be partially polarised.
  - the degree of polarisation depends on the angle of incidence
  - at a special angle, called Brewster's angle, the reflected light is 100% polarised. Thus shining unpolarised light on a dielectric, we can get polarised light from the reflected component. (The transmitted light will only be partially polarised)
  - Brewster's angle occurs when $\vartheta_i + \vartheta_t = 90^0$:



(from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html) (local copy)

From Snell's law:

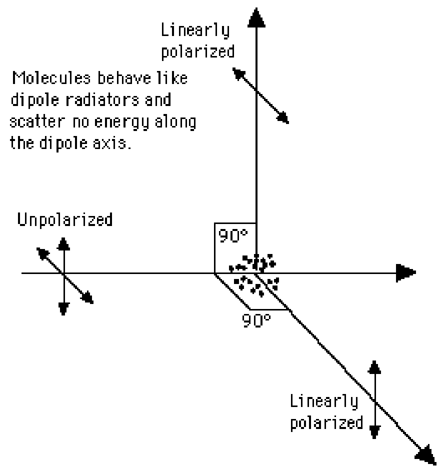


(from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html) (local copy)

Note:

- By measuring Brewster's angle we can determine the refractive index (for example, n_1=1 (air), n_2= 1.5 (glass) $\vartheta_i = \tan^{-1}(1.5) = 56^0$)
- Brewster windows often used in lasers

- **Polarisation by scattering:**



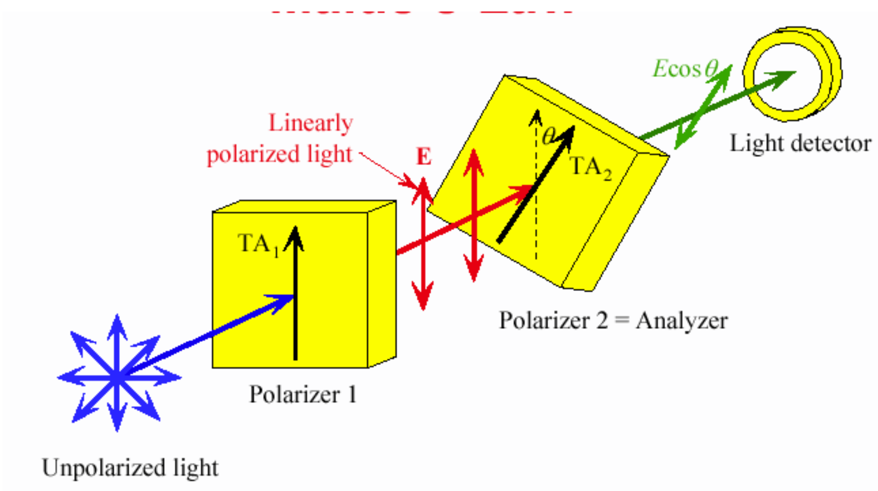
(From http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html) (local copy)

  - just like polarisation by reflection, polarisation by scattering produces linearly polarised light
  - air (other) molecules can be thought of as small antennas which reradiate perpendicular to their line of oscillation.
  - For example, the blue sky is (partially) polarised.

How much light gets through a linear polariser?

Malus's Law:

$$I = I_0 \cos^2 \vartheta$$



(from http://ece-classweb.ucsd.edu/archive/spring02/ece183/lecture17.pdf)