Physics 130 General Relativity Seminar

Assignment 4   February 11, 2013

General topic: **Curved Spacetimes in General Relativity**

Part 1: Readings

Hartle: Ch 7-Sections 7-9 - The Description of Curved Spacetime
Hartle: Ch 8 - Geodesics
Hartle: Ch 8 Supplement - Derivation of Geodesic Equation(WEB)

Part 2: Problems Hartle Problems

1. Hartle 7.20 Embedding diagrams - get the book cover!
2. Hartle 7.25 Toy model of a wormhole
3. Hartle 8.02 Christoffel symbols and great circles
4. Hartle 8.03 Lagrangians and Christoffel symbols
5. Hartle 8.04 Rotating frames
6. Hartle 8.05 Work out Christoffel symbols (check errata)
7. Hartle 8.08 Killing vectors (make sure we all understand them)
8. Hartle 8.09 All the timelike geodesics
9. Hartle 8.11 Null geodesics in flat 3-dimensional spacetime
10. Hartle 8.12 The hyperbolic plane
11. Hartle 8.14 Fermat's Principle of Least Time

Boccio Extra Problems

1. **Null and Orthogonal**
   (a) Show that the sum of any two orthogonal (scalar product is zero) spacelike vectors is spacelike.
   (b) Show that a timelike vector and a null vector cannot be orthogonal.
2. **Killing Vectors in Flat Space**

   Find the Killing vectors for flat space $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$, i.e., write out Killings equation in flat space, differentiate it once and then solve the resulting differential equation.

3. **Timelike Geodesics**

   Find the timelike geodesics for the metric

$$ds^2 = \frac{1}{t^2}\left(-dt^2 + dx^2\right)$$

4. **Light Cones**

   Consider the 2-dimensional metric

$$ds^2 = -x dw^2 + 2 dw dx$$

   (a) Calculate the light cone at a point (w, x), i.e., find dw/dx for the light cone. Sketch a (w, x) spacetime diagram showing how the light cones change with x. What can you say about the motion of particles, and in particular, about whether they can cross from positive to negative x and vice versa.

   (b) Find a new system of coordinates in which the metric is diagonal.

5. **Light Cones and Embedding**

   A certain spacetime is describe by the metric

$$ds^2 = -(1 - H^2 r^2)dt^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

   (a) Describe the lightcone structure in the (r, t) plane using both equations and a spacetime diagram. Think carefully about the lightcone structure for $r > H^{-1}$ versus $r < H^{-1}$.

   (b) Construct an embedding diagram for this spacetime. The following steps will guide you through the process:

      (1) Argue that it is sufficient to consider the 2-dimensional slice

$$d\Sigma^2 = (1 - H^2 r^2)^{-1} dr^2 + r^2 d\varphi^2$$

      (2) Pick one of the three common forms for the 3-dimensional flat space line element:

$$ds^2_{3D} = dx^2 + dy^2 + dz^2$$
$$ds^2_{3D} = d\rho^2 + \rho^2 d\varphi^2 + dz^2$$
$$ds^2_{3D} = dw^2 + w^2(d\Theta^2 + \sin^2\Theta d\Phi^2)$$

and find the equations that describe the 2-dimensional surface corresponding to the 2-dimensional slice metric above. What is the geometry of this surface?