Physics 130 General Relativity Seminar

Assignment 3 February 04, 2013

General topic: **Curved Spacetimes in General Relativity**

Part 1: Readings

Hartle: Ch 6 - Gravity as Geometry

Hartle: Ch 7-Sections 1-6 - The Description of Curved Spacetime

Part 2: Problems Hartle Problems

1. Hartle 6.06 A uniformly accelerating Newtonian frame
2. Hartle 6.07 Properties of an accelerated laboratory
3. Hartle 6.14 Worldlines around the Earth
4. Hartle 7.01 A singular line element
5. Hartle 7.02 Show the space-time is flat
6. Hartle 7.05 Light cone structure in 2D spacetime
7. Hartle 7.10 4-velocity in a 2D spacetime
8. Hartle 7.11,7.12 Properties of warp-drive spacetime
9. Hartle 7.14 Distance, area, 3D and 4D volume in curved spacetime
10. Hartle 7.18 Distances and volumes in Schwarzschild metric

Boccio Extra Problems

1. **Multiple Lorentz Transformations**

   A Lorentz transformation is the product of a boost with rapidity ζ in the direction $\hat{n}_1$, followed by a boost with rapidity ζ in the direction $\hat{n}_2$, followed by a boost with rapidity ζ in the direction $\hat{n}_3$, where ζ is the same in each case, and the three directions $\hat{n}_1$, $\hat{n}_2$, and $\hat{n}_3$ lie in the same plane separated by $120°$. What is the resulting transformation? To lowest order for small ζ, is it a boost or a rotation? At what order does the other (boost or rotation) enter?

2. **An Invisible Sphere**

   A hollow sphere has density ρ, inner radius a and outer radius b. Find the gravitational field in the region $r < a$. Suppose now that the sphere were invisible. Could an observer at the center deduce its existence without leaving the region $r < a$?

3. **Gravitational Fields**

   (a) Compute the gradient of the gravitational field $\partial g_i / \partial x_j$ (a nine component object) corresponding to a sphere of density ρ and radius R centered at the origin.

   (b) Find a mass distribution $\rho(x, y, z)$ on a bounded domain, that is, zero whenever $x^2 + y^2 + z^2 > R^2$ for some positive constant R; uniformly bounded, i.e., $|\rho(x, y, z)| < C$ for some positive constant C independent of position; and for which at least one component of the gradient of the gravitational field is infinite at some point.