

The topic this week is **Complex Variables**.

Readings: Riley, Hobson, and Bence - Chapter 24
Boccio - 17_ComplexVariables

Problems:

EP-11 Some Elementary Functions - Write answers in form $a+bi$

$$(a) (-1)^{1/3} \quad (b) \sqrt{2-i} \quad (c) (-1-i)^{-1/2}$$

EP-12 Find the Taylor series expansion for $f(z) = \ln(1+z)$

EP-13 Use known Taylor series expansions about the origin to determine

$$f(z) = \frac{\sin z}{1-z}$$

EP-14 For each function, find all the Taylor series and Laurent series expansions about the point $z = a$ and state the region of convergence

$$(a) \frac{1}{z}, \quad a=1$$

$$(b) \frac{1}{z^2+1}, \quad a=i$$

$$(c) \frac{1}{(z+1)(z-2)}, \quad a=2$$

EP-15 Find the residue of each function at each pole.

$$(a) \frac{1}{z^2} \sin 2z \quad (b) \frac{\cos z}{z^2 + 2z + 1}$$

EP-16 Evaluate each integral around the circle $|z|=2$.

$$(a) \oint \frac{\sin z}{z^3} dz \quad (b) \oint \frac{\sin z}{z^3 - z^2} dz \quad (c) \oint \frac{z^2 + 1}{z(z+1)^3} dz$$

EP-17 Determine the value of each real integral.

$$(a) \int_0^{2\pi} \frac{d\theta}{(2 + \cos \theta)^2} \quad (b) \int_0^{2\pi} \frac{\sin 2\theta d\theta}{5 + 4 \cos \theta}$$

EP-18 Evaluate each integral.

$$(a) \int_{-\infty}^{\infty} \frac{1+x}{1+x^3} dx \quad (b) \int_{-\infty}^{\infty} \frac{\cos x}{(1+x^2)^2} dx \quad (c) \int_0^{\infty} \frac{\cos x}{1+x^4} dx$$

EP-19 Find the value of

$$\int_{-\infty}^{\infty} \frac{1}{x^4 - 1} dx$$

using a closed path where you have a large semicircle in the upper half-plane and you integrate around (small semicircles in upper half-plane) the two poles on the real axis.