Physics 50 Assignments #12 Spring 2009

Readings: Riley, Hobson and Bence - Chapter 21
Boccio - 14_PDEs, 15_PDEs, 16_PDEs

B. Steady-State Temperature in a Cylinder

3. Separate Laplace's equation in two dimensions in polar coordinates and solve the r and θ equations. Remember that for the θ equation, only periodic solutions are of interest. Use your results to solve the problem of the steady state temperature in a circular plate (radius = a) if the upper semi-circular boundary is held at 100° and the lower is held at 0°.

Answer is:

$$T = 50 + \frac{200}{\pi}\sum_{n\ odd}\left(\frac{r}{a}\right)^n \frac{\sin n\theta}{n} \qquad a = disk\ radius$$

4. Find the steady-state temperature distribution in a circular annulus of inner radius r = 1 and outer radius r = 2 if the inner circle is held at 0° and the outer circle has half of its circumference at 0° and half at 100°.(Hint: you cannot neglect r solutions corresponding to k = 0).

Answer is:

$$T = 50\frac{\log(r)}{\log(2)} + \frac{200}{\pi}\sum_{n\ odd}\frac{1}{n}\left(\frac{r^n - r^{-n}}{2^n - 2^{-n}}\right)\sin n\theta \qquad \log = natural\ \log arithm$$

5. A right circular cylinder is 1 m long and 2 m in diameter. Its left end and lateral surface are maintained at a temperature of 0° and its right end at 100°. Find the temperature at any interior point. Calculate the first three coefficients in the series expansion.

Answer is:

$$T(r,z) = 29.4J_0(2.4r)\sinh(2.4z) - 0.86J_0(5.52r)\sinh(5.52z) + 0.03J_0(8.65r)\sinh(8.65z) - ....$$

C. Steady-State Temperature in a Sphere

1. Find the steady-state temperature distribution inside a sphere of radius r = 1 when the surface temperatures are given by:

(a) $35(\cos\theta)^4$

(b) $\begin{cases} \cos\theta & 0 < \theta < \pi/2 \\ 0 & \pi/2 < \theta < \pi \end{cases}$

(c) $\sin^2\theta\cos\theta\cos 2\phi - \cos\theta$

Answers are:

(a) $T(r,\theta) = 8r^4 P_4(\cos\theta) + 20r^2 P_2(\cos\theta) + 7P_0(\cos\theta)$

(b) $T(r,\theta) = \frac{1}{4}P_0(\cos\theta) + \frac{1}{2}rP_1(\cos\theta) + \frac{5}{16}r^2 P_2(\cos\theta) + ...$

(c) $T(r,\theta,\phi) = \frac{1}{15}r^3 P_3^2(\cos\theta)\cos 2\phi - rP_1(\cos\theta)$

2. A sphere (radius = a) initially at $0°$ has its surface kept at $100°$ from t = 0 on (for example, a frozen spherical potato is tossed into in boiling water). Find the time-dependent temperature distribution. (Hint: subtract 100 from all temperatures, solve the problem and then add 100 to the solution; can you justify this procedure?). Show that the Legendre function required for this problem is P_0 and the r solution is

$$\frac{1}{\sqrt{r}}J_{1/2} \to j_0$$

Answer is:

$$T = 100 + \frac{200a}{\pi r}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi r}{a}\right)e^{-(n\pi\alpha/a)^2 t}$$

IV. Electric Potentials

1. Electric Potentials Shells - Find the potential between two concentric spheres if the outer sphere is maintained at V = 100 and the potential on the inner sphere is maintained at zero. The radii are 2 m and 1 m, respectively.

Answer is:

$$V(r,\theta) = 200\left[1 - \frac{1}{r}\right]$$

2. **Text Example 21.17.** A conducting sphere of radius a is cut around its equator and the two haves are connected to voltages of $+V$ and $-V$. Show that an expression for the potential at a point (r,θ,ϕ) anywhere inside the two hemispheres is

Answer is:

$$u(r,\theta,\phi) = V\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!(4n+3)}{2^{2n+1}n!(n+1)!}\left(\frac{r}{a}\right)^{2n+1} P_{2n+1}(\cos\theta)$$

You only need to modify some of the example in the text.