Polarization Experiments Using Jones Calculus

Reference

http://chaos.swarthmore.edu/courses/Physics50_2008/P50_Optics/04_Polariz_Matrices.pdf

Theory

In Jones calculus, the polarization state of light is given by a two-element column vector representing the electric field of the light

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix},$$

where E_x and E_y are in general complex quantities. The intensity of the light is given by $|E_x|^2 + |E_y|^2$. The effect of optical components on the polarization state of the light is described using two-by-two matrices that operate on the column vector. It is assumed that the direction of propagation is along the z-axis, and angles in the x-y plane are measured starting from the positive x-axis and moving toward the positive y-axis.

The table below gives the Jones matrices for ideal linear polarizers, linear retarders, rotation of axes and circular retarders. The angle θ specifies how the pass-plane of a polarizer or the optic(fast) axis of a linear retarder is oriented with respect to the x-axis.

Type of device	$\theta = 0$	$\theta = \pm\pi/4$	$\theta = \pi/2$	$\theta =$ any value
Ideal linear polarizer at angle θ	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} C_1^2 & C_1 S_1 \\ C_1 S_1 & S_1^2 \end{bmatrix}$
Quarter-wave linear retarder with optic axis at angle θ	$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1-i & \pm(1+i) \\ \pm(1+i) & 1-i \end{bmatrix}$	$\begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} C_1^2 - iS_1^2 & C_1 S_1(1+i) \\ C_1 S_1(1+i) & -iC_1^2 + S_1^2 \end{bmatrix}$

$C_1 = \cos\theta \quad , \quad S_1 = \sin\theta$

The matrix for $\theta = \pm\pi/4$ can be multiplied by $e^{i\pi/4}$ to give

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \pm i \\ \pm i & 1 \end{bmatrix}$$

Type of device	$\theta = 0$	$\theta = \pm\pi/4$	$\theta = \pi/2$	$\theta =$ any value
Half-wave linear retarder with optic axis at angle θ	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} C_2 & S_2 \\ S_2 & -C_2 \end{bmatrix}$
Linear retarder with retardation δ and with optic axis at angle θ	$\begin{bmatrix} 1 & 0 \\ 0 & e^{-i\delta} \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} e^{-i\delta}+1 & \pm\left(1-e^{-i\delta}\right) \\ \pm\left(1-e^{-i\delta}\right) & e^{-i\delta}+1 \end{bmatrix}$ *or* $e^{-i\delta/2}\begin{bmatrix} \cos\frac{\delta}{2} & \pm i\sin\frac{\delta}{2} \\ \pm i\sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{bmatrix}$	$\begin{bmatrix} e^{-i\delta} & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} C_1^2 + S_1^2 e^{-i\delta} & C_1 S_1(1-e^{-i\delta}) \\ C_1 S_1(1-e^{-i\delta}) & C_1^2 e^{-i\delta} + S_1^2 \end{bmatrix}$

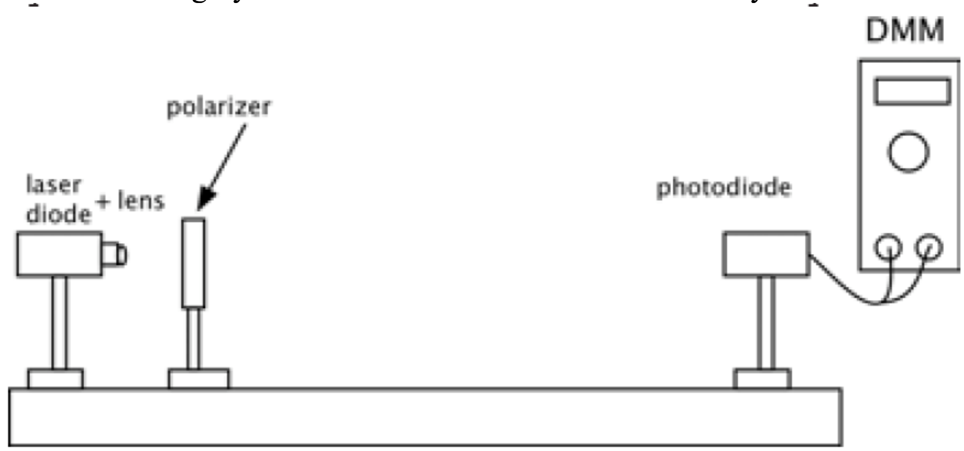
$C_2 = \cos 2\theta \quad , \quad S_2 = \sin 2\theta$

Experiment

Please handle all the optics by the edges. Do not put optical elements on top of each other or slide them across the bench, as they scratch easily.

We consider the experimental set-up shown below. The general experimental layout consists of a light source and a detector.

The light source is a laser diode in a holder with an attached lens. It produces light at a wavelength of 635 nm. The (laser-diode + lens + polarizer) system as shown below produces a polarized beam of roughly constant diameter and uniform intensity.



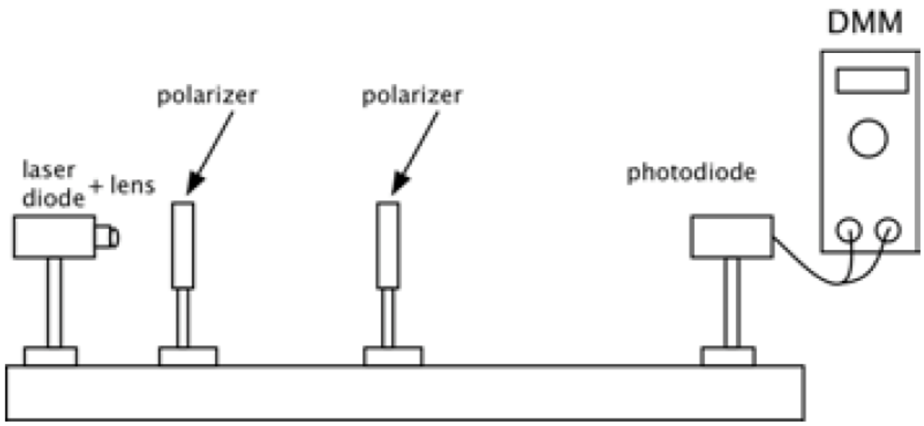
Set the polarizer to allow horizontally polarized light to pass (0° is horizontal, 90° is vertical). Some laser diodes produce partially polarized light. It has been set in its holder so its polarization is also horizontal. The other holders that you will be using are then set up between the first polarizer and the photodiode.

The light detector is a photodiode with a filter that allows only light near the laser diode wavelength to pass. It is a semiconductor device that is connected in series with a battery and a large resistor. When light falls on the semiconductor it creates charge carriers in the material, which then flow through a resistor to produce a voltage. This voltage is amplified so that the detector voltage is proportional to the light intensity. In this experiment, there is no reason to find the proportionality factor between light intensity and detector voltage.

Investigation #1

The first exercise is to measure the transmission of the polarized beam through a linear polarizer as it is rotated about the propagation direction (the beam axis).

We assume that the light exiting the first polarizer, which is linearly polarized, is polarized horizontally (long the x-axis). By convention, the y-axis is vertical and the z-axis is along the propagation direction. Determine the direction of the x-axis knowing the coordinate system is right-handed. The set-up is shown in the figure below.



We expect a maximum transmission when the polarizer axis is aligned with the polarization of the beam (along the x-axis). The polarizer axis can be set at any chosen angle θ. Remember, θ represents a counter-clockwise rotation about the z-axis starting at the x-axis. Using Jones matrices, show that the intensity of the light at the photodiode should follow a $\cos^2\theta$ dependence.

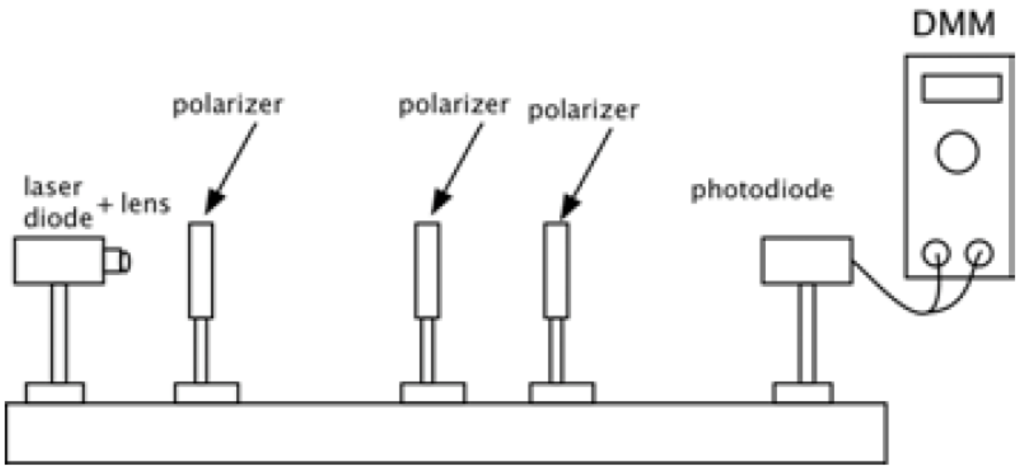
To check this relationship, position a second polarizer in the beam as in the above figure. Measure the transmitted intensity as a function of rotation angle for the second polarizer. You can make a plot of the intensity versus angle, which should follow the expression

$$I(\theta) = I_0 \cos^2(\theta - \theta_0) + I_b,$$

where θ_0 represents any misalignment of the zero of the polarizer scale and the polarization direction of the incident light, and where I_b represents the background light intensity. You should first try a fit without θ_0 and I_b as fitting parameters. Introduce them only if necessary.

Investigation #2

A more striking demonstration of how polarizers work can be done with two polarizers configured as shown in the figure.



The polarizer closest to the photodiode is oriented orthogonal to the input polarization, that is, along the y-axis. The middle polarizer can be set at any angle θ. Using Jones matrices, show that the intensity of the light at the photodiode should follow a $\sin^2\theta\cos^2\theta$ dependence.

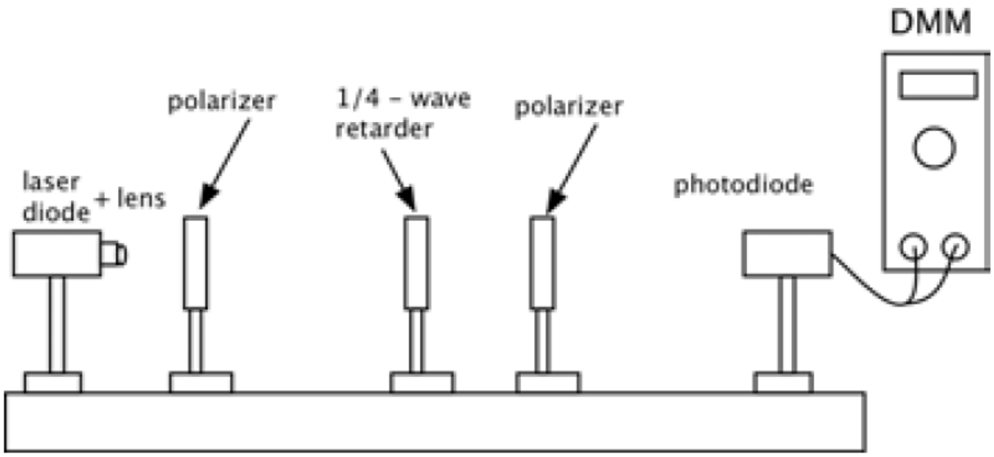
Set up the apparatus by installing the polarizer closest to the photodiode and rotating it to get minimum transmission. Then install the middle polarizer and measure the transmitted intensity as you rotate it. Plot your data and fit it to the theoretical relationship, introducing θ_0 and I_b only if necessary.

$$I(\theta) = I_0 \sin^2(\theta - \theta_0)\cos^2(\theta - \theta_0) + I_b = \frac{I_0}{8}\left\{1 - \cos\left[4(\theta - \theta_0)\right]\right\} + I_b.$$

Investigation #3

If a $\lambda/4$ retarder is illuminated with polarized light, the emerging beam will usually be elliptically polarized. Circular polarization (pure left or right) is a special case occurring when the optic axis (fast axis) of the retarder is at ± 45° to the input polarization and the retardation is exactly $\pi/2$ radians or 1/4 wavelength. Our light source operates at 635 nm. The retarder we will use is a 15th order $\lambda/4$ plate for 633 nm light. Because it is not exactly a $\lambda/4$ retarder, let's use the Jones matrix for an optical component that introduces a retardation of δ radians.

The experimental set-up is shown below, where the polarizer closest to the laser diode is oriented horizontally (along the x-axis) and the polarizer closest to the photodiode is oriented vertically (along the y-axis).



Using Jones matrices, show that the intensity at the photodiode goes as $4\sin^2\theta\cos^2\theta\sin^2(\delta/2)$ when the δ-retarder is rotated through an angle θ.

Take data on the intensity as the δ-retarder is rotated through an angle θ. Complete the measurement by plotting intensity versus angle and doing a computer fit to the proper function. Notice that the $\sin^2(\delta/2)$ term must be combined with I_0 to form a single fitting parameter. That is, without an independent measurement of I_0, the retardation δ cannot be measured this way.

Investigation #4

If the optic axis of the δ-retarder is fixed at θ = 45°, use Jones matrices to show that the intensity at the photodiode should follow a function of the form $[1+\cos\delta\cos(2\phi)]/2$ when the polarizer closest to the photodiode is rotated through an angle ϕ (measured from the x-axis).

With the polarizer axis perpendicular (along the y-axis) to the input polarization (along the x-axis), rotate the retarder to get maximum transmission. That sets the retarder axis at ± 45° to the input polarization and gives an output that is as close to circular polarization as the retarder can produce with the wavelength used. Measure the intensity at the photodiode as the polarizer is rotated and fit the data to the appropriate function. Make δ one of your fitting parameters.

For the retarder we are using, the retardation for 633 nm light is (15 x 360°) + 90° = 5,490° (for a 1st order quarter-wave plate, the retardation is 90°). The retardation depends on the wavelength λ as follows,

$$\delta = \frac{(360^\circ)\Delta n d}{\lambda},$$

where Δn is the difference in the indices of refraction for the two orthogonal polarizations in the retarder and d is the thickness of the retarder. Assuming the indices of refraction do not change for such a small change in wavelength, then the retardation for 635 nm light should be less by a factor of 633/635, namely 5,473°. This equals (15 x 360°) + 73°, meaning the λ/4 plate should exhibit a retardation of 73° for 635 nm light.