37 Diffraction

Georges Seurat painted *Sunday Afternoon on the Island of La Grande Jatte* using not brush strokes in the usual sense, but rather a myriad of small colored dots, in a style of painting now known as pointillism. You can see the dots if you stand close enough to the painting, but as you move away from it, they eventually blend and cannot be distinguished. Moreover, the color that you see at any given place on the painting changes as you move away—which is why Seurat painted with the dots.

What causes this change in color?

The answer is in this chapter.

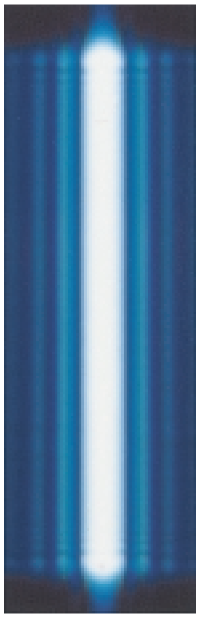


FIGURE 37-1 ■ This diffraction pattern appeared on a viewing screen when light that had passed through a narrow but tall vertical slit reached the screen. Diffraction causes light to flare out perpendicular to the long sides of the slit. That produces an interference pattern consisting of a broad central maximum less intense and narrower secondary (or side) maxima, with minima between them.

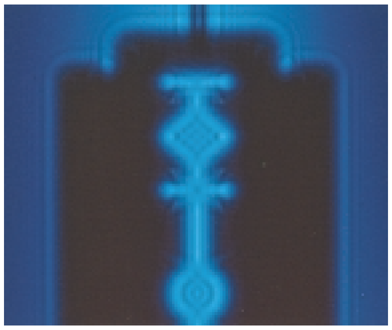


FIGURE 37-2 ■ The diffraction pattern produced by a razor blade in monochromatic light. Note the lines of alternating maximum and minimum intensity.

37-1 Diffraction and the Wave Theory of Light

In Chapter 36 we defined diffraction rather loosely as the flaring of light as it emerges from a narrow slit. More than just flaring occurs, however, because the light produces an interference pattern called a **diffraction pattern.** For example, when monochromatic light from a distant source (or a laser) passes through a narrow slit and is then intercepted by a viewing screen, the light produces on the screen a diffraction pattern like that in Fig. 37-1. This pattern consists of a broad and intense (very bright) central maximum and a number of narrower and less intense maxima (called **secondary** or **side** maxima) to both sides. In between the maxima are minima.

Such a pattern would be totally unexpected in geometrical optics: If light traveled in straight lines as rays, then the slit would allow some of those rays through and they would form a sharp, bright rendition of the slit on the viewing screen. As in Chapter 36, we again must conclude that geometrical optics is only an approximation.

Diffraction of light is not limited to situations of light passing through a narrow opening (such as a slit or pinhole). It also occurs when light passes an edge, such as the edges of the razor blade whose diffraction pattern is shown in Fig. 37-2. Note the lines of maxima and minima that run approximately parallel to the edges, at both the inside edges of the blade and the outside edges. As the light passes, say, the vertical edge at the left, it flares left and right and undergoes interference, producing the pattern along the left edge. The rightmost portion of that pattern actually lies within what would have been the shadow of the blade if geometrical optics prevailed.

You encounter a common example of diffraction when you look at a clear blue sky and see tiny specks and hair-like structures floating in your view. These *floaters,* as they are called, are produced when light passes the edges of tiny deposits in the vitreous humor, the transparent material filling most of your eyeball. What you are seeing when a floater is in your field of vision is the diffraction pattern produced on the retina by one of these deposits. If you sight through a pinhole in an otherwise opaque sheet so as to make the light entering your eye approximately a plane wave, you can distinguish individual maxima and minima in the patterns.

The Fresnel Bright Spot

Diffraction finds a ready explanation in the wave theory of light. However, this theory, originally advanced in the late 1600s by Huygens and used 123 years later by Young to explain double-slit interference, was very slow in being adopted, largely because it ran counter to Newton's theory that light was a stream of particles.

Newton's view was the prevailing view in French scientific circles of the early 19th century, when Augustin Fresnel was a young military engineer. Fresnel, who believed in the wave theory of light, submitted a paper to the French Academy of Sciences describing his experiments with light and his wave-theory explanations of them.

In 1819, the Academy, dominated by supporters of Newton and thinking to challenge the wave point of view, organized a prize competition for an essay on the subject of diffraction. Fresnel won. The Newtonians, however, were neither converted nor silenced. One of them, S. D. Poisson, pointed out the "strange result" that if Fresnel's theories were correct, then light waves should flare into the shadow region of a sphere as they pass the edge of the sphere, producing a bright spot at the center of the shadow. The prize committee arranged to have Dominique Argo test the famous mathematician's prediction. He discovered (see Fig. 37-3) that the predicted *Fresnel bright spot,* as we call it today, was indeed there!* Nothing builds confidence in a

* Since Poisson predicted the spot and Argo discovered it, an alternate name is the Poisson-Argo bright spot.

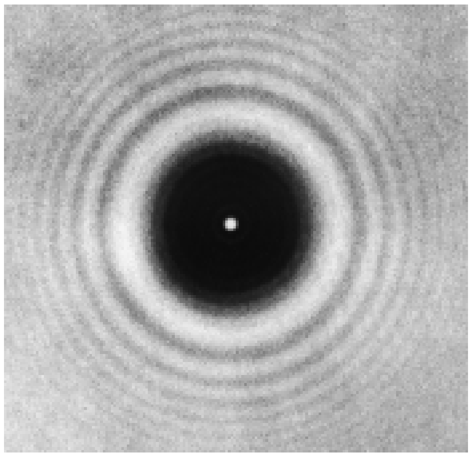


FIGURE 37-3 ■ A photograph of the diffraction pattern of a disk. Note the concentric diffraction rings and the Fresnel bright spot at the center of the pattern. This experiment is essentially identical to that arranged by the committee testing Fresnel's theories, because both the sphere they used and the disk used here have a cross section with a circular edge.

theory so much as having one of its unexpected and counterintuitive predictions verified by experiment.

37-2 Diffraction by a Single Slit: Locating the Minima

Let us now examine the diffraction pattern of plane waves of light of wavelength λ that are diffracted by a single, long, narrow slit of width a in an otherwise opaque screen B, as shown in cross section in Fig. 37-4*a*. (In that figure, the slit's length extends into and out of the page, and the incoming wavefronts are parallel to screen B.) When the diffracted light reaches viewing screen C, waves from different points within the slit undergo interference and produce a diffraction pattern of bright and dark fringes (interference maxima and minima) on the screen. To locate the fringes, we shall use a procedure somewhat similar to the one we used to locate the fringes in a two-slit interference pattern. However, diffraction is more mathematically challenging, and here we shall be able to find equations for only the dark fringes.

Before we do that, however, we can justify the central bright fringe seen in Fig. 37-1 by noting that the Huygens wavelets from all points in the slit travel about the same distance to reach the center of the pattern and thus are in phase there. As for the other bright fringes, we can say only that they are approximately halfway between adjacent dark fringes.

To find the dark fringes, we shall use a clever (and simplifying) strategy that involves pairing up all the rays coming through the slit and then finding what conditions cause the wavelets of the rays in each pair to cancel each other. Figure 37-4*a* shows how we apply this strategy to locate the first dark fringe, at point P_1. First, we mentally divide the slit into two zones of equal widths $a/2$. Then we extend to P_1 a light ray r_1 from the top point of the top zone and a light ray r_2 from the top point of the bottom zone. A central axis is drawn from the center of the slit to screen C, and P_1 is located at an angle θ to that axis.

The wavelets of the pair of rays r_1 and r_2 are in phase within the slit because they originate from the same wavefront passing through the slit, along the width of the slit. However, to produce the first dark fringe they must be out of phase by $\lambda/2$ when they reach P_1; this phase difference is due to their path length difference, with the wavelet of r_2 traveling a longer path to reach P_1 than the wavelet of r_1. To display this path length difference, we find a point b on ray r_2 such that the path length from b to P_1 matches the path length of ray r_1. Then the path length difference between the two rays is the distance from the center of the slit to b.

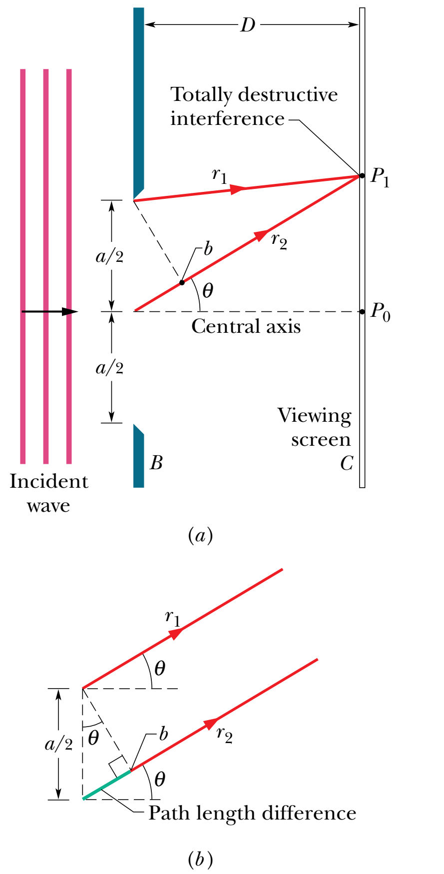


FIGURE 37-4 ■ (*a*) Waves from the top points of two zones of width $a/2$ undergo totally destructive interference at point P_1 on viewing screen C. (*b*) For $D \gg a$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

When viewing screen C is near screen B, as in Fig. 37-4a, the diffraction pattern on C is difficult to describe mathematically. However, we can simplify the mathematics considerably if we arrange for the distance between the slit and screen D to be much larger than the slit width a. Then we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis (Fig. 37-4b). We can also approximate the triangle formed by point b, the top point of the slit, and the center point of the slit as being a right triangle, and one of the angles inside that triangle as being θ. The path length difference between rays r_1 and r_2 (which is still the distance from the center of the slit to point b) is then equal to $(a/2)\sin\theta$.

We can repeat this analysis for any other pair of rays originating at corresponding points in the two zones (say, at the midpoints of the zones) and extending to point P_1. Each such pair of rays has the same path length difference $(a/2)\sin\theta$. Setting this common path length difference equal to $\lambda/2$ (our condition for the first dark fringe), we have

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

which gives us

$$a\sin\theta = \lambda \qquad \text{(first minimum for } D \gg a\text{).} \tag{37-1}$$

Given slit width a and wavelength λ, Eq. 37-1 tells us the angle θ of the first dark fringe above and (by symmetry) below the central axis.

Note that if we begin with $a > \lambda$ and then narrow the slit while holding the wavelength constant, we increase the angle at which the first dark fringes appear; that is, the extent of the diffraction (the extent of the flaring and the width of the pattern) is *greater* for a *narrower* slit. When we have reduced the slit width to the wavelength (that is, $a = \lambda$), the angle of the first dark fringes is 90°. Since the first dark fringes mark the two edges of the central bright fringe, that bright fringe must then cover the entire viewing screen.

We find the second dark fringes above and below the central axis as we found the first dark fringes, except that we now divide the slit into *four* zones of equal widths $a/4$, as shown in Fig. 37-5a. We then extend rays r_1, r_2, r_3, and r_4 from the top points of the zones to point P_2, the location of the second dark fringe above the central axis. To produce that fringe, the path length difference between r_1 and r_2, that between r_2 and r_3, and that between r_3 and r_4 must all be equal to $\lambda/2$.

For $D \gg a$, we can approximate these four rays as being parallel, at angle θ to the central axis. To display their path length differences, we extend a perpendicular line through each adjacent pair of rays, as shown in Fig. 37-5b, to form a series of right triangles, each of which has a path length difference as one side. We see from the top triangle that the path length difference between r_1 and r_2 is $(a/4)\sin\theta$. Similarly, from the bottom triangle, the path length difference between r_3 and r_4 is also $(a/4)\sin\theta$. In fact, the path length difference for any two rays that originate at corresponding points in two adjacent zones is $(a/4)\sin\theta$. Since in each such case the path length difference is equal to $\lambda/2$, we have

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2},$$

which gives us

$$a\sin\theta = 2\lambda \qquad \text{(second minimum for } D \gg a\text{).} \tag{37-2}$$

We could now continue to locate dark fringes in the diffraction pattern by splitting up the slit into more zones of equal width. We would always choose an even num-

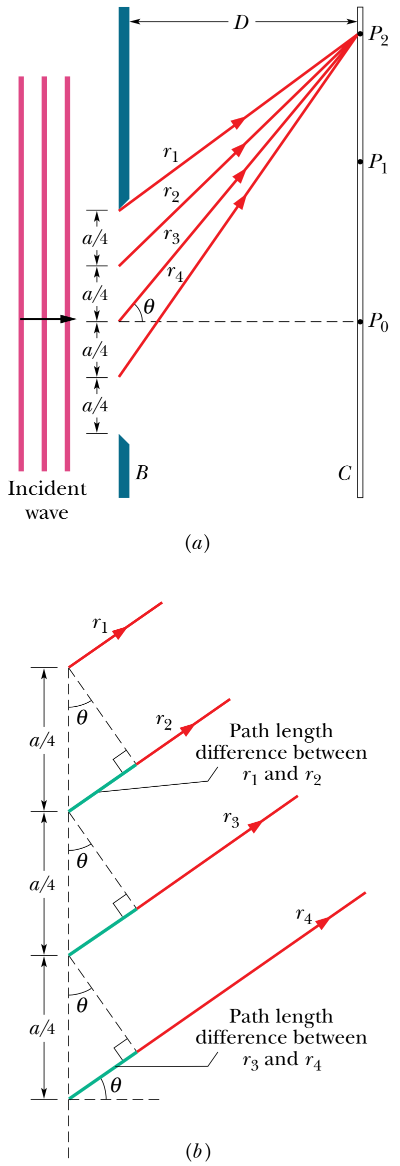


FIGURE 37-5 ■ (a) Waves from the top points of four zones of width $a/4$ undergo totally destructive interference at point P_2. (b) For $D \gg a$, we can approximate rays r_1, r_2, r_3, and r_4 as being parallel, at angle θ to the central axis.

ber of zones so that the zones (and their waves) could be paired as we have been doing. We would find that the dark fringes above and below the central axis can be located with the following general equation:

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \qquad \text{(single slit minima—dark fringes)}. \qquad (37\text{-}3)$$

You can remember this result in the following way. Draw a triangle like the one in Fig. 37-4*b*, but for the full slit width *a*, and note that the path length difference between the top and bottom rays from the slit equals $a \sin\theta$. Thus, Eq. 37-3 says:

> In a single-slit diffraction experiment, dark fringes are produced where the path length differences ($a \sin\theta$) between the top and bottom rays are equal to $\lambda, 2\lambda, 3\lambda \ldots$.

This may seem to be wrong, because the waves of those two particular rays will be exactly in phase with each other when their path length difference is an integer number of wavelengths. However, they each will still be part of a pair of waves that are exactly out of phase with each other; thus, *each* will be canceled by some other wave.

READING EXERCISE 37-1: We produce a diffraction pattern on a viewing screen by means of a long narrow slit illuminated by blue light. Does the pattern expand away from the bright center (the maxima and minima shift away from the center) or contract toward it if we (a) switch to yellow light or (b) decrease the slit width? ■

TOUCHSTONE EXAMPLE 37-1: White Light, Red Light

A slit of width *a* is illuminated by white light (which consists of all the wavelengths in the visible range).

(a) For what value of *a* will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15°$?

SOLUTION ■ The **Key Idea** here is that diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 37-3 ($a \sin\theta = m\lambda$). When we set $m = 1$ (for the first minimum) and substitute the given values of θ and λ, Eq. 37-3 yields

$$a = \frac{m\lambda}{\sin\theta} = \frac{(1)(650\text{ nm})}{\sin 15°}$$

$$= 2511\text{ nm} \approx 2.5\ \mu\text{m}. \qquad \text{(Answer)}$$

For the incident light to flare out that much ($\pm 15°$ to the first minima) the slit has to be very fine indeed—about four times the wavelength. For comparison, note that a fine human hair may be about 100 μm in diameter.

(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15°, thus coinciding with the first minimum for the red light?

SOLUTION ■ The **Key Idea** here is that the first side maximum for any wavelength is about halfway between the first and second minima for that wavelength. Those first and second minima can be located with Eq. 37-3 by setting $m = 1$ and $m = 2$, respectively. Thus, the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 37-3 becomes

$$a \sin\theta = 1.5\lambda'.$$

Solving for λ' and substituting known data yield

$$\lambda' = \frac{a \sin\theta}{1.5} = \frac{(2511\text{ nm})(\sin 15°)}{1.5}$$

$$= 430\text{ nm}. \qquad \text{(Answer)}$$

Light of this wavelength is violet. The first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width is. If the slit is relatively narrow, the angle θ at which this overlap occurs will be relatively large, and conversely for a wide slit the angle is small.

37-3 Intensity in Single-Slit Diffraction, Qualitatively

In Section 37-2 we saw how to find the positions of the minima and the maxima in a single-slit diffraction pattern. Now we turn to a more general problem: Find an expression for the intensity I of the pattern as a function of θ, the angular position of a point on a viewing screen.

To do this, we divide the slit of Fig. 37-4*a* into N zones of equal widths Δx small enough that we can assume each zone acts as a source of Huygens wavelets. We wish to superimpose the wavelets arriving at an arbitrary point P on the viewing screen, at angle θ to the central axis, so that we can determine the amplitude E_θ of the magnitude of the electric field of the resultant wave at P. The intensity of the light at P is then proportional to the square of that amplitude.

To find E_θ, we need the phase relationships among the arriving wavelets. The phase difference between wavelets from adjacent zones is given by

$$(\text{phase difference}) = \left(\frac{2\pi}{\lambda}\right)(\text{path length difference}).$$

For point P at angle θ, the path length difference between wavelets from adjacent zones is $\Delta x \sin\theta$, so the phase difference $\Delta\phi$ between wavelets from adjacent zones is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta). \tag{37-4}$$

We assume that the wavelets arriving at P all have the same amplitude ΔE. To find the amplitude E_θ of the resultant wave at P, we add the amplitudes ΔE via phasors. To do this, we construct a diagram of N phasors, one corresponding to the wavelet from each zone in the slit.

For point P_0 at $\theta = 0$ on the central axis of Fig. 37-4*a*, Eq. 37-4 tells us that the phase difference $\Delta\phi$ between the wavelets is zero; that is, the wavelets all arrive in phase. Figure 37-6*a* is the corresponding phasor diagram; adjacent phasors represent wavelets from adjacent zones and are arranged head to tail. Because there is zero phase difference between the wavelets, there is zero angle between each pair of adjacent phasors. The amplitude E_θ of the net wave at P_θ is the vector-like sum of these phasors. This arrangement of the phasors turns out to be the one that gives the greatest value for the amplitude E_θ. We call this value $E^{\max}$; that is, $E^{\max}$ is the value of E_θ for $\theta = 0$.

We next consider a point P that is at a small angle θ to the central axis. Equation 37-4 now tells us that the phase difference $\Delta\phi$ between wavelets from adjacent zones is no longer zero. Figure 37-6*b* shows the corresponding phasor diagram; as before,

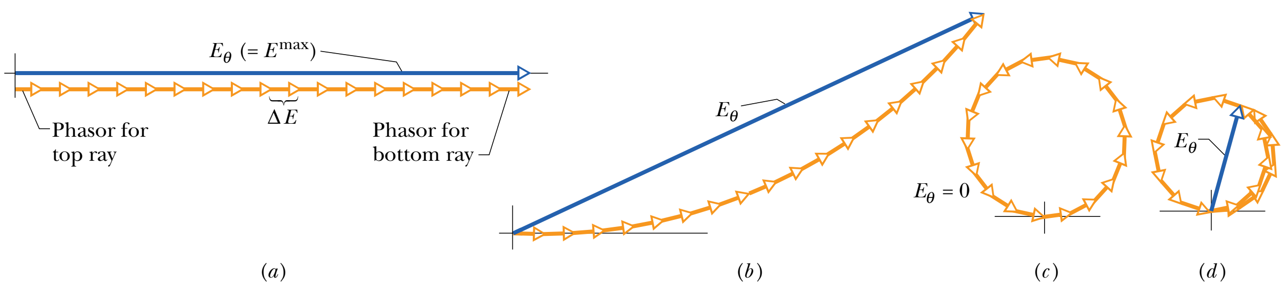


FIGURE 37-6 Phasor diagrams for $N = 18$ phasors, corresponding to the division of a single slit into 18 zones. Resultant amplitudes E_θ are shown for (*a*) the central maximum at $\theta = 0$, (*b*) a point on the screen lying at a small angle θ to the central axis, (*c*) the first minimum, and (*d*) the first side maximum.

the phasors are arranged head to tail, but now there is an angle $\Delta\phi$ between adjacent phasors. The amplitude E_θ at this new point is still the vector sum of the phasors, but it is smaller than the amplitude in Fig. 37-6*a*, which means that the intensity of the light is less at this new point P than at P_θ.

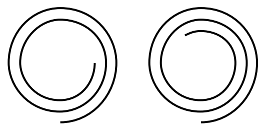
If we continue to increase θ, the angle $\Delta\phi$ between adjacent phasors increases, and eventually the chain of phasors curls completely around so that the head of the last phasor just reaches the tail of the first phasor (Fig. 37-6*c*). The amplitude E_θ is now zero, which means that the intensity of the light is also zero. We have reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of 2π rad, which means that the path length difference between the top and bottom rays through the slit equals one wavelength. Recall that this is the condition we determined for the first diffraction minimum.

As we continue to increase θ, the angle $\Delta\phi$ between adjacent phasors continues to increase, the chain of phasors begins to wrap back on itself, and the resulting coil begins to shrink. Amplitude E_θ now increases until it reaches a maximum value in the arrangement shown in Fig. 37-6*d*. This arrangement corresponds to the first side maximum in the diffraction pattern.

If we increase θ a bit more, the resulting shrinkage of the coil decreases E_θ, which means that the intensity also decreases. When θ is increased enough, the head of the last phasor again meets the tail of the first phasor. We have then reached the second minimum.

We could continue this qualitative method of determining the maxima and minima of the diffraction pattern but, instead, we shall now turn to a quantitative method.

READING EXERCISE 37-2: The figures represent, in smoother form (with more phasors) than Fig. 37-6, the phasor diagrams for two points of a diffraction pattern that are on opposite sides of a certain diffraction maximum. (a) Which maximum is it? (b) What is the approximate value of m (in Eq. 37-3) that corresponds to this maximum?



(*a*) (*b*)

37-4 Intensity in Single-Slit Diffraction, Quantitatively

Equation 37-3 tells us how to locate the minima of the single-slit diffraction pattern on screen C of Fig. 37-4*a* as a function of the angle θ in that figure. Here we wish to derive an expression for the intensity I_θ of the pattern as a function of θ. We state, and shall prove below, that the intensity is given by

$$I_\theta = I^{\max}\left(\frac{\sin\alpha}{\alpha}\right)^2, \qquad (37\text{-}5)$$

where

$$\alpha = \frac{1}{2}\Delta\phi = \frac{\pi a}{\lambda}\sin\theta. \qquad (37\text{-}6)$$

The symbol α is just a convenient connection between the angle θ that locates a point on the viewing screen and the light intensity I_θ at that point. $I^{\max}$ is the greatest value of the intensity I_θ in the pattern and occurs at the central maximum (where $\theta = 0$), and $\Delta\phi$ is the phase difference (in radians) between the top and bottom rays from the slit width a.

Study of Eq. 37-5 shows that intensity minima will occur where

$$\alpha = m\pi, \qquad \text{for } m = 1, 2, 3, \ldots. \qquad (37\text{-}7)$$

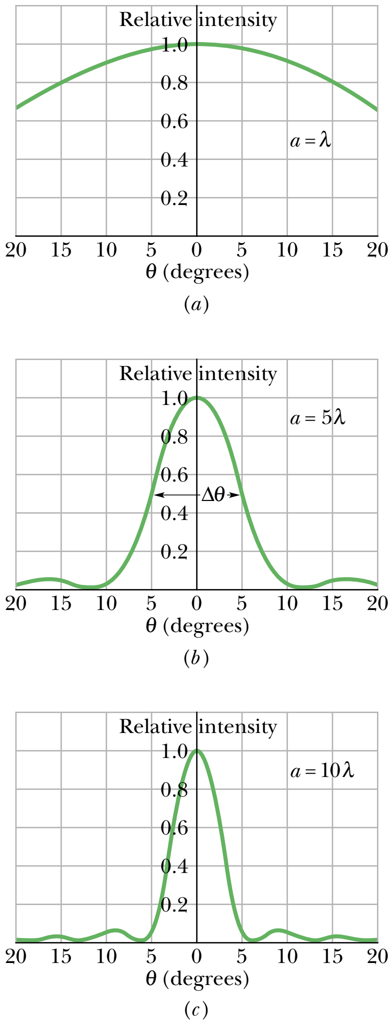


FIGURE 37-7 ■ The relative intensity in single-slit diffraction for three values of the ratio a/λ. The wider the slit is, the narrower is the central diffraction maximum.

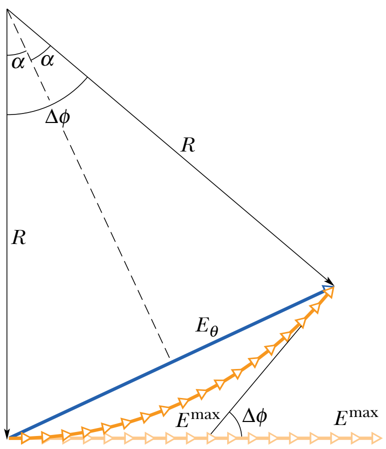


FIGURE 37-8 ■ A construction used to calculate the intensity in single-slit diffraction. The situation shown corresponds to that of Fig. 37-6*b*.

If we put this result into Eq. 37-6 we find

$$m\pi = \frac{\pi a}{\lambda}\sin\theta, \qquad \text{for } m = 1, 2, 3, \ldots,$$

or

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ \ldots \qquad \text{(minima—dark fringes)}, \tag{37-8}$$

which is exactly Eq. 37-3, the expression that we derived earlier for the location of the minima.

Figure 37-7 shows plots of the intensity of a single-slit diffraction pattern, calculated with Eqs. 37-5 and 37-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$. Note that as the slit width increases (relative to the wavelength), the width of the *central diffraction maximum* (the central hill-like region of the graphs) decreases; that is, the light undergoes less flaring by the slit. The secondary maxima also decrease in width (and become weaker). In the limit of slit width a being much greater than wavelength λ, the secondary maxima due to the slit disappear; we then no longer have single-slit diffraction (but we still have diffraction due to the edges of the wide slit, like that produced by the edges of the razor blade in Fig. 37-2).

Proof of Eqs. 37-5 and 37-6

The arc of phasors in Fig. 37-8 represents the wavelets that reach an arbitrary point P on the viewing screen of Fig. 37-4, corresponding to a particular small angle θ. The amplitude E_θ of the resultant wave at P is the vector sum of these phasors. If we divide the slit of Fig. 37-4 into infinitesimal zones of width Δx, the arc of phasors in Fig. 37-8 approaches the arc of a circle; we call its radius R as indicated in that figure. The length of the arc must be E^{max}, the amplitude at the center of the diffraction pattern, because if we straightened out the arc we would have the phasor arrangement of Fig. 37-6*a* (shown lightly in Fig. 37-8).

The angle $\Delta\phi$ in the lower part of Fig. 37-8 is the difference in phase between the infinitesimal vectors at the left and right ends of arc E^{max}. From the geometry, $\Delta\phi$ is also the angle between the two radii marked R in Fig. 37-8. The dashed line in that figure, which bisects $\Delta\phi$, then forms two congruent right triangles. From either triangle we can write

$$\sin\tfrac{1}{2}\Delta\phi = \frac{E_\theta}{2R}. \tag{37-9}$$

In radian measure, $\Delta\phi$ is (with E^{max} considered to be a circular arc)

$$\Delta\phi = \frac{E^{max}}{R}.$$

Solving this equation for R, substituting the result into Eq. 37-9 and re-arranging terms yields

$$E_\theta = \frac{E^{max}}{\frac{1}{2}\Delta\phi}\sin\tfrac{1}{2}\Delta\phi. \tag{37-10}$$

In Section 34-4 we saw that the intensity of an electromagnetic wave is proportional to the square of the amplitude of its electric field. Here, this means that the maximum intensity I^{max} (which occurs at the center of the diffraction pattern) is proportional to $(E^{max})^2$ and the intensity I_θ at angle θ is proportional to E_θ^2. Thus, we may write

$$\frac{I_\theta}{I^{\max}} = \frac{E_\theta^2}{(E^{\max})^2}. \tag{37-11}$$

Substituting for E_θ with Eq. 37-10 and then substituting $\alpha = \frac{1}{2}\Delta\phi$, we are led to the following expression for the intensity as a function of θ:

$$I_\theta = I^{\max}\left(\frac{\sin\alpha}{\alpha}\right)^2.$$

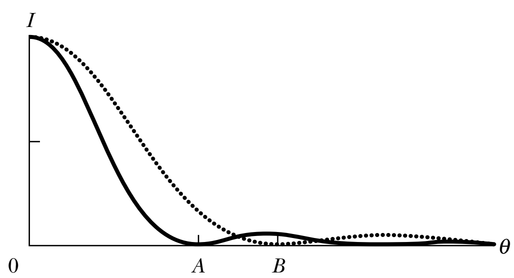
This is exactly Eq. 37-5, one of the two equations we set out to prove.

The second equation we wish to prove relates α to θ: The phase difference $\Delta\phi$ between the rays from the top and bottom of the entire slit may be related to a path length difference with Eq. 37-4; it tells us that

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta),$$

where a is the sum of the widths Δx of the infinitesimal zones. However, $\Delta\phi = 2\alpha$, so this equation reduces to Eq. 37-6.

READING EXERCISE 37-3: Two wavelengths, 650 and 430 nm, are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity I versus angle θ for the two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at (a) angle A and (b) angle B?



TOUCHSTONE EXAMPLE 37-2: Maxima Intensities

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 37-1, measured relative to the intensity of the central maximum.

SOLUTION ■ One **Key Idea** here is that the secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 37-7 ($\alpha = m\pi$). The locations of the secondary maxima are then given (approximately) by

$$\alpha = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 1, 2, 3, \ldots,$$

with α in radian measure.

A second **Key Idea** is that we can relate the intensity I at any point in the diffraction pattern to the intensity $I^{\max}$ of the central maximum via Eq. 37-5. Thus, we can substitute the approximate values of α for the secondary maxima into Eq. 37-5 to obtain the relative intensities at those maxima. We get

$$\frac{I}{I^{\max}} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(m + \frac{1}{2})\pi}{(m + \frac{1}{2})\pi}\right)^2, \qquad \text{for } m = 1, 2, 3, \ldots.$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

$$\frac{I_1}{I^{\max}} = \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2$$
$$= 4.50 \times 10^{-2} \approx 4.5\%. \qquad \text{(Answer)}$$

For $m = 2$ and $m = 3$ we find that

$$\frac{I_2}{I^{\max}} = 1.6\% \qquad \text{and} \qquad \frac{I_3}{I^{\max}} = 0.83\%. \qquad \text{(Answer)}$$

Successive secondary maxima decrease rapidly in intensity. Figure 37-1 was deliberately overexposed to reveal them.

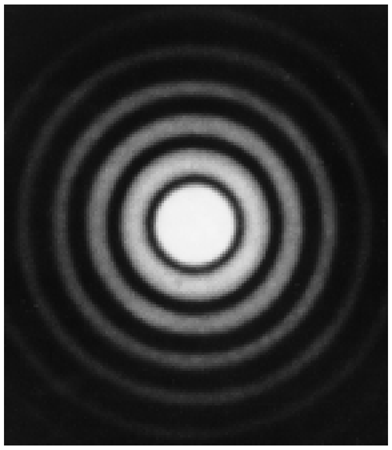


FIGURE 37-9 ■ The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

37-5 Diffraction by a Circular Aperture

Here we consider diffraction by a circular aperture—that is, a circular opening such as a circular lens, through which light can pass. Figure 37-9 shows the image of a distant point source of light (a star, for instance) formed on photographic film placed in the focal plane of a converging lens. This image is not a point, as geometrical optics would suggest, but a circular disk surrounded by several progressively fainter secondary rings. Comparison with Fig. 37-1 leaves little doubt that we are dealing with a diffraction phenomenon. Here, however, the aperture is a circle of diameter d rather than a rectangular slit.

The analysis of such patterns is complex. It shows, however, that the first minimum for the diffraction pattern of a circular aperture of diameter d is located by

$$\sin\theta = 1.22\frac{\lambda}{d} \qquad \text{(first minimum—circular aperture).} \qquad (37\text{-}12)$$

The angle θ here is the angle from the central axis to any point on that (circular) minimum. Compare this with Eq. 37-1,

$$\sin\theta = \frac{\lambda}{a} \qquad \text{(first minimum—single slit),} \qquad (37\text{-}13)$$

which locates the first minimum for a long narrow slit of width a. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

Resolvability

The fact that lens images are diffraction patterns is important when we wish to *resolve* (distinguish) two distant point objects whose angular separation is small. Figure 37-10 shows, in three different cases, the visual appearance and corresponding intensity pattern for two distant point objects (stars, say) with small angular separation. In Figure 37-10*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished from a single point object. In Fig. 37-10*b* the objects are barely resolved, and in Fig. 37-10*c* they are fully resolved.

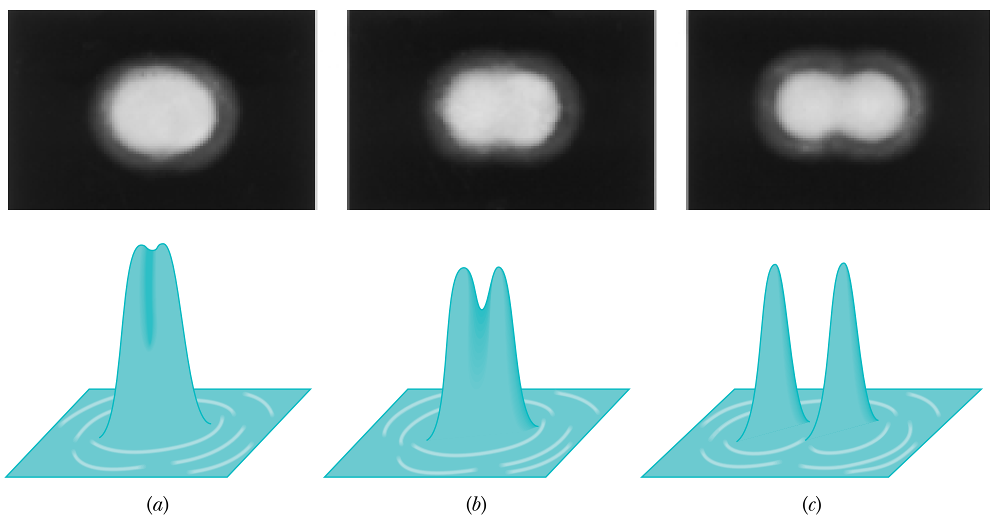


FIGURE 37-10 ■ At the top, the images of two point sources (stars), formed by a converging lens. At the bottom, representations of the image intensities. In (*a*) the angular separation of the sources is too small for them to be distinguished; in (*b*) they can be marginally distinguished, and in (*c*) they are clearly distinguished. Rayleigh's criterion is just satisfied in (*b*), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

In Fig. 37-10*b* the angular separation of the two point sources is such that the central maximum of the diffraction pattern of one source is centered on the first minimum of the diffraction pattern of the other, a condition called **Rayleigh's criterion** for resolvability. From Eq. 37-12, two objects that are barely resolvable by this criterion must have an angular separation θ_R of

$$\theta_R = \sin^{-1}\frac{1.22\lambda}{d}.$$

Since the angles involved are small, we can replace $\sin\theta_R$ with θ_R expressed in radians:

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion—circular aperture).} \tag{37-14}$$

Rayleigh's criterion for resolvability is only an approximation, because resolvability depends on many factors, such as the relative brightness of the sources and their surroundings, turbulence in the air between the sources and the observer, and the functioning of the observer's visual system. Experimental results show that the least angular separation that can actually be resolved by a person is generally somewhat greater than the value given by Eq. 37-14. However, for the sake of calculations here, we shall take Eq. 37-14 as being a precise criterion: If the angular separation θ between the sources is greater than θ_R, we can resolve the sources; if it is less, we cannot.

Rayleigh's criterion can explain the colors in Seurat's *Sunday Afternoon on the Island of La Grande Jatte* (or any other pointillistic painting). When you stand close enough to the painting, the angular separations θ of adjacent dots are greater than θ_R and thus the dots can be seen individually. Their colors are the colors of the paints Seurat used. However, when you stand far enough from the painting, the angular separations θ are less than θ_R and the dots cannot be seen individually. The resulting blend of colors coming into your eye from any group of dots can then cause your brain to "make up" a color for that group—a color that may not actually exist in the group. In this way, Seurat uses your visual system to create the colors of his art.

When we wish to use a lens instead of our visual system to resolve objects of small angular separation, it is desirable to make the diffraction pattern as small as possible. According to Eq. 37-14, this can be done either by increasing the lens diameter or by using light of a shorter wavelength.

For this reason ultraviolet light is often used with microscopes; because of its shorter wavelength, it permits finer detail to be examined than would be possible for the same microscope operated with visible light. It turns out that under certain circumstances, a beam of electrons behaves like a wave. In an *electron microscope* such beams may have an effective wavelength that is 10^{-5} of the wavelength of visible light. They permit the detailed examination of tiny structures, like that in Fig. 37-11, that would be blurred by diffraction if viewed with an optical microscope.

FIGURE 37-11 ■ A false-color scanning electron micrograph of red blood cells traveling through an arterial branch.

READING EXERCISE 37-4: Suppose you can barely resolve two red dots, due to diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.) ■

TOUCHSTONE EXAMPLE 37-3: Circular Converging Lens

A circular converging lens, with diameter $d = 32$ mm and focal length $f = 24$ cm, forms images of distant point objects in the focal plane of the lens. Light of wavelength $\lambda = 550$ nm is used.

(a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh's criterion?

SOLUTION ■ Figure 37-12 shows two distant point objects P_1 and P_2, the lens, and a viewing screen in the focal plane of the lens. It also shows, on the right, plots of light intensity I versus position on the screen for the central maxima of the images formed by the lens. Note that the angular separation θ_o of the objects equals the angular separation θ_i of the images. Thus, the **Key Idea** here is that if the images are to satisfy Rayleigh's criterion for resolvability, the angular separations on both sides of the lens must be given by Eq. 37-14 (assuming small angles). Substituting the given data, we obtain from Eq. 37-14

$$\theta_o = \theta_i = \theta_R = 1.22\,\frac{\lambda}{d}$$
$$= \frac{(1.22)(550 \times 10^{-9}\text{ m})}{32 \times 10^{-3}\text{ m}} = 2.1 \times 10^{-5}\text{ rad.} \quad \text{(Answer)}$$

At this angular separation, each central maximum in the two intensity curves of Fig. 37-12 is centered on the first minimum of the other curve.

(b) What is the separation Δx of the centers of the *images* in the focal plane? (That is, what is the separation of the *central* peaks in the two curves?)

SOLUTION ■ The **Key Idea** here is to relate the separation Δx to the angle θ_i, which we now know. From either triangle between the lens and the screen in Fig. 37-12, we see that tan $\theta_i/2 = \Delta x/2f$. Rearranging this and making the approximation tan $\theta<\theta$, we find

$$\Delta x = f\theta_i, \qquad (37\text{-}15)$$

where θ_i is in radian measure. Substituting known data then yields

$$\Delta x = (0.24\text{ m})(2.1 \times 10^{-5}\text{ rad}) = 5.0\ \mu\text{m}. \quad \text{(Answer)}$$

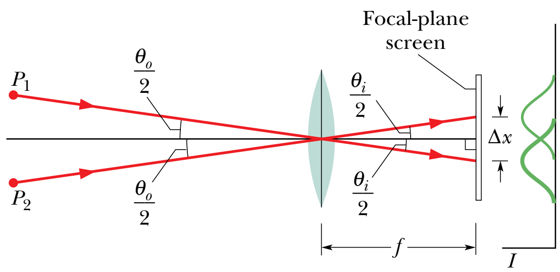


FIGURE 37-12 ■ Light from two distant point objects P_1 and P_2 passes through a converging lens and forms images on a viewing screen in the focal plane of the lens. Only one representative ray from each object is shown. The images are not points but diffraction patterns, with intensities approximately as plotted at the right. The angular separation of the objects is θ_o and that of the images is θ_i; the central maxima of the images have a separation Δx.

37-6 Diffraction by a Double Slit

In the double-slit experiments of Chapter 36, we implicitly assumed that the slits were narrow compared to the wavelength of the light illuminating them; that is, $a \ll \lambda$. For such narrow slits, the central maximum of the diffraction pattern of either slit covers the entire viewing screen. Moreover, the interference of light from the two slits produces bright fringes that all have approximately the same intensity (Fig. 36-9).

In practice with visible light, however, the condition $a \ll \lambda$ is rarely met. For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity. That is, the intensities of the fringes produced by double-slit interference (as discussed in Chapter 36) are modified by diffraction of the light passing through each slit (as discussed in this chapter).

As an example, the intensity plot of Fig. 37-13*a* (like that in Fig. 36-9) suggests the double-slit interference pattern that would occur if the slits were infinitely narrow (for $a \ll \lambda$); all the bright interference fringes would have the same intensity. The intensity plot of Fig. 37-13*b* is that for diffraction by a single actual slit; the diffraction pattern has a broad central maximum and weaker secondary maxima at $\pm 1.7°$. The plot of Fig. 37-13*c* suggests the interference pattern for two actual slits. That plot was constructed by using the curve of Fig. 37-13*b* as an *envelope* on the intensity plot in Fig. 37-13*a*. The positions of the fringes are not changed; only the intensities are affected.

Figure 37-14*a* shows an actual pattern in which both double-slit interference and diffraction are evident. If one slit is covered, the single-slit diffraction pattern of Fig. 37-14*b* results. Note the correspondence between Figs. 37-14*a* and 37-13*c* and between Figs. 37-14*b* and 37-13*b*. In comparing these figures, bear in mind that 37-14 has been deliberately overexposed to bring out the faint secondary maxima and that two secondary maxima (rather than one) are shown.

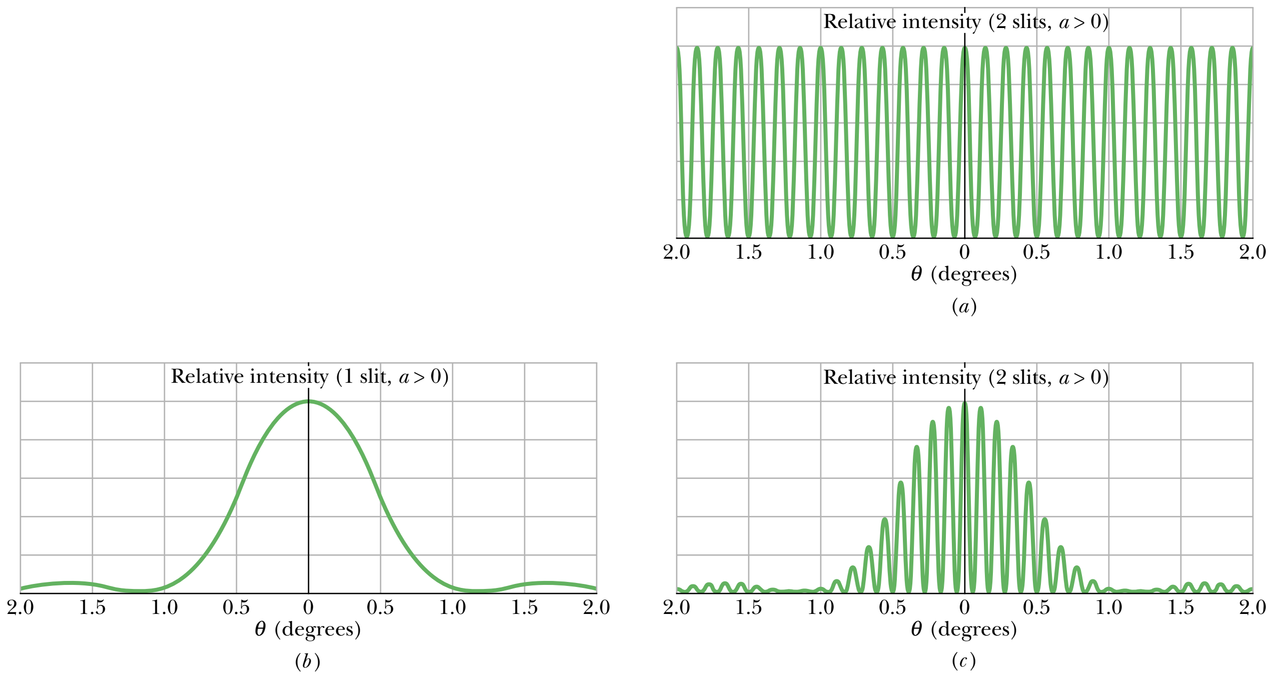


FIGURE 37-13 ■ (*a*) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits (here the distance between the center of the slits is $d = 25$ mm and the incident light is reddish-orange with $\lambda = 623$ mm). (*b*) The intensity plot for diffraction by a typical slit of width $a = 0.031$ mm (not vanishingly narrow). (*c*) The intensity plot to be expected for two slits of width $a = 0.031$ mm. The curve of (*b*) acts as an envelope, limiting the intensity of the double-slit fringes in (*a*). Note that the first minima of the diffraction pattern of (*b*) eliminate the double-slit fringes that would occur near 1.2° in (*c*).

With diffraction effects taken into account, the intensity of a double-slit interference pattern is given by

$$I(\theta) = I^{\max}(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)}, \qquad (37\text{-}16)$$

in which

$$\beta = \frac{\pi d}{\lambda}\sin\theta \qquad (37\text{-}17)$$

and

$$\alpha = \frac{\pi a}{\lambda}\sin\theta \qquad (37\text{-}18)$$

Here d is the distance between the centers of the slits, and a is the slit width. Note carefully that the right side of Eq. 37-16 is the product of $I^{\max}$ and two factors. (1) The

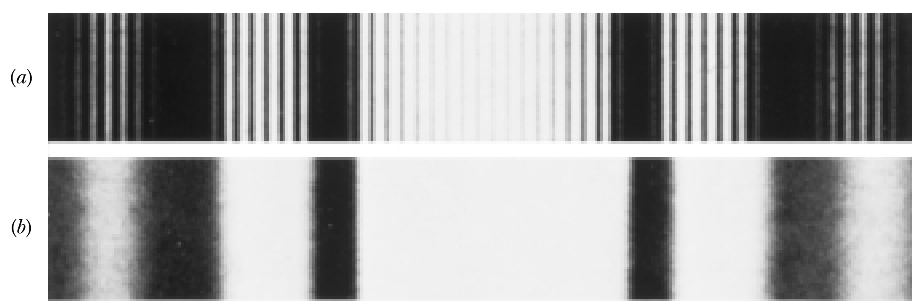


FIGURE 37-14 ■ (*a*) Interference fringes for an actual double-slit system; compare with Fig. 37-13*c*. (*b*) The diffraction pattern of a single slit; compare with Fig. 37-13*b*.

interference factor $\cos^2\beta$ is due to the interference between two slits with slit separation d (as given by Eqs. 36-17 and 36-18). (2) The *diffraction factor* $[(\sin\alpha)/\alpha]^2$ is due to diffraction by a single slit of width a (as given by Eqs. 37-5 and 37-6).

Let us check these factors. If we let $a \to 0$ in Eq. 37-18, for example, then $\alpha \to 0$ and using L'Hopital's rule, we find that $(\sin\alpha)/\alpha \to 1$. Equation 37-16 then reduces, as it must, to an equation describing the interference pattern for a pair of vanishingly narrow slits with slit separation d. Similarly, putting $d = 0$ in Eq. 37-17 is equivalent physically to causing the two slits to merge into a single slit of width a. Then Eq. 37-17 yields $\beta = 0$ and $\cos^2\beta = 1$. In this case Eq. 37-16 reduces, as it must, to an equation describing the diffraction pattern for a single slit of width a.

The double-slit pattern described by Eq. 37-16 and displayed in Fig. 37-14*a* combines interference and diffraction in an intimate way. Both are superposition effects, in that they result from the combining of waves with different phases at a given point. If the combining waves originate from a small number of elementary coherent sources—as in a double-slit experiment with $a \ll \lambda$—we call the process *interference*. If the combining waves originate in a single wavefront—as in a single-slit experiment—we call the process *diffraction*. This distinction between interference and diffraction (which is somewhat arbitrary and not always adhered to) is a convenient one, but we should not forget that both are superposition effects and usually both are present simultaneously (as in Fig. 37-14*a*).

TOUCHSTONE EXAMPLE 37-4: Bright Fringes

Let's consider a double slit with an unusually small spacing. Suppose the wavelength λ of the light source is 405 nm, the slit separation d is 19.44 μm, and the slit width a is 4.050 μm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope?

SOLUTION Let us first analyze the two basic mechanisms responsible for the optical pattern produced in the experiment:

Single-slit diffraction: The **Key Idea** here is that the limits of the central peak are the first minima in the diffraction pattern due to either slit, individually. (See Fig. 37-13.) The angular locations of those minima are given by Eq. 37-3 ($a\sin\theta = m\lambda$). Let us write this equation as $a\sin\theta = m_1\lambda$, with the subscript 1 referring to the one-slit diffraction. For the first minima in the diffraction pattern, we substitute $m_1 = 1$, obtaining

$$a\sin\theta = \lambda. \tag{37-19}$$

Double-slit interference: The **Key Idea** here is that the angular locations of the bright fringes of the double-slit interference pattern are given by Eq. 36-14, which we can write as

$$d\sin\theta = m_2\lambda, \qquad \text{for } m_2 = 1, 2, 3, \ldots. \tag{37-20}$$

Here the subscript 2 refers to the double-slit interference.

We can locate the first diffraction minimum within the double-slit fringe pattern by dividing Eq. 37-20 by Eq. 37-19 and solving for m_2. By doing so and then substituting the given data, we obtain

$$m_2 = \frac{d}{a} = \frac{19.44\ \mu\text{m}}{4.050\ \mu\text{m}} = 4.8.$$

This tells us that the bright interference fringe for $m_2 = 4$ fits into the central peak of the one-slit diffraction pattern, but the fringe for

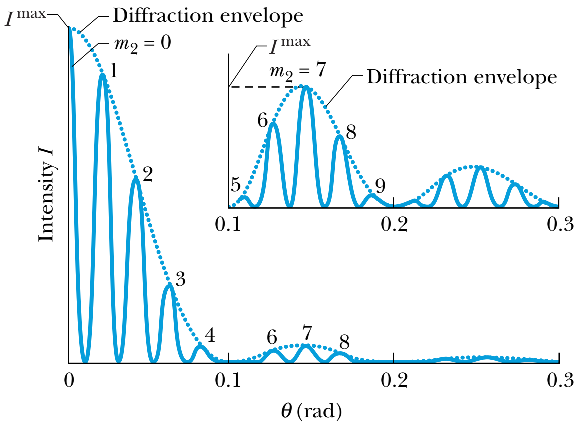


FIGURE 37-15 One side of the intensity plot for a two-slit interference experiment; the diffraction envelope is indicated by the dotted curve. The smaller inset shows (vertically expanded) the intensity plot within the first and second side peaks of the diffraction envelope.

$m_2 = 5$ does not fit. Within the central diffraction peak we have the central bright fringe ($m_2 = 0$), and four bright fringes (up to $m_2 = 4$) on each side of it. Thus, a total of nine bright fringes of the double-slit interference pattern are within the central peak of the diffraction envelope. The bright fringes to one side of the central bright fringe are shown in Fig. 37-15.

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

SOLUTION ■ The **Key Idea** here is that the outer limits of the first side diffraction peaks are the second diffraction minima, each of which is at the angle θ given by $a \sin\theta = m_1\lambda$ with $m_1 = 2$:

$$a \sin\theta = 2\lambda \qquad (37\text{-}21)$$

Dividing Eq. 37-20 by Eq. 37-21, we find

$$m_2 = \frac{2d}{a} = \frac{(2)(19.44\ \mu\text{m})}{4.050\ \mu\text{m}} = 9.6.$$

This tells us that the second diffraction minimum occurs just before the bright interference fringe for $m_2 = 10$ in Eq. 37-20. Within either first side diffraction peak we have the fringes from $m_2 = 5$ to $m_2 = 9$ for a total of five bright fringes of the double-slit interference pattern (shown in the inset of Fig. 37-15). However, if the $m_2 = 5$ bright fringe, which is almost eliminated by the first diffraction minimum, is considered too dim to count, then only four bright fringes are in the first side diffraction peak.

37-7 Diffraction Gratings

One of the most useful tools in the study of light and of objects that emit and absorb light is the **diffraction grating.** A diffraction grating is a device that uses **interference** phenomena to seperate a beam of light by wavelength. A diffraction grating is a more elaborate form of the double-slit arrangement of Fig. 36-8. This device has a much greater number N of slits, often called *rulings,* perhaps as many as several thousand per millimeter. An idealized grating consisting of only five slits is represented in Fig. 37-16. When monochromatic light is sent through the slits, it forms narrow interference fringes that can be analyzed to determine the wavelength of the light. (Diffraction gratings can also be opaque surfaces with narrow parallel grooves arranged like the slits in Fig. 37-16. Light then scatters back from the grooves to form interference fringes rather than being transmitted through open slits.)

With monochromatic light incident on a diffraction grating, if we gradually increase the number of slits from two to a large number N, the intensity plot changes from the typical double-slit plot of Fig. 37-13*c* to a much more complicated one and then eventually to a simple graph like that shown in Fig. 37-17*a*. The pattern you would see on a viewing screen using monochromatic red light from, say, a helium-neon laser, is shown in Fig. 37-17*b*. The maxima are now very narrow (and so are called *lines*); they are separated by relatively wide dark regions.

We use a familiar procedure to find the locations of the bright lines on the viewing screen. We first assume that the screen is far enough from the grating so that the rays reaching a particular point P on the screen are approximately parallel when they leave the grating (Fig. 37-18). Then we apply to each pair of adjacent rulings the same reasoning we used for double-slit interference. The separation d between rulings is called the *grating spacing.* (If N rulings occupy a total width w, then $d = w/N$.) The path length difference between adjacent rays is again $d\sin\theta$ (Fig. 37-18), where θ is the angle from the central axis of the grating (and of the diffraction pattern) to point P. A line will be located at P if the path length difference between adjacent rays is an integer number of wavelengths—that is, if

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \qquad \text{(maxima—lines)}, \qquad (37\text{-}22)$$

where λ is the wavelength of the light. Each integer m represents a different line; hence these integers can be used to label the lines, as in Fig. 37-17. The integers are then called the *order numbers,* and the lines are called the zeroth-order line (the central line, with $m = 0$), the first-order line, the second-order line, and so on.

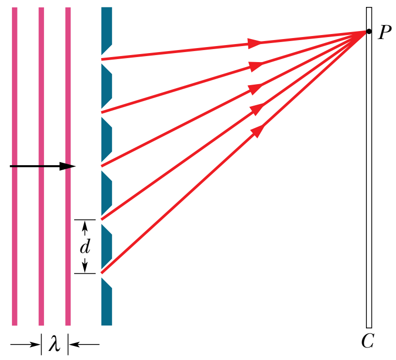


FIGURE 37-16 ■ An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen C.

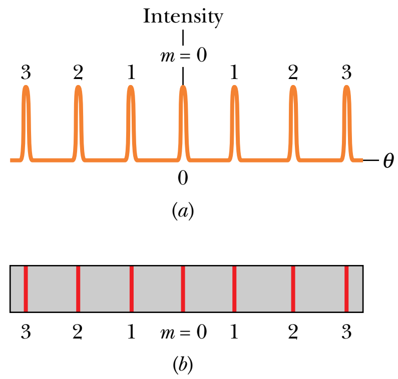


FIGURE 37-17 ■ A diffraction grating illuminated with a single wavelength of light. (*a*) The intensity plot produced by a diffraction grating with a great many rulings consists of narrow peaks, here labeled with their order numbers m. (*b*) The corresponding bright fringes seen on the screen are called lines and are here also labeled with order numbers m. Lines of the zeroth, first, second, and third orders are shown.

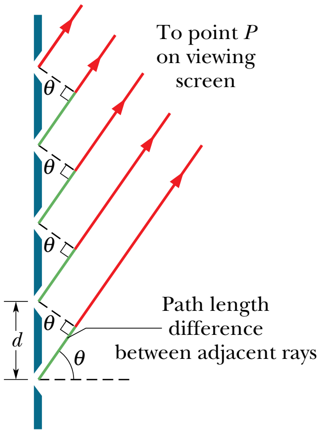


FIGURE 37-18 ■ The rays from the rulings in a diffraction grating to a distant point P are approximately parallel. The path length difference between each two adjacent rays is $d \sin\theta$, where θ is measured as shown. (The rulings extend into and out of the page.)

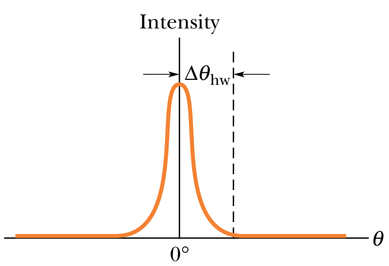


FIGURE 37-19 ■ The half-width $\Delta\theta_{hw}$ of the central line is measured from the center of that line to the adjacent minimum on a plot of I versus θ like Fig. 37-17*a*.

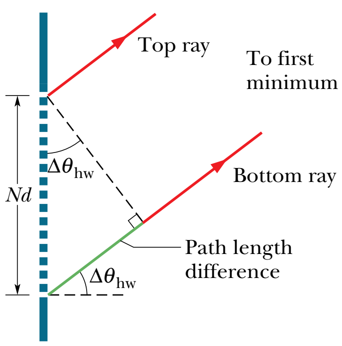


FIGURE 37-20 ■ The top and bottom rulings of a diffraction grating of N rulings are separated by distance Nd. The top and bottom rays passing through these rulings have a path length difference of $Nd \sin \Delta\theta_{hw}$, where $\Delta\theta_{hw}$ is the angle to the first minimum. (The angle is here greatly exaggerated for clarity.)

If we rewrite Eq. 37-22 as $\theta = \sin^{-1}(m\lambda/d)$ we see that, for a given diffraction grating, the angle from the central axis to any line (say, the third-order line) depends on the wavelength of the light being used. Thus, when light of an unknown wavelength is sent through a diffraction grating, measurements of the angles to the higher-order lines can be used in Eq. 37-22 to determine the wavelength. Even light of several unknown wavelengths can be distinguished and identified in this way. We cannot do that with the double-slit arrangement of Section 36-4, even though the same equation and wavelength dependence apply there. In double-slit interference, the bright fringes due to different wavelengths overlap too much to be distinguished.

Width of the Lines

A grating's ability to resolve (separate) lines of different wavelengths depends on the width of the lines. We shall here derive an expression for the *half-width* of the central line (the line for which $m = 0$) and then state an expression for the half-widths of the higher-order lines. We measure the half-width of the central line as the angle $\Delta\theta_{hw}$ from the center of the line at $\theta = 0$ outward to where the line effectively ends and darkness effectively begins with the first minimum (Fig. 37-19). At such a minimum, the N rays from the N slits of the grating cancel one another. (The actual width of the central line is, of course $2(\Delta\theta_{hw})$, but line widths are usually compared via half-widths.)

In Section 37-2 we were also concerned with the cancellation of a great many rays, there due to diffraction through a single slit. We obtained Eq. 37-3, which, because of the similarity of the two situations, we can use to find the first minimum here. It tells us that the first minimum occurs where the path length difference between the top and bottom rays equals λ. For single-slit diffraction, this difference is $a \sin \theta$. For a grating of N rulings, each separated from the next by distance d, the distance between the top and bottom rulings is Nd (Fig. 37-20), so the path length difference between the top and bottom rays here is $Nd \sin\Delta\theta_{hw}$. Thus, the first minimum occurs where

$$Nd \sin \Delta\theta_{hw} = \lambda. \qquad (37\text{-}23)$$

Because $\Delta\theta_{hw}$ is small, $\sin\Delta\theta_{hw} \approx \Delta\theta_{hw}$ (in radian measure). Substituting this in Eq. 37-23 gives the half-width of the central line as

$$\Delta\theta_{hw} = \frac{\lambda}{Nd} \quad \text{(half-width of central line).} \qquad (37\text{-}24)$$

We state without proof that the half-width of any other line depends on its location relative to the central axis and is

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half-width of line at } \theta\text{).} \qquad (37\text{-}25)$$

Note that for light of a given wavelength λ and a given ruling separation d, the widths of the lines decrease with an increase in the number N of rulings. Thus, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap. But the line width of a monochromatic light beam is determined by the number of slits that the beam encounters. In a diffraction grating spectrometer, a collimating telescope can be used to illuminate all N slits of the grating.

The Diffraction Grating Spectrometer

Diffraction gratings are widely used to determine the wavelengths that are emitted by sources of light ranging from lamps to stars. Figure 37-21 shows a simple *grating spectroscope* in which a grating is used for this purpose. Light from source S is focused by lens L_1 on a vertical slit S_1 placed in the focal plane of lens L_2. The light emerging from tube C (called a *collimator*) is a plane wave and is incident perpendicularly on grating G, where it is diffracted into a diffraction pattern, with the $m = 0$ order diffracted at angle $\theta = 0$ along the central axis of the grating.

We can view the diffraction pattern that would appear on a viewing screen at any angle θ simply by orienting telescope T in Fig. 37-21 to that angle. Lens L_3 of the telescope then focuses the light diffracted at angle θ (and at slightly smaller and larger angles) onto a focal plane FF' within the telescope. When we look through eyepiece E, we see a magnified view of this focused image.

By changing the angle θ of the telescope, we can examine the entire diffraction pattern. For any order number other than $m = 0$, the original light is spread out according to wavelength (or color) so that we can determine, with Eq. 37-22, just what wavelengths are being emitted by the source. If the source emits a number of discrete wavelengths, what we see as we rotate the telescope horizontally through the angles corresponding to an order m is a vertical line of color for each wavelength, with the shorter-wavelength line at a smaller angle $m = 0$ than the longer-wavelength line.

For example, the light emitted by a hydrogen lamp, which contains hydrogen gas, has four discrete wavelengths in the visible range. If our eyes intercept this light directly, it appears to be white. If, instead, we view it through a grating spectroscope, we can distinguish, in several orders, the lines of the four colors corresponding to these visible wavelengths. (Such lines are called *emission lines.*) Four orders are represented in Fig. 37-22. In the central order ($m = 0$), the lines corresponding to all four wavelengths are superimposed, giving a single white line at $\theta = 0$. The colors are separated in the higher orders.

The third order is not shown in Fig. 37-22 for the sake of clarity; it actually overlaps the second and fourth orders. The fourth-order red line is missing because it is not formed by the grating used here. That is, when we attempt to solve Eq. 37-22 for the angle θ for the red wavelength when $m = 4$, we find that sin θ is greater than unity, which is not possible. The fourth order is then said to be *incomplete* for this grating; it might not be incomplete for a grating with greater spacing d, which will spread the lines less than in Fig. 37-22. Figure 37-23 is a photograph of the visible emission lines produced by cadmium.

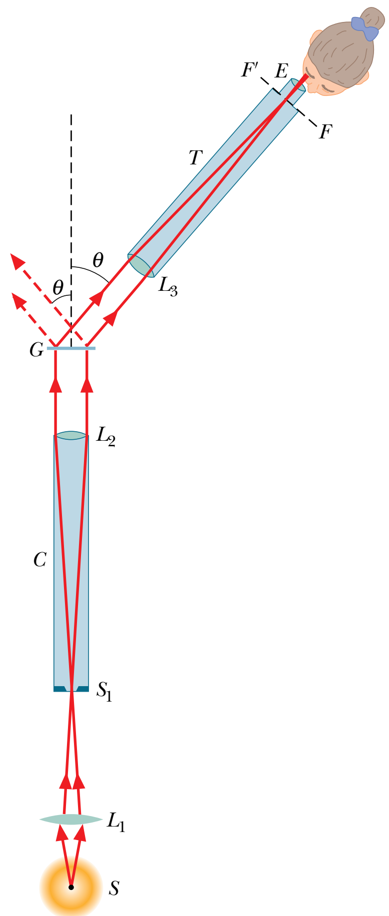


FIGURE 37-21 ■ A simple type of grating spectroscope used to analyze the wavelengths of light emitted by source S.

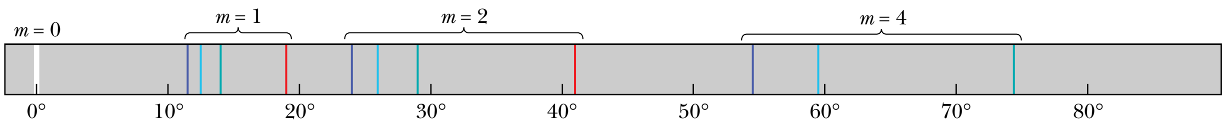


FIGURE 37-22 ■ The zeroth, first, second, and fourth orders of the visible emission lines from hydrogen. Note that the lines are farther apart at greater angles. (The lines are also dimmer and wider, although that is not shown here. Also, the third order line is eliminated for clarity.)

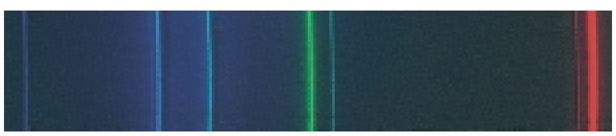


FIGURE 37-23 ■ The visible emission lines of cadmium, as seen through a grating spectroscope.

READING EXERCISE 37-5: The figure shows lines of different orders produced by a diffraction grating in monochromatic red light. (a) Is the center of the pattern to the left or right? (b) If we switch to monochromatic green light, will the half-widths of the lines then produced in the same orders be greater than, less than, or the same as the half-widths of the lines shown?

37-8 Gratings: Dispersion and Resolving Power

The fine rulings, each 0.5 μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are the composite of the diffraction patterns from the rulings.

There are two characteristics that are important in the design of a diffraction grating spectrometer. First, the different wavelengths of light in a beam should be spread out. This characteristic is called **dispersion.** The second characteristic is the **resolving power** of the spectrometer. It should have a narrow line width for each wavelength so the lines are sharp.

Dispersion

To be useful in distinguishing wavelengths that are close to each other (as in a grating spectroscope), a grating must spread apart the diffraction lines associated with the various wavelengths. This spreading, called **dispersion,** is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda} \quad \text{(dispersion defined).} \tag{37-26}$$

Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$. The greater D is, the greater is the distance between two emission lines whose wavelengths differ by $\Delta\lambda$. We show below that the dispersion of a grating at angle θ is given by

$$D = \frac{m}{d\cos\theta} \quad \text{(dispersion of a grating).} \tag{37-27}$$

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing d and work in a higher order m. Note that the dispersion does not depend on the number of rulings. The SI unit for D is the degree per meter or the radian per meter.

Proof of Eq. 37-27

Let us start with Eq. 37-22, the expression for the locations of the lines in the diffraction pattern of a grating:

$$d\sin\theta = m\lambda.$$

Let us regard θ and λ as variables and take differentials of this equation. We find

$$d\cos\theta\,(d\theta) = m\,(d\lambda),$$

where the differentials $d\theta$ and $d\lambda$ are placed in parentheses to distinguish them from the product of the center to center slit spacing d and the angle θ or wavelength λ.

For small enough angles, we can write these differentials as small differences, obtaining

$$d\cos\theta\,(\Delta\theta) = m(\Delta\lambda), \tag{37-30}$$

or

$$\frac{(\Delta\theta)}{(\Delta\lambda)} = \frac{m}{d\cos\theta}.$$

The ratio on the left is simply D (see Eq. 37-26), so we have indeed derived Eq. 37-27.

Resolving Power

To *resolve* lines whose wavelengths are close together (that is, to make the lines distinguishable), the line should also be as narrow as possible. Expressed otherwise, the grating should have a high **resolving power *R*,** defined as

$$R = \frac{\langle\lambda\rangle}{\Delta\lambda} \quad \text{(resolving power defined).} \tag{37-28}$$

Here $\langle\lambda\rangle$ is the mean wavelength of two emission lines that can barely be recognized as separate, and $\Delta\lambda$ is the wavelength difference between them. The greater R is, the closer two emission lines can be and still be resolved. We shall show below that the resolving power of a grating is given by the simple expression

$$R = Nm \quad \text{(resolving power of a grating).} \tag{37-29}$$

To achieve high resolving power, we must spread out the light beam so it is incident on many rulings (large N in Eq. 37-29).

Proof of Eq. 37-29

We start with Eq. 37-30, which was derived from Eq. 37-22, the expression for the locations of the lines in the diffraction pattern formed by a grating. Here $\Delta\lambda$ is the small wavelength difference between two waves that are diffracted by the grating, and $\Delta\theta$ is the angular separation between them in the diffraction pattern. If $\Delta\theta$ is to be the smallest angle that will permit the two lines to be resolved, it must (by Rayleigh's criterion) be equal to the half-width of each line, which is given by Eq. 37-25:

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd\cos\theta}.$$

If we substitute $\Delta\theta_{\text{hw}}$ as given here for $\Delta\theta$ in Eq. 37-30, we find that

$$\frac{\lambda}{N} = m\,\Delta\lambda,$$

from which it readily follows that

$$R = \frac{\lambda}{\Delta\lambda} = Nm.$$

This is Eq. 37-29, which we set out to derive.

Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused with its dispersion. Table 37-1 shows the characteristics of three gratings, all illuminated with light of wavelength $\lambda = 589$ nm, whose diffracted light is viewed in the first order ($m = 1$ in Eq. 37-22). You should verify that the values of D and R as given in the table can be calculated with Eqs. 37-27 and 37-29, respectively. (In the calculations for D, you will need to convert radians per meter to degrees per micrometer.)

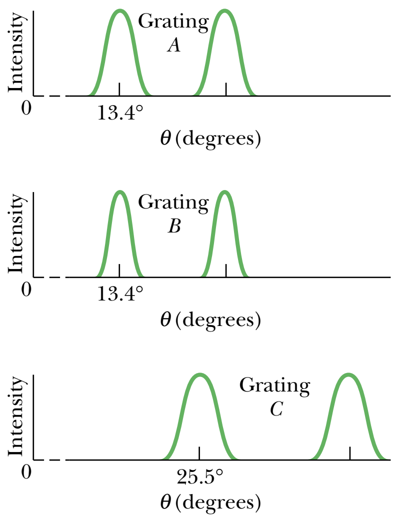


FIGURE 37-24: The intensity patterns for light of two wavelengths sent through the gratings of Table 37-1. Grating B has the highest resolving power and grating C the highest dispersion.

TABLE 37-1
Three Gratings[a]

	Specifications		Calculated Values		
Grating	N	d (nm)	θ	D (°/μm)	R
A	10 000	2540	13.4°	23.2	10 000
B	20 000	2540	13.4°	23.2	20 000
C	10 000	1370	25.5°	46.3	10 000

[a]Data are for λ = 589 nm and m = 1.

For the conditions noted in Table 37-1, gratings A and B have the same *dispersion* and A and C have the same *resolving power.*

Figure 37-24 shows the intensity patterns (also called *line shapes*) that would be produced by these gratings for two lines of wavelengths λ_1 and λ_2, in the vicinity of λ = 589 nm. Grating B, with the higher resolving power, produces narrower lines and thus is capable of distinguishing lines that are much closer together in wavelength than those in the figure. Grating C, with the higher dispersion, produces the greater angular separation between the lines.

TOUCHSTONE EXAMPLE 37-5: Diffraction Grating

A diffraction grating has 1.26×10^4 rulings uniformly spaced over width w = 25.4 mm (so that it has 496 lines/mm). It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

(a) At what angle does the first-order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?

SOLUTION ■ The **Key Idea** here is that the maxima produced by the diffraction grating can be located with Eq. 37-22 ($d \sin\theta = m\lambda$). The grating spacing d for this diffraction grating is

$$d = \frac{w}{N} = \frac{25.4 \times 10^{-3}\text{ m}}{1.26 \times 10^4}$$
$$= 2.016 \times 10^{-6}\text{ m} = 2016\text{ nm}.$$

The first-order maximum corresponds to m = 1. Substituting these values for d and m into Eq. 37-22 leads to

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(589.00\text{ nm})}{2016\text{ nm}}$$
$$= 16.99° \approx 17.0°. \qquad \text{(Answer)}$$

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

SOLUTION ■ One **Key Idea** here is that the angular separation $\Delta\theta$ between the two lines in the first order depends on their wavelength difference $\Delta\lambda$ and the dispersion D of the grating, according to Eq. 37-26 ($D = \Delta\theta/\Delta\lambda$). A second **Key Idea** is that the dispersion D depends on the angle θ at which it is to be evaluated. We can assume that, in the first order, the two sodium lines occur close enough to each other for us to evaluate D at the angle θ = 16.99° we found in part (a) for one of those lines. Then Eq. 37-27 gives the dispersion as

$$D = \frac{m}{d\cos\theta} = \frac{1}{(2016\text{ nm})(\cos 16.99°)}$$
$$= 5.187 \times 10^{-4}\text{ rad/nm}.$$

From Eq. 37-26, we then have

$$\Delta\theta = D\,\Delta\lambda = (5.187 \times 10^{-4}\text{ rad/nm})(589.59\text{ nm} - 589.00\text{ nm})$$
$$= 3.06 \times 10^{-4}\text{ rad} = 0.0175°. \qquad \text{(Answer)}$$

You can show that this result depends on the grating spacing d but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

SOLUTION ■ One **Key Idea** here is that the resolving power of a grating in any order m is physically set by the number of rulings N in the grating according to Eq. 37-29 ($R = Nm$). A second **Key Idea** is that the least wavelength difference $\Delta\lambda$ that can be resolved depends on the average wavelength involved and the resolving power R of the grating, according to Eq. 37-28 ($R = \langle\lambda\rangle/\Delta\lambda$).

For the sodium doublet to be barely resolved, $\Delta\lambda$ must be their wavelength separation of 0.59 nm, and $\langle\lambda\rangle$ must be their average wavelength of 589.30 nm.

Putting these ideas together, we find that the least number of rulings for a grating to resolve the sodium doublet is

$$N = \frac{R}{m} = \frac{\langle\lambda\rangle}{m\Delta\lambda} = \frac{589.30\text{ nm}}{(1)(0.59\text{ nm})} = 999\text{ rulings.} \qquad \text{(Answer)}$$

37-9 X-Ray Diffraction

X rays are electromagnetic radiation whose wavelengths are of the order of 1 Å (= 0.1 nm = 10^{-10} m). Compare this with a wavelength of 550 nm (= 5.5×10^{-7} m) at the center of the visible spectrum. Figure 37-25 shows that x rays are produced when electrons escaping from a heated filament F are accelerated by a potential difference V and strike a metal target T.

A standard optical diffraction grating cannot be used to discriminate between different wavelengths in the x-ray wavelength range. For $\lambda = 1$ Å (= 0.1 nm) and d = 3000 nm, for example, Eq. 37-22 shows that the first-order maximum occurs at

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(0.1\text{ nm})}{3000\text{ nm}} = 0.0019°.$$

This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is desirable, but, since x-ray wavelengths are about equal to atomic diameters, such gratings cannot be constructed mechanically.

In 1912, it occurred to German physicist Max von Laue that a crystalline solid, which consists of a regular array of atoms, might form a natural three-dimensional "diffraction grating" for x rays. The idea is that, in a crystal such as sodium chloride (NaCl), a basic unit of atoms (called the *unit cell*) repeats itself throughout the array. In NaCl four sodium ions and four chlorine ions are associated with each unit cell. Figure 37-26a represents a section through a crystal of NaCl and identifies this basic unit. The unit cell is a cube measuring a_0 on each side.

When an x-ray beam enters a crystal such as NaCl, x rays are *scattered*—that is, redirected—in all directions by the crystal structure. In some directions the scattered

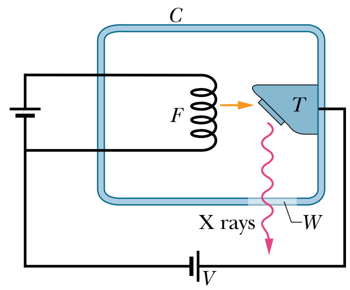


FIGURE 37-25: X rays are generated when electrons leaving heated filament F are accelerated through a potential difference V and strike a metal target T. The "window" W in the evacuated chamber C is transparent to x rays.

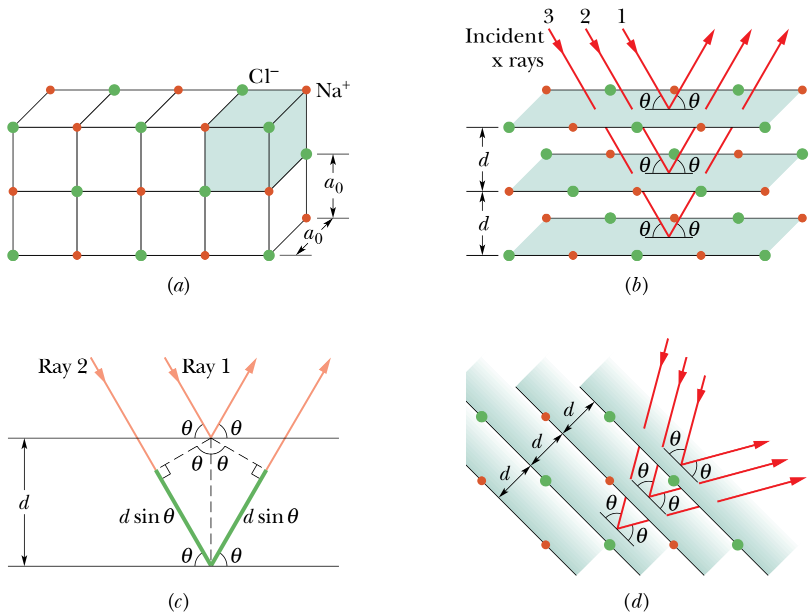


FIGURE 37-26: (a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x rays undergo diffraction by the structure of (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with the angle of reflection equal to the angle of incidence, both angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is $2d \sin\theta$. (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

waves undergo destructive interference, resulting in intensity minima; in other directions the interference is constructive, resulting in intensity maxima. This process of scattering and interference is a form of diffraction, although it is unlike the diffraction of light traveling through a slit or past an edge as we discussed earlier.

Although the process of diffraction of x rays by a crystal is complicated, the maxima turn out to be in directions as if the x rays were reflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend through the atoms within the crystal and that contain regular arrays of the atoms. (The x rays are not actually reflected; we use these fictional planes only to simplify the analysis of the actual diffraction process.)

Figure 37-26*b* shows three of the family of planes, with *interplanar spacing d*, from which the incident rays shown are said to reflect. Rays 1, 2, and 3 reflect from the first, second, and third planes, respectively. At each reflection the angle of incidence and the angle of reflection are represented with θ. Contrary to the custom in optics, these angles are defined relative to the *surface* of the reflecting plane rather than a normal to that surface. For the situation of Fig. 37-26*b*, the interplanar spacing happens to be equal to the unit cell dimension a_0.

Figure 37-26*c* shows an edge-on view of reflection from an adjacent pair of planes. The waves of rays 1 and 2 arrive at the crystal in phase. After they are reflected, they must again be in phase, because the reflections and the reflecting planes have been defined solely to explain the intensity maxima in the diffraction of x rays by a crystal. Unlike light rays, the x rays have negligible refraction when entering the crystal; moreover, we do not define an index of refraction for this situation. Thus, the relative phase between the waves of rays 1 and 2 as they leave the crystal is set solely by their path length difference. For these rays to be in phase, the path length difference must be equal to an integer multiple of the wavelength λ of the x rays.

By drawing the dashed perpendiculars in Fig. 37-26*c*, we find that the path length difference is $2d \sin \theta$. In fact, this is true for any pair of adjacent planes in the family of planes represented in Fig. 37-26*b*. Thus, we have, as the criterion for intensity maxima for x-ray diffraction,

$$2d \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \qquad \text{(Bragg's law)}, \tag{37-31}$$

where m is the order number of an intensity maximum. Equation 37-31 is called **Bragg's law** after British physicist W. L. Bragg, who first derived it. (He and his father shared the 1915 Nobel Prize for their use of x rays to study the structures of crystals.) The angle of incidence and reflection in Eq. 37-31 is called a *Bragg angle.*

Regardless of the angle at which x rays enter a crystal, there is always a family of planes from which they can be said to reflect so that we can apply Bragg's law. In Fig. 37-26*d*, the crystal structure has the same orientation as it does in Fig. 37-26*a*, but the angle at which the beam enters the structure differs from that shown in Fig. 37-26*b*. This new angle requires a new family of reflecting planes, with a different interplanar spacing d and different Bragg angle θ, in order to explain the x-ray diffraction via Bragg's law.

Figure 37-27 shows how the interplanar spacing d can be related to the unit cell dimension a_0. For the particular family of planes shown there, the Pythagorean theorem gives

$$5d = \sqrt{5}a_0,$$

or

$$d = \frac{a_0}{\sqrt{5}}. \tag{37-32}$$

Figure 37-27 suggests how the dimensions of the unit cell can be found once the interplanar spacing has been measured by means of x-ray diffraction.

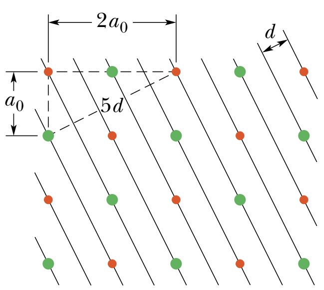


FIGURE 37-27: A family of planes through the structure of Fig. 37-26*a*, and a way to relate the edge length a_0 of a unit cell to the interplanar spacing d.

44026_37_p1083_1110 1/19/04 1:55 PM Page 1105

X-ray diffraction is a powerful tool for studying both x-ray spectra and the arrangement of atoms in crystals. To study spectra, a particular set of crystal planes, having a known spacing d, is chosen. These planes effectively reflect different wavelengths at different angles. A detector that can discriminate one angle from another can then be used to determine the wavelength of radiation reaching it. The crystal itself can be studied with a monochromatic x-ray beam, to determine not only the spacing of various crystal planes but also the structure of the unit cell.

Problems

SEC. 37-2 ■ DIFFRACTION BY A SINGLE SLIT: LOCATING THE MINIMA

1. Narrow Slit Light of wavelength 633 nm is incident on a narrow slit. The angle between the first diffraction minimum on one side of the central maximum and the first minimum on the other side is 1.20°. What is the width of the slit?

2. Distance Between Monochromatic light of wavelength 441 nm is incident on a narrow slit. On a screen 2.00 m away, the distance between the second diffraction minimum and the central maximum is 1.50 cm. (a) Calculate the angle of diffraction θ of the second minimum. (b) Find the width of the slit.

3. Single Slit A single slit is illuminated by light of wavelengths λ_a and λ_b, chosen so the first diffraction minimum of the λ_a component coincides with the second minimum of the λ_b component. (a) What relationship exists between the two wavelengths? (b) Do any other minima in the two diffraction patterns coincide?

4. First and Fifth The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, when light of wavelength 550 nm is used. (a) Find the slit width. (b) Calculate the angle θ of the first diffraction minimum.

5. Plane Wave A plane wave of wavelength 590 nm is incident on a slit with a width of $a = 0.40$ nm. A thin converging lens of focal length +70 cm is placed between the slit and a viewing screen and focuses the light on the screen. (a) How far is the screen from the lens? (b) What is the distance on the screen from the center of the diffraction pattern to the first minimum?

6. Sound Waves Sound waves with frequency 3000 Hz and speed 343 m/s diffract through the rectangular opening of a speaker cabinet and into a large auditorium. The opening, which has a horizontal width of 30.0 cm, faces a wall 100 m away (Fig. 37-28). Where along that wall will a listener be at the first diffraction minimum and thus have difficulty hearing the sound? (Neglect reflections).

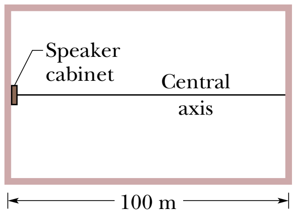


FIGURE 37-28 ■ Problem 6.

7. Central Maximum A slit 1.00 mm wide is illuminated by light of wavelength 589 nm. We see a diffraction pattern on a screen 3.00 m away. What is the distance between the first two diffraction minima on the same side of the central diffraction maximum?

SEC. 37-4 ■ INTENSITY IN SINGLE-SLIT DIFFRACTION, QUANTITATIVELY

8. Off Central Axis A 0.10-mm-wide slit is illuminated by light of wavelength 589 nm. Consider a point P on a viewing screen on which the diffraction pattern of the slit is viewed; the point is at 30° from the central axis of the slit. What is the phase difference between the Huygens wavelets arriving at point P from the top and midpoint of the slit? (*Hint:* See Eq. 37-4.)

9. Explain Quantitatively If you double the width of a single slit, the intensity of the central maximum of the diffraction pattern increases by a factor of 4, even though the energy passing through the slit only doubles. Explain this quantitatively.

10. Monochromatic Monochromatic light with wavelength 538 nm is incident on a slit with width 0.025 mm. The distance from the slit to a screen is 3.5 m. Consider a point on the screen 1.1 cm from the central maximum. (a) Calculate θ for that point. (b) Calculate α. (c) Calculate the ratio of the intensity at this point to the intensity at the central maximum.

11. FWHM The full width at half-maximum (FWHM) of a central diffraction maximum is defined as the angle between the two points in the pattern where the intensity is one-half that at the center of the pattern. (See Fig. 37-7*b*.) (a) Show that the intensity drops to one-half the maximum value when $\sin^2 \alpha = \alpha^2/2$. (b) Verify that $\alpha = 1.39$ rad (about 80°) is a solution to the transcendental equation of (a). (c) Show that the FWHM is $\Delta\theta = 2\sin^{-1}(0.443\lambda/a)$, where a is the slit width. (d) Calculate the FWHM of the central maximum for slits whose widths are 1.0, 5.0, and 10 wavelengths.

12. Babinet's Principle A monochromatic beam of parallel light is incident on a "collimating" hole of diameter $x \gg \lambda$. Point P lies in the geometrical shadow region on a *distant* screen (Fig. 37-29*a*). Two diffracting objects, shown in Fig. 37-29*b*, are placed in turn

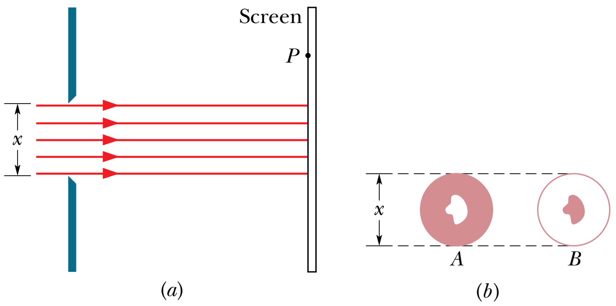


FIGURE 37-29 ■ Problem 12.

over the collimating hole. A is an opaque circle with a hole in it and B is the "photographic negative" of A. Using superposition concepts, show that the intensity at P is identical for the two diffracting objects A and B.

13. Values of α (a) Show that the values of α at which intensity maxima for single-slit diffraction occur can be found exactly by differentiating Eq. 37-5 with respect to α and equating the result to zero, obtaining the condition $\tan\alpha = \alpha$. (b) Find the values of α satisfying this relation by plotting the curve $y = \tan\alpha$ and the straight line $y = \alpha$ and finding their intersections or by using a calculator with an equation solver to find an appropriate value of α (or by using trial and error). (c) Find the (noninteger) values of m corresponding to successive maxima in the single-slit pattern. Note that the secondary maxima do not lie exactly halfway between minima.

SEC. 37-5 ■ DIFFRACTION BY A CIRCULAR APERTURE

14. Entopic Halos At night many people see rings (called *entopic halos*) surrounding bright outdoor lamps in otherwise dark surroundings. The rings are the first of the side maxima in diffraction patterns produced by structures that are thought to be within the cornea (or possibly the lens) of the observer's eye. (The central maxima of such patterns overlap the lamp.) (a) Would a particular ring become smaller or larger if the lamp were switched from blue to red light? (b) If a lamp emits white light, is blue or red on the outside edge of the ring? (c) Assume that the lamp emits light at wavelength 550 nm. If a ring has an angular diameter of 2.5°, approximately what is the (linear) diameter of the structure in the eye that causes the ring?

15. Headlights The two headlights of an approaching automobile are 1.4 m apart. At what (a) angular separation and (b) maximum distance will the eye resolve them? Assume that the pupil diameter is 5.0 mm, and use a wavelength of 550 nm for the light. Also assume that diffraction effects alone limit the resolution so that Rayleigh's criterion can be applied.

16. An Astronaut An astronaut in a space shuttle claims she can just barely resolve two point sources on the Earth's surface, 160 km below. Calculate their (a) angular and (b) linear separation, assuming ideal conditions. Take $\lambda = 540$ nm and the pupil diameter of the astronaut's eye to be 5.0 mm.

17. Moon's Surface Find the separation of two points on the Moon's surface that can just be resolved by the 200 in. (= 5.1 m) telescope at Mount Palomar, assuming that this separation is determined by diffraction effects. The distance from the Earth to the Moon is 3.8×10^5 km. Assume a wavelength of 550 nm for the light.

18. Large Room The wall of a large room is covered with acoustic tile in which small holes are drilled 5.0 mm from center to center. How far can a person be from such a tile and still distinguish the individual holes, assuming ideal conditions, the pupil diameter of the observer's eye to be 4.0 mm, and the wavelength of the room light to be 550 nm?

19. Estimate Linear Separation Estimate the linear separation of two objects on the planet Mars that can just be resolved under ideal conditions by an observer on Earth (a) using the naked eye and (b) using the 200 in. (= 5.1 m) Mount Palomar telescope. Use the following data: distance to Mars = 8.0×10^7 km, diameter of pupil = 5.0 mm, wavelength of light = 550 nm.

20. Radar System The radar system of a navy cruiser transmits at a wavelength of 1.6 cm, from a circular antenna with a diameter of 2.3 m. At a range of 6.2 km, what is the smallest distance that two speedboats can be from each other and still be resolved as two separate objects by the radar system?

21. Tiger Beetles The wings of tiger beetles (Fig. 37-30) are colored by interference due to thin cuticle-like layers. In addition, these layers are arranged in patches that are 60 μm across and produce different colors. The color you see is a pointillistic mixture of thin-film interference colors that varies with perspective. Approximately what viewing distance from a wing puts you at the limit of resolving the different colored patches according to Rayleigh's criterion? Use 550 nm as the wavelength of light and 3.00 mm as the diameter of your pupil.

FIGURE 37-30 ■ Problem 21. Tiger beetles are colored by pointillistic mixtures of thin-film interference colors.

22. Discovery In June 1985, a laser beam was sent out from the Air Force Optical Station on Maui, Hawaii, and reflected back from the shuttle *Discovery* as it sped by, 354 km overhead. The diameter of the central maximum of the beam at the shuttle position was said to be 9.1 m, and the beam wavelength was 500 nm. What is the effective diameter of the laser aperture at the Maui ground station? (*Hint:* A laser beam spreads only because of diffraction; assume a circular exit aperture.)

23. Millimeter-Wave Radar Millimeter-wave radar generates a narrower beam than conventional microwave radar, making it less vulnerable to antiradar missiles. (a) Calculate the angular width of the central maximum, from first minimum to first minimum, produced by a 220 GHz radar beam emitted by a 55.0-cm-diameter circular antenna. (The frequency is chosen to coincide with a low-absorption atmospheric "window.") (b) Calculate the same quantity for the ship's radar described in Problem 20.

24. Circular Obstacle A circular obstacle produces the same diffraction pattern as a circular hole of the same diameter (except very near $\theta = 0$). Airborne water drops are examples of such obstacles. When you see the Moon through suspended water drops, such as in a fog, you intercept the diffraction pattern from many drops.

FIGURE 37-31 ■ Problem 24. The corona around the Moon is a composite of the diffraction patterns of airborne water drops.

The composite of the central diffraction maxima of those drops forms a white region that surrounds the Moon and may obscure it. Figure 37-31 is a photograph in which the Moon is obscured. There are two, faint, colored rings around the Moon (the larger one may be too faint to be seen in your copy of the photograph). The smaller ring is on the outer edge of the central maxima from the drops; the somewhat larger ring is on the outer edge of the smallest of the secondary maxima from the drops (see Fig. 37-3). The color is visible because the rings are adjacent to the diffraction minima (dark rings) in the patterns. (Colors in other parts of the pattern overlap too much to be visible.)

(a) What is the color of these rings on the outer edges of the diffraction maxima? (b) The colored ring around the central maxima in Fig. 37-31 has an angular diameter that is 1.35 times the angular diameter of the Moon, which is 0.50°. Assume that the drops all have about the same diameter. Approximately what is that diameter?

25. Allegheny Observatory (a) What is the angular separation of two stars if their images are barely resolved by the Thaw refracting telescope at the Allegheny Observatory in Pittsburgh? The lens diameter is 76 cm and its focal length is 14 m. Assume $\lambda = 550$ nm. (b) Find the distance between these barely resolved stars if each of them is 10 light-years distant from Earth. (c) For the image of a single star in this telescope, find the diameter of the first dark ring in the diffraction pattern, as measured on a photographic plate placed at the focal plane of the telescope lens. Assume that the structure of the image is associated entirely with diffraction at the lens aperture and not with lens "errors".

26. Soviet–French Experiment In a joint Soviet–French experiment to monitor the Moon's surface with a light beam, pulsed radiation from a ruby laser ($\lambda = 0.69$ μm) was directed to the Moon through a reflecting telescope with a mirror radius of 1.3 m. A reflector on the Moon behaved like a circular plane mirror with radius 10 cm, reflecting the light directly back toward the telescope on the Earth. The reflected light was then detected after being brought to a focus by this telescope. What fraction of the original light energy was picked up by the detector? Assume that for each direction of travel all the energy is in the central diffraction peak.

SEC. 37-6 ■ DIFFRACTION BY A DOUBLE SLIT

27. Bright Fringes Suppose that the central diffraction envelope of a double-slit diffraction pattern contains 11 bright fringes and the first diffraction minima eliminate (are coincident with) bright fringes. How many bright fringes lie between the first and second minima of the diffraction envelope?

28. Slit Separation In a double-slit experiment, the slit separation d is 2.00 times the slit width w. How many bright interference fringes are in the central diffraction envelope?

29. Eliminate Bright Fringes (a) In a double-slit experiment, what ratio of d to a causes diffraction to eliminate the fourth bright side fringe? (b) What other bright fringes are also eliminated?

30. Two Slits Two slits of width a and separation d are illuminated by a coherent beam of light of wavelength λ. What is the linear separation of the bright interference fringes observed on a screen that is at a distance D away?

31. How Many (a) How many bright fringes appear between the first diffraction-envelope minima to either side of the central maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm, and $a = 30.0$ μm? (b) What is the ratio of the intensity of the third bright fringe to the intensity of the central fringe?

32. Intensity Vs. Position Light of wavelength 440 nm passes through a double slit, yielding a diffraction pattern whose graph of intensity I versus angular position θ is shown in Fig. 37-32. Calculate the (a) slit width and (b) slit separation. (c) Verify the displayed intensities of the $m = 1$ and $m = 2$ interference fringes.

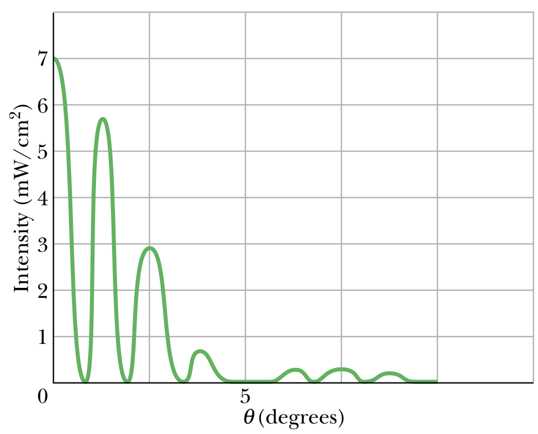


FIGURE 37-32 ■ Problem 32.

SEC. 37-7 ■ DIFFRACTION GRATINGS

33. Calculate *d* A diffraction grating 20.0 mm wide has 6000 rulings. (a) Calculate the distance d between adjacent rulings. (b) At what angles θ will intensity maxima occur on a viewing screen if the radiation incident on the grating has a wavelength of 589 nm?

34. Visible Spectrum A grating has 315 rulings/mm. For what wavelengths in the visible spectrum can fifth-order diffraction be observed when this grating is used in a diffraction experiment?

35. How Many Orders A grating has 400 lines/mm. How many orders of the entire visible spectrum (400–700 nm) can it produce in a diffraction experiment, in addition to the $m = 0$ order?

36. Confuse a Predator Perhaps to confuse a predator, some tropical gyrinid beetles (whirligig beetles) are colored by optical interference that is due to scales whose alignment forms a diffraction grating (which scatters light instead of transmiting it). When the incident light rays are perpendicular to the grating, the angle between the first-order maxima (on opposite sides of the zeroth-order maximum) is about 26° in light with a wavelength of 550 nm. What is the grating spacing of the beetle?

37. Two Adjacent Maxima Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent maxima occur at angles given by $\sin\theta = 0.2$ and $\sin\theta = 0.3$. The fourth-order maxima are missing. (a) What is the separation between adjacent slits? (b) What is the smallest slit width this grating can have? (c) Which orders of intensity maxima are produced by the grating, assuming the values derived in (a) and (b)?

38. Normal Incidence A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength $\lambda = 600$ nm at normal incidence. (a) How many maxima are there in the full diffraction pattern? (b) What is the width of a spectral line observed in the first order if the grating has 1000 slits?

39. Visible Spectrum Assume that the limits of the visible spectrum are arbitrarily chosen as 430 and 680 nm. Calculate the number of rulings per millimeter of a grating that will spread the first-order spectrum through an angle of 20°.

40. Gaseous Discharge Tube With light from a gaseous discharge tube incident normally on a grating with slit separation 1.73 μm, sharp maxima of green light are produced at angles $\theta = \pm 17.6°$, $37.3°$, $-37.1°$, $65.2°$, and $-65.0°$. Compute the wavelength of the green light that best fits these data.

41. Show That Light is incident on a grating at an angle ψ as shown in Fig. 37-33. Show that bright fringes occur at angles θ that satisfy the equation

$$d(\sin\psi + \sin\theta) = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots .$$

(Compare this equation with Eq. 37-22.) Only the special case $\psi = 0$ has been treated in this chapter.

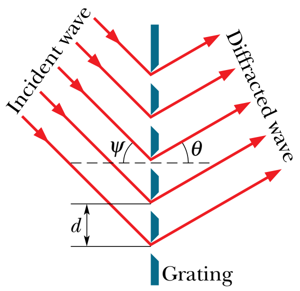


FIGURE 37-33 ■ Problem 41.

42. Plot A grating with $d = 1.50$ μm is illuminated at various angles of incidence by light of wavelength 600 nm. Plot, as a function of the angle of incidence (0 to 90°), the angular deviation of the first-order maximum from the incident direction. (See Problem 41.)

43. Derive Derive Eq. 37-25, the expression for the half-widths of lines in a grating's diffraction pattern.

44. Spectrum Is Formed A grating has 350 rulings per millimeter and is illuminated at normal incidence by white light. A spectrum is formed on a screen 30 cm from the grating. If a hole 10 mm square is cut in the screen, its inner edge being 50 mm from the central maximum and parallel to it, what is the range in the wavelengths of the light that passes through the hole?

45. Derive Two Derive this expression for the intensity pattern for a three-slit grating (ignore diffraction effects);

$$I_\theta = \tfrac{1}{9} I^{\max}(1 + 4\cos\phi + 4\cos^2\phi),$$

where $\phi = (2\pi d \sin\theta)/\lambda$. Assume that $a \ll \lambda$; be guided by the derivation of the corresponding double-slit formula (Eq. 36-21).

SEC. 37-8 ■ GRATINGS: DISPERSION AND RESOLVING POWER

46. D Line The D line in the spectrum of sodium is a doublet with wave-lengths 589.0 and 589.6 nm. Calculate the minimum number of lines needed in a grating that will resolve this doublet in the second-order spectrum. See Touchstone Example 37-5.

47. Hydrogen–Deuterium Mix A source containing a mixture of hydrogen and deuterium atoms emits red light at two wavelengths whose mean is 656.3 nm and whose separation is 0.180 nm. Find the minimum number of lines needed in a diffraction grating that can resolve these lines in the first order.

48. Smallest Wavelength A grating has 600 rulings/mm and is 5.0 mm wide. (a) What is the smallest wavelength interval it can resolve in the third order at $\lambda = 500$ nm? (b) How many higher orders of maxima can be seen?

49. Dispersion Show that the dispersion of a grating is $D = (\tan\theta)/\lambda$.

50. Sodium Doublet With a particular grating the sodium doublet (see Touchstone Example 37-5) is viewed in the third order at 10° to the normal and is barely resolved. Find (a) the grating spacing and (b) the total width of the rulings.

51. Resolving Power A diffraction grating has resolving power $R = \langle\lambda\rangle/\Delta\lambda = Nm$. (a) Show that the corresponding frequency range Δf that can just be resolved is given by $\Delta f = c/Nm\lambda$. (b) From Fig. 37-18, show that the times required for light to travel along the ray at the bottom of the figure and the ray at the top differ by an amount $\Delta t = (Nd/c)\sin\theta$. (c) Show that $(\Delta f)(\Delta t) = 1$, this relation being independent of the various grating parameters. Assume $N \gg 1$.

52. Product (a) In terms of the angle θ locating a line produced by a grating, find the product of that line's half-width and the resolving power of grating. (b) Evaluate that product for the grating of Problem 38, for the first order.

SEC. 37-9 ■ X-RAY DIFFRACTION

53. Second-Order Reflection X rays of wavelength 0.12 nm are found to undergo second-order reflection at a Bragg angle of 28° from a lithium fluoride crystal. What is the interplanar spacing of the reflecting planes in the crystal?

54. Diffraction by Crystal Figure 37-34 is a graph of intensity versus angular position θ for the diffraction of an x-ray beam by a crystal. The beam consists of two wavelengths, and the spacing between the reflecting planes is 0.94 nm. What are the two wavelengths?

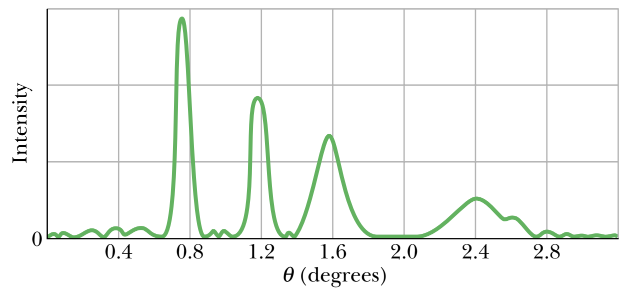


FIGURE 37-34 ■ Problem 54.

55. NaCl Crystal An x-ray beam of a certain wavelength is incident on a NaCl crystal, at 30.0° to a certain family of reflecting planes of spacing 39.8 pm. If the reflection from those planes is of the first order, what is the wavelength of the x rays?

56. Two Beams An x-ray beam of wavelength A undergoes first-order reflection from a crystal when its angle of incidence to a crystal face is 23°, and an x-ray beam of wavelength 97 pm undergoes third-order reflection when its angle of incidence to that face is 60°. Assuming that the two beams reflect from the same family of reflecting planes, find the (a) interplanar spacing and (b) wavelength A.

57. Not Possible Prove that it is not possible to determine both wavelength of incident radiation and spacing of reflecting planes in a crystal by measuring the Bragg angles for several orders.

58. Reflection Planes In Fig. 37-35, first-order reflection from the reflection planes shown occurs when an x-ray beam of wavelength 0.260 nm makes an angle of 63.8° with the top face of the crystal. What is the unit cell size a_0?

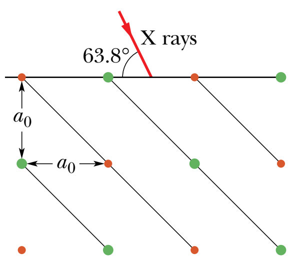


FIGURE 37-35 ■ Problem 58.

59. Square Crystal Consider a two-dimensional square crystal structure, such as one side of the structure shown in Fig. 37-26*a*. One interplanar spacing of reflecting planes is the unit cell size a_0. (a) Calculate and sketch the next five smaller interplanar spacings. (b) Show that your results in (a) are consistent with the general formula

$$d = \frac{a_0}{h^2 + k^2},$$

where h and k are relatively prime integers (they have no common factor other than unity).

60. X-Ray Beam In Fig. 37-36, an x-ray beam of wavelengths from 95.0 pm to 140 pm is incident at 45° to a family of reflecting planes with spacing $d = 275$ pm. At which wavelengths will these planes produce intensity maxima in their reflections?

61. NaCl In Fig. 37-36, let a beam of x-rays of wavelength 0.125 nm be incident on an NaCl crystal at an angle of 45.0° to the top face of the crystal and a family of reflecting planes. Let the reflecting planes have separation $d = 0.252$ nm. Through what angles must the crystal be turned about an axis that is perpendicular to the plane of the page for these reflecting planes to give intensity maxima in their reflections?

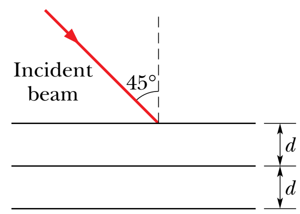


FIGURE 37-36 ■ Problems 60 and 61.

Additional Problems

62. Changing Interference Consider a plane wave of monochromatic green light, $\lambda = 500$ nm, that is incident normally upon two identical narrow slits (the widths of the individual slits are much less than λ). The slits are separated by a distance $d = 30$ μm. An interference pattern is observed on a screen located a distance L away from the slits. On the screen, the location nearest the central maximum where the intensity is zero (i.e., the first dark fringe) is found to be 1.5 cm from this central point. Let this particular position on the screen be referred to as P_1. (a) Calculate the distance, L, to the screen. Show all work. (b) In each of the parts below, one change has been made to the problem above (in each case, all parameters not explicitly mentioned have the value or characteristics stated above). For each case, explain briefly whether the light intensity at location P_1 remains zero or not. If not, does P_1 become the location of a maximum constructive interference (bright) fringe? In each case, explain your reasoning.

(1) One of the two slits is made slightly narrower, so that the amount of light passing through it is less than that through the other.

(2) The wavelength is doubled so that $\lambda = 1000$ nm.

(3) The two slits are replaced by a single slit whose width is exactly 60 μm.

63. Hearing and Seeing Around a Corner We can make the observation that we can hear around corners (somewhat) but not see around corners. Estimate why this is so by considering a doorway and two kinds of waves passing through it: (1) a beam of red light ($\lambda = 660$ nm), and (2) a sound wave playing an "A" ($f = 440$ Hz). (See Fig. 37-37.) Treat these two waves as plane waves passing through a slit whose width equals the width of the door. (a) Find the angle that gives the position of the first dark diffraction fringe. (b) From that, assuming you are 2 m back from the door, estimate how far outside the door you could be and still detect the wave. (See the picture for a clarification. The distance x is desired.)

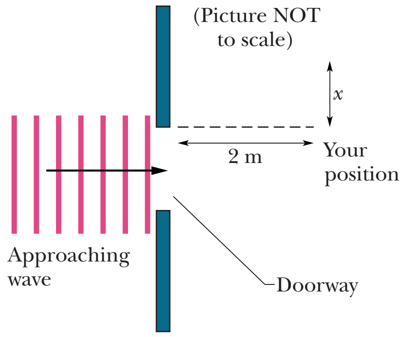


FIGURE 37-37 ■ Problem 63.