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A MODIFIED GRAVITY AND ITS CONSEQUENCES FOR THE SOLAR SYSTEM, ASTROPHYSICS AND COSMOLOGY*

J. W. MOFFAT

Perimeter Institute for Theoretical Physics,
31 Caroline St. North, Waterloo, Ontario, N2L 2Y5, Canada

Department of Physics, University of Waterloo, Address
Waterloo, Ontario, N2L 3G1, Canada
john.moffat@utoronto.ca

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A relativistic modified gravity (MOG) theory leads to a self-consistent, stable gravity theory that can describe the solar system, galaxy and clusters of galaxies data and cosmology.

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1. Introduction

A relativistic modified gravity (MOG) called Scalar-Tensor-Vector Gravity (STVG) desribes a self-consistent, stable gravity theory that contains Einstein's general relativity in a well-defined limit.[1] The theory has an extra degree of freedom, a vector field called a "phion" field whose curl is a skew symmetric field that couples to matter ("fifth force"). The spacetime geometry is described by a symmetric Einstein metric. An alternative relativistic gravity theory called Metric-Skew-Tensor Gravity (MSTG) has also been formulated[2] in which the spacetime is described by a symmetric metric, and the extra degree of freedom is a skew symmetric second rank tensor field. Both of these theories yield the same weak field consequences for physical systems.

The classical STVG theory allows the gravitational coupling "constant" G and the coupling of the phion field and its effective mass to vary with space and time as scalar fields.

A MOG should explain the following physical phenomena:

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(1) Galaxy rotation curve data;
(2) Mass profiles of x-ray clusters;
(3) Gravitational lensing data for galaxies and clusters of galaxies;
(4) The cosmic microwave background (CMB) including the acoustical oscillation power spectrum data;
(5) The formation of proto-galaxies in the early universe and the growth of galaxies;
(6) N-body simulations of galaxy surveys;
(7) The accelerating expansion of the universe.

We seek a unified description of solar system, astrophysical and large-scale cosmological data without exotic non-baryonic dark matter. Dark matter in the form of particles has until now not been discovered in spite of large-scale experimental efforts.[3] The accelerating expansion of the universe should be explained by the MOG theory without postulating a cosmological constant.

2. Action and Field Equations

Our MOG action takes the form[1]:

$$S = S_{\rm Grav} + S_\phi + S_S + S_M, \tag{1}$$

where

$$S_{\rm Grav} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\biggl[\frac{1}{G}(R + 2\Lambda)\biggr], \tag{2}$$

$$S_\phi = -\int d^4x\sqrt{-g}\biggl[\omega\biggl(\frac{1}{4}B^{\mu\nu}B_{\mu\nu} + V(\phi)\biggr)\biggr], \tag{3}$$

and

$$S_S = \int d^4x\sqrt{-g}(\mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3), \tag{4}$$

where

$$\mathcal{F}_1 = \frac{1}{G^3}\biggl(\frac{1}{2}g^{\mu\nu}\nabla_\mu G\nabla_\nu G - V(G)\biggr), \tag{5}$$

$$\mathcal{F}_2 = \frac{1}{G}\biggl(\frac{1}{2}g^{\mu\nu}\nabla_\mu\omega\nabla_\nu\omega - V(\omega)\biggr), \tag{6}$$

$$\mathcal{F}_3 = \frac{1}{\mu^2 G}\biggl(\frac{1}{2}g^{\mu\nu}\nabla_\mu\mu\nabla_\nu\mu - V(\mu)\biggr). \tag{7}$$

We have chosen units with $c = 1$, ∇_μ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$. We adopt the metric signature $\eta_{\mu\nu} = {\rm diag}(1, -1, -1, -1)$ where $\eta_{\mu\nu}$ is the Minkowski spacetime metric, and $R = g^{\mu\nu}R_{\mu\nu}$ where $R_{\mu\nu}$ is the symmetric Ricci tensor. Moreover, $V(\phi)$ denotes a potential for the vector field ϕ^μ, while $V(G), V(\omega)$ and $V(\mu)$ denote the three potentials associated with the three scalar fields $G(x), \omega(x)$ and $\mu(x)$, respectively. The field $\omega(x)$ is dimensionless and Λ denotes the cosmological constant. Moreover,

$$B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu. \tag{8}$$

The field equations and the test particle equations of motion are derived in Ref. 1.

The action for the field $B_{\mu\nu}$ is of the Maxwell-Proca form for a massive vector field ϕ_μ. It can be proved that this MOG possesses a stable vacuum and the Hamiltonian is bounded from below. Even though the action is not gauge invariant, it can be shown that the longitudinal mode ϕ_0 (where $\phi_\mu = (\phi_0, \phi_i)\,(i = 1, 2, 3)$) does not propagate and the theory is free of ghosts. Similar arguments apply to the MSTG theory.[2][a]

3. Modified Newtonian Acceleration Law and Galaxy Dynamics

The modified acceleration law can be written as[1]:

$$a(r) = -\frac{G(r)M}{r^2}, \tag{9}$$

where

$$G(r) = G_N\left[1 + \sqrt{\frac{M_0}{M}}\left(1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right)\right] \tag{10}$$

is an *effective* expression for the variation of G with respect to r, and G_N denotes Newton's gravitational constant. A good fit to a large number of galaxies has been achieved with the parameters[5]:

$$M_0 = 9.60 \times 10^{11}\, M_\odot, \quad r_0 = 13.92\,\mathrm{kpc} = 4.30 \times 10^{22}\,\mathrm{cm}. \tag{11}$$

In the fitting of the galaxy rotation curves for both LSB and HSB galaxies, using photometric data to determine the mass distribution $\mathcal{M}(r)$,[5] only the mass-to-light ratio $\langle M/L\rangle$ is employed, once the values of M_0 and r_0 are fixed universally for all LSB and HSB galaxies. Dwarf galaxies are also fitted with the parameters[5]:

$$M_0 = 2.40 \times 10^{11}\, M_\odot, \quad r_0 = 6.96\,\mathrm{kpc} = 2.15 \times 10^{22}\,\mathrm{cm}. \tag{12}$$

By choosing universal values for the parameters $G_\infty = G_N(1 + \sqrt{M_0/M})$, $(M_0)_{\mathrm{clust}}$ and $(r_0)_{\mathrm{clust}}$, we are able to obtain satisfactory fits to a large sample of X-ray cluster data.[6]

4. Solar System and Binary Pulsar

Let us assume that we are in a distance scale regime for which the fields G, ω and μ take their approximate renormalized constant values:

$$G \sim G_0(1 + Z), \quad \omega \sim \omega_0 A, \quad \mu \sim \mu_0 B, \tag{13}$$

[a] For a detailed discussion of possible instabilities and pathological behavior of vector-gravity theories, see Ref. 4.

where G_0, ω_0 and μ_0 denote the "bare" values of G, ω and μ, respectively, and Z, A and B are the associated renormalization constants. We obtain from the equations of motion of a test particle the orbital equation (we reinsert the speed of light c)[1]:

$$\frac{d^2u}{d\phi^2}+u=\frac{GM}{c^2J^2}-\frac{K}{c^2J^2}\exp(-r/r_0)\left[1+\left(\frac{r}{r_0}\right)\right]+\frac{3GM}{c^2}u^2. \tag{14}$$

where $u=1/r$, $K=G_N\sqrt{MM_0}$ and J denotes the orbital angular momentum. Using the large r weak field approximation, we obtain the orbit equation for $r \ll r_0$:

$$\frac{d^2u}{d\phi^2}+u=N+3\frac{GM}{c^2}u^2, \tag{15}$$

where J_N denotes the Newtonian value of J and

$$N=\frac{GM}{c^2J_N^2}-\frac{K}{c^2J_N^2}. \tag{16}$$

We can solve Eq.(15) by perturbation theory and find for the perihelion advance of a planetary orbit

$$\Delta\omega=\frac{6\pi}{c^2L}(GM_\odot-K_\odot), \tag{17}$$

where $J_N=(GM_\odot L/c^2)^{1/2}$, $L=a(1-e^2)$ and a and e denote the semimajor axis and the eccentricity of the planetary orbit, respectively.

For the solar system $r \ll r_0$ and from the running of the effective gravitational coupling constant, $G=G(r)$, we have $G\sim G_N$ within the experimental errors for the measurement of Newton's constant G_N. We choose for the solar system

$$\frac{K_\odot}{c^2}\ll 1.5\,\mathrm{km} \tag{18}$$

and use $G=G_N$ to obtain from (17) a perihelion advance of Mercury in agreement with GR. The bound (18) requires that the coupling constant ω varies with distance in such a way that it is sufficiently small in the solar system regime and determines a value for M_0 that is in accord with the bound (18).

For terrestrial experiments and orbits of satellites, we have also that $G\sim G_N$ and for $K_\oplus$ sufficiently small, we then achieve agreement with all gravitational terrestrial experiments including Eötvös free-fall experiments and "fifth force" experiments.

For the binary pulsar PSR 1913+16 the formula (17) can be adapted to the periastron shift of a binary system. Combining this with the STVG gravitational wave radiation formula, which will approximate closely the GR formula, we can obtain agreement with the observations for the binary pulsar. The mean orbital radius for the binary pulsar is equal to the projected semi-major axis of the binary, $\langle r\rangle_N=7\times 10^{10}\,\mathrm{cm}$, and we choose $\langle r\rangle_N \ll r_0$. Thus, for $G=G_N$ within the experimental errors, we obtain agreement with the binary pulsar data for the periastron shift when

$$\frac{K_N}{c^2}\ll 4.2\,\mathrm{km}. \tag{19}$$

For a massless photon we have

$$\frac{d^2u}{d\phi^2}+u=3\frac{GM}{c^2}u^2. \tag{20}$$

For the solar system using $G\sim G_N$ within the experimental errors gives the light deflection:

$$\Delta_\odot=\frac{4G_NM_\odot}{c^2R_\odot} \tag{21}$$

in agreement with GR.

5. Pioneer Anomaly

The radio tracking data from the Pioneer 10/11 spacecraft during their travel to the outer parts of the solar system have revealed an anomalous acceleration. The Doppler data obtained at distances r from the Sun between 20 and 70 astronomical units (AU) showed the anomaly as a deviation from Newton's and Einstein's gravitational theories. The anomaly is observed in the Doppler residuals data, as the differences of the observed Doppler velocity from the modelled Doppler velocity, and can be represented as an anomalous acceleration directed towards the Sun, with an approximately constant amplitude over the range of distance, $20\,\mathrm{au} < r < 70\,\mathrm{au}$[7,8,9,10]:

$$a_P=(8.74\pm1.33)\times10^{-8}\,\mathrm{cm}\,s^{-2}. \tag{22}$$

After a determined attempt to account for all *known* sources of systematic errors, the conclusion has been reached that the anomalous acceleration towards the Sun could be a real physical effect that requires a physical explanation.[7,8,9,10][b]

We can rewrite the acceleration in the form

$$a(r)=-\frac{G_NM}{r^2}\biggl\{1+\alpha(r)\biggl[1-\exp(-r/\lambda(r))\biggl(1+\frac{r}{\lambda(r)}\biggr)\biggr]\biggr\}. \tag{23}$$

We postulate a gravitational solution that the Pioneer 10/11 anomaly is caused by the difference between the running of $G(r)$ and the Newtonian value, G_N. So the Pioneer anomalous acceleration directed towards the center of the Sun is given by

$$a_P=-\frac{\delta G(r)M_\odot}{r^2}, \tag{24}$$

where

$$\delta G(r)=G_N\alpha(r)\biggl[1-\exp(-r/\lambda(r))\biggl(1+\frac{r}{\lambda(r)}\biggr)\biggr]. \tag{25}$$

[b]It is possible that a heat transfer mechanism from the spacecraft transponders could produce a non-gravitational explanation for the anomaly.

Lacking at present a solution for the variations of $\alpha(r)$ and $\lambda(r)$ in the solar system, we adopt the following parametric representations of the "running" of $\alpha(r)$ and $\lambda(r)$:

$$\alpha(r) = \alpha_\infty(1-\exp(-r/\bar{r}))^{b/2}, \tag{26}$$

$$\lambda(r) = \lambda_\infty(1-\exp(-r/\bar{r}))^{-b}. \tag{27}$$

Here, $\bar{r}$ is a non-running distance scale parameter and b is a constant.

In Ref. 11, a best fit to the acceleration data extracted from Figure 4 of Ref. 10 was obtained using a nonlinear least-squares fitting routine including estimated errors from the Doppler shift observations[8]. The best fit parameters are

$$\begin{aligned} \alpha_\infty &= (1.00 \pm 0.02)\times 10^{-3}, \\ \lambda_\infty &= 47 \pm 1\,\mathrm{au}, \\ \bar{r} &= 4.6 \pm 0.2\,\mathrm{au}, \\ b &= 4.0. \end{aligned} \tag{28}$$

The small uncertainties in the best fit parameters are due to the remarkably low variance of residuals corresponding to a reduced χ^2 per degree of freedom of 0.42 signalling a good fit. An important result obtained from our fit to the anomalous acceleration data is that the anomalous acceleration kicks-in at the orbit of Saturn.

Fifth force experimental bounds plotted for $\log_{10}\alpha$ versus $\log_{10}\lambda$ are shown in Fig. 1 of Ref. 12 for fixed values of α and λ. The updated 2003 observational data for the bounds obtained from the planetary ephemerides is extrapolated to $r = 10^{15}\,\mathrm{m} = 6,685\,\mathrm{au}$[13]. However, this extrapolation is based on using fixed universal values for the parameters α and λ. Since known reliable data from the ephemerides of the outer planets ends with the data for Pluto at a distance from the Sun, $r = 39.52\,\mathrm{au} = 5.91\times 10^{12}\,m$, we could claim that for our range of values $47\,\mathrm{au} < \lambda(r) < \infty$, we predict $\alpha(r)$ and $\lambda(r)$ values consistent with the *un-extrapolated* fifth force bounds.

A consequence of a variation of G and $GM_\odot$ for the solar system is a modification of Kepler's third law:

$$a^3_{PL} = G(a_{PL})M_\odot\left(\frac{T_{PL}}{2\pi}\right)^2, \tag{29}$$

where T_{PL} is the planetary sidereal orbital period and a_{PL} is the physically measured semi-major axis of the planetary orbit. For given values of a_{PL} and T_{PL}, (29) can be used to determine $G(r)M_\odot$.

For several planets such as Mercury, Venus, Mars and Jupiter there are planetary ranging data, spacecraft tracking data and radiotechnical flyby observations available, and it is possible to measure a_{PL} directly. For a distance varying $GM_\odot$ we derive[14,15]:

$$\left(\frac{a_{PL}}{\bar{a}_{PL}}\right) = 1 + \eta_{PL} = \left[\frac{G(a_{PL})M_\odot}{G(a_\oplus)M_\odot}\right]^{1/3}. \tag{30}$$

Here, it is assumed that $GM_\odot$ varies with distance such that η_{PL} can be treated as a constant for the orbit of a planet. We obtain

$$\eta_{PL} = \left[\frac{G(a_{PL})}{G(a_\oplus)}\right]^{1/3} - 1. \tag{31}$$

The results for $\Delta\eta_{PL}$ due to the uncertainty in the planetary ephemerides are presented in Ref. 11 for the nine planets and are consistent with the solar ephemerides.

The validity of the bounds on a possible fifth force obtained from the ephemerides of the outer planets Uranus, Neptune and Pluto are critical in the exclusion of a parameter space for our fits to the Pioneer anomaly acceleration. Beyond the outer planets, the theoretical prediction for $\eta(r)$ approaches an asymptotic value:

$$\eta_\infty \equiv \lim_{r\to\infty} \eta(r) = 3.34 \times 10^{-4}. \tag{32}$$

We see that the variations ("running") of $\alpha(r)$ and $\lambda(r)$ with distance play an important role in interpreting the data for the fifth force bounds. This is in contrast to the standard non-modified Yukawa correction to the Newtonian force law with fixed universal values of α and λ and for the range of values $0 < \lambda < \infty$, for which the equivalence principle and lunar laser ranging and radar ranging data to planetary probes exclude the possibility of a gravitational and fifth force explanation for the Pioneer anomaly.[16,17,18]

A study of the Shapiro time delay prediction in our MOG is found to be consistent with time delay observations and predicts a measurable deviation from GR for the outer planets Neptune and Pluto.[19]

6. Gravitational Lensing

The bending angle of a light ray as it passes near a massive system along an approximately straight path is given to lowest order in v^2/c^2 by

$$\theta = \frac{2}{c^2}\int |a^\perp| dz, \tag{33}$$

where $\perp$ denotes the perpendicular component to the ray's direction, and dz is the element of length along the ray and a denotes the acceleration.

From (20), we obtain the light deflection

$$\Delta = \frac{4GM}{c^2R} = \frac{4G_N\overline{M}}{c^2R}, \tag{34}$$

where

$$\overline{M} = M\left(1 + \sqrt{\frac{M_0}{M}}\right). \tag{35}$$

The value of $\overline{M}$ follows from (10) for clusters as $r \gg r_0$ and

$$G(r) \to G_\infty = G_N\left(1 + \sqrt{\frac{M_0}{M}}\right). \tag{36}$$

We choose for a cluster $M_0 = 3.6\times 10^{15}\,M_\odot$ and a cluster mass $M_{\rm clust}\sim 10^{14}\,M_\odot$, and obtain

$$\biggl(\sqrt{\frac{M_0}{M}}\biggr)_{\rm clust}\sim 6. \tag{37}$$

We see that $\overline{M}\sim 7M$ and we can explain the increase in the light bending without exotic dark matter.

For $r\gg r_0$ we get

$$a(r) = -\frac{G_N\overline{M}}{r^2}. \tag{38}$$

We expect to obtain from this result a satisfactory description of lensing phenomena using Eq.(33).

7. Modified Friedmann Equations in Cosmology

We shall base our results for the cosmic microwave background (CMB) power spectrum on our MOG without a second component of cold dark matter (CDM). Our description of the accelerating universe[20,21] is based on Λ_G in Eq.(62) derived from our varying gravitational constant.[22]

We adopt a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) background geometry with the line element

$$ds^2 = dt^2 - a^2(t)\biggl(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\biggr), \tag{39}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $k = 0, -1, +1$ for a spatially flat, open and closed universe, respectively. Due to the symmetry of the FLRW background spacetime, we have $\phi_0\equiv\phi\neq 0$, $\phi_i = 0$ and $B_{\mu\nu} = 0$.

We define the energy-momentum tensor for a perfect fluid by

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}, \tag{40}$$

where $u^\mu = dx^\mu/ds$ is the 4-velocity of a fluid element and $g_{\mu\nu}u^\mu u^\nu = 1$. Moreover, we have

$$\rho = \rho_m + \rho_\phi + \rho_S,\quad p = p_m + p_\phi + p_S, \tag{41}$$

where ρ_i and p_i denote the components of density and pressure associated with the matter, the ϕ^μ field and the scalar fields G, ω and μ, respectively.

The modified Friedmann equations take the form[1]:

$$\frac{\dot a^2(t)}{a^2(t)} + \frac{k}{a^2(t)} = \frac{8\pi G(t)\rho(t)}{3} + f(t) + \frac{\Lambda}{3}, \tag{42}$$

$$\frac{\ddot a(t)}{a(t)} = -\frac{4\pi G(t)}{3}[\rho(t) + 3p(t)] + h(t) + \frac{\Lambda}{3}, \tag{43}$$

where $\dot{a} = da/dt$ and

$$f(t) = \frac{\dot{a}(t)}{a(t)}\frac{\dot{G}(t)}{G(t)}, \tag{44}$$

$$h(t) = \frac{1}{2}\biggl(\frac{\ddot{G}(t)}{G(t)} - \frac{\dot{G}^2(t)}{G^2(t)} + 2\frac{\dot{a}(t)}{a(t)}\frac{\dot{G}(t)}{G(t)}\biggr). \tag{45}$$

From (42) we obtain

$$\rho a^3 = \frac{3}{8\pi G}a\biggl(\dot{a}^2 + k - a^2 f - \frac{1}{3}a^2\Lambda\biggr). \tag{46}$$

This leads by differentiation with respect to t to the expression:

$$\dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) + \mathcal{I} = 0, \tag{47}$$

where

$$\mathcal{I} = \frac{3a^2}{8\pi G}(2\dot{a}f + a\dot{f} - 2\dot{a}h). \tag{48}$$

An approximate solution to the field equations for the variation of G in Ref. 1 in the background FLRW spacetime is given by

$$\ddot{\mathcal{G}} + 3H\dot{\mathcal{G}} + V'(\mathcal{G}) = \frac{1}{2}G_N\mathcal{G}^2\biggl(\rho - 3p + \frac{\Lambda}{4\pi G_N\mathcal{G}}\biggr), \tag{49}$$

where $\mathcal{G}(t) = G(t)/G_N$ and $H = \dot{a}/a$. A solution for $\mathcal{G}$ in terms of a given potential $V(\mathcal{G})$ and for given values of ρ, p and Λ can be obtained from (49).[22]

The solution for $\mathcal{G}$ must satisfy a constraint at the time of big bang nucleosynthesis.[23] The number of relativistic degrees of freedom is very sensitive to the cosmic expansion rate at 1 MeV. This can be used to constrain the time dependence of G. Measurements of the ^4He mass fraction and the deuterium abundance at 1 MeV lead to the constraint $G(t) \sim G_N$. We impose the condition $\mathcal{G}(t) \rightarrow 1$ as $t \rightarrow t_{BBN}$ where t_{BBN} denotes the time of the big bang nucleosynthesis. Moreover, *locally in the solar system* we must satisfy the observational bound from the Cassini spacecraft measurements[24]:

$$|\dot{G}/G| \leq 10^{-12}\mathrm{yr}^{-1}. \tag{50}$$

We shall now impose the approximate conditions at the epoch of recombination:

$$2\dot{a}f + a\dot{f} \sim 2\dot{a}h, \tag{51}$$

$$\frac{d}{dt}\biggl(\frac{\dot{G}}{G}\biggr) < 2\frac{\dot{a}}{a}\frac{\dot{G}}{G}. \tag{52}$$

We find from (45) and (52) that $f \sim h$, and from the condition (51) we obtain

$$\dot{f} \equiv \frac{d\Lambda_G}{dt} \sim 0, \tag{53}$$

where

$$\Lambda_G = \frac{\dot{a}}{a}\frac{\dot{G}}{G}. \tag{54}$$

By setting the cosmological constant $\Lambda = 0$, we get the generalized Friedmann equations

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \Lambda_G, \tag{55}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \Lambda_G. \tag{56}$$

We now have from (47), (48) and (51) at the epoch of recombination $\mathcal{I} \sim 0$ and

$$\dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) \sim 0. \tag{57}$$

We adopt the equation of state: $p(t) = w\rho(t)$ and derive from (57) the approximate solution for $\rho(t)$:

$$\rho(t) \sim \rho(t_0)\left(\frac{a_0}{a(t)}\right)^{3(1+w)}, \tag{58}$$

where $a/a_0 = 1/(1+z)$ and z denotes the red shift. For the matter and radiation densities ρ_m and ρ_r, we have $w = 0$ and $w = 1/3$, respectively. This gives

$$\rho_m(t) \sim \rho_m(t_0)(1+z)^3, \quad \rho_r(t) \sim \rho_r(t_0)(1+z)^4. \tag{59}$$

Let us expand $G(t)$ in a power series

$$G(t) = G_{\rm eff}(t_r) + (t - t_r)\dot{G}(t_r) + (t - t_r)^2\ddot{G}(t_r) + ... \tag{60}$$

where $t \sim t_r$ is the time of recombination and $G_{\rm eff}(t_r) = G_N(1+Z) = \text{const}$. We write the generalized Friedmann equation for flat space, $k = 0$, in the approximate form

$$H^2 = \frac{8\pi G_{\rm eff}\rho_m}{3} + \Lambda_G, \tag{61}$$

where

$$\Lambda_G = H\frac{\dot{G}}{G} > 0 \tag{62}$$

and $\dot{\Lambda}_G \sim 0$. It follows from (61) that for a spatially flat universe:

$$\Omega_m + \Omega_G = 1, \tag{63}$$

where

$$\Omega_m = \frac{8\pi G_{\rm eff}\rho_m}{3H^2}, \quad \Omega_G = \frac{\Lambda_G}{H^2}. \tag{64}$$

We shall postulate that the matter density ρ_m is dominated by the baryon density, $\rho_m \sim \rho_b$, and we have

$$\Omega_m \sim \Omega_{b\rm eff}, \tag{65}$$

where

$$\Omega_{b\text{eff}} = \frac{8\pi G_{\text{eff}}\rho_b}{3H^2}. \tag{66}$$

Thus, we assume that *the baryon-photon fluid dominates matter before recombination and at the surface of last scattering without a cold dark matter fluid component.*

From the current value: $H_0 = 7.5 \times 10^{-11}\,\text{yr}^{-1}$ and (62) and (64), we obtain for $\Omega_G \sim 0.7$:

$$|\dot{G}/G| \sim 5 \times 10^{-11}\,\text{yr}^{-1}, \tag{67}$$

valid at cosmological scales for red shifts $z > 0.1$. In the *local solar system* and for the binary pulsar PSR 1913+16 for $z \sim 0$, the experimental bound is

$$|\dot{G}/G| < 5 \times 10^{-12}\,\text{yr}^{-1}. \tag{68}$$

We can explain the accelerated expansion of the universe deduced from supernovae measurements in the range $0.1 < z < 1.7$ using the cosmologically scaled value of $\dot{G}/G$ in (67) with Einstein's cosmological constant $\Lambda = 0$.

8. Acoustical Peaks in the CMB Power Spectrum

Mukhanov[25] has obtained an analytical solution to the amplitude of fluctuations in the CMB power spectrum for $l \gg 1$:

$$l(l+1)C_l \sim \frac{B}{\pi}(O + N). \tag{69}$$

Here, O denotes the oscillating part of the spectrum, while the non-oscillating contribution can be written as the sum of three parts

$$N = N_1 + N_2 + N_3. \tag{70}$$

The oscillating contributions can be calculated from the formula

$$O \sim \sqrt{\frac{\pi}{r_h l}}\left[A_1 \cos\left(l r_p + \frac{\pi}{4}\right) + A_2 \cos\left(2 l r_p + \frac{\pi}{4}\right)\right]\exp(-(l/l_s)^2), \tag{71}$$

where r_h and r_p are parameters that determine predominantly the heights and positions of the peaks, respectively. The A_1 and A_2 are constant coefficients given in the range $100 < l < 1200$ for $\Omega_m \sim \Omega_{b\text{eff}}$ by

$$A_1 \sim 0.1\xi\frac{((\mathcal{P} - 0.78)^2 - 4.3)}{(1+\xi)^{1/4}}\exp\left(\frac{1}{2}(l_s^{-2} - l_f^{-2})l^2\right), \tag{72}$$

$$A_2 \sim 0.14\frac{(0.5 + 0.36\mathcal{P})^2}{(1+\xi)^{1/2}}, \tag{73}$$

where

$$\mathcal{P} = \ln\left(\frac{lI}{200(\Omega_{b\text{eff}})^{1/2}}\right), \tag{74}$$

and I is given by the ratio

$$\frac{\eta_x}{\eta_0} \sim \frac{I}{z_x^{1/2}} = 3\left(\frac{\Omega_G}{\Omega_{\rm beff}}\right)^{1/6}\left(\int_0^y \frac{dx}{(\sinh x)^{2/3}}\right)^{-1}\frac{1}{z_x^{1/2}}. \tag{75}$$

Here, η_x and z_x denote a conformal time $\eta = \eta_x$ and a redshift in the range $\eta_0 > \eta_x > \eta_r$ when radiation can be neglected and $y = \sinh^{-1}(\Omega_G/\Omega_{\rm beff})^{1/2}$. To determine η_x/η_0, we use the exact solution for a flat dust-dominated universe with a constant Λ_G:

$$a(t) = a_0\left(\sinh\left(\frac{3}{2}\right)H_0 t\right)^{2/3}, \tag{76}$$

where a_0 and H_0 denote the present values of a and the Hubble parameter H.

The l_f and l_s in (72) denote the finite thickness and Silk damping scales, respectively, given by

$$l_f^2 = \frac{1}{2\sigma^2}\left(\frac{\eta_0}{\eta_r}\right)^2, \quad l_s^2 = \frac{1}{2(\sigma^2 + 1/(k_D\eta_r)^2)}\left(\frac{\eta_0}{\eta_r}\right)^2, \tag{77}$$

where

$$\sigma \sim 1.49\times 10^{-2}\left[1 + \left(1 + \frac{z_{\rm eq}}{z_r}\right)^{-1/2}\right], \quad k_D(\eta) = \left(\frac{2}{5}\int_0^\eta d\eta c_s^2\frac{\tau_\gamma}{a}\right)^{-1/2}, \tag{78}$$

and τ_γ is the photon mean-free time.

A numerical fitting formula gives[25]:

$$\mathcal{P} \sim \ln\left(\frac{l}{200(\Omega_{\rm beff}^{0.59})}\right), \quad r_p = \frac{1}{\eta_0}\int d\eta c_s(\eta). \tag{79}$$

Moreover,

$$\xi \equiv \frac{1}{3c_s^2} - 1 = \frac{3}{4}\left(\frac{\rho_b}{\rho_\gamma}\right), \tag{80}$$

where $c_s(\eta)$ is the speed of sound:

$$c_s(\eta) = \frac{1}{\sqrt{3}}\left[1 + \xi\left(\frac{a(\eta)}{a(\eta_r)}\right)\right]^{-1/2}. \tag{81}$$

We note that ξ does not depend on the value of $G_{\rm eff}$. For the matter-radiation universe:

$$a(\eta) = \bar{a}\left[\left(\frac{\eta}{\eta_*}\right)^2 + 2\left(\frac{\eta}{\eta_*}\right)\right], \tag{82}$$

where for radiation-matter equality $z = z_{eq}$:

$$\frac{z_{eq}}{z_r} \sim \left(\frac{\eta_r}{\eta_*}\right)^2 + 2\left(\frac{\eta_r}{\eta_*}\right), \tag{83}$$

and $\eta_{eq} = \eta_*(\sqrt{2}-1)$ follows from $\bar{a} = a(\eta_{eq})$.

For the non-oscillating parts, we have

$$N_1 \sim 0.063\xi^2 \frac{(\mathcal{P} - 0.22(l/l_f)^{0.3} - 2.6)^2}{1 + 0.65(l/l_f)^{1.4}} \exp(-(l/l_f)^2), \tag{84}$$

$$N_2 \sim \frac{0.037}{(1+\xi)^{1/2}} \frac{\mathcal{P} - 0.22(l/l_s)^{0.3} + 1.7)^2}{1 + 0.65(l/l_f)^{1.4}} \exp(-(l/l_s)^2), \tag{85}$$

$$N_3 \sim \frac{0.033}{(1+\xi)^{3/2}} \frac{\mathcal{P} - 0.5(l/l_s)^{0.55} + 2.2)^2}{1 + 2(l/l_s)^2} \exp(-(l/l_s)^2). \tag{86}$$

Mukhanov's formula[25] for the oscillating spectrum is given by

$$C(l) \equiv \frac{l(l+1)C_l}{[l(l+1)C_l]_{\mathrm{low}\, l}} = \frac{100}{9}(O + N), \tag{87}$$

where we have normalized the power spectrum by using for a flat spectrum with a constant amplitude B:

$$[l(l+1)C_l]_{\mathrm{low}\, l} = \frac{9B}{100\pi}. \tag{88}$$

We adopt the parameters

$$\Omega_{bN} \sim 0.04, \quad \Omega_{b\mathrm{eff}} \sim 0.3, \quad \Omega_G \sim 0.7, \quad \xi \sim 0.6, \tag{89}$$

and

$$r_h = 0.03, \quad r_p = 0.01 \quad l_f \sim 1580, \quad l_s \sim 1100, \tag{90}$$

where $\Omega_{bN} = 8\pi G_N \rho_b / 3H^2$.

The fluctuation spectrum determined by Mukhanov's analytical formula is displayed in Fig. 1 for the choice of cosmological parameters given in (89) and (90).

The role played by CDM in the standard scenario is replaced in the modified gravity theory by the significant deepening of the gravitational potential well by the effective gravitational constant, $G_{\mathrm{eff}} \sim 7G_N$, that traps the non-relativistic baryons before recombination. The deepening of the gravitational well reduces the baryon dissipation due to the photon coupling pressure and the third and higher peaks in the acoustic oscillation spectrum are not erased by finite thickness and baryon drag effects. The effective baryon density $\Omega_{b\mathrm{eff}} = (1+Z)\Omega_{bN} \sim 7\Omega_{bN} \sim 0.3$ dominates the fluid before recombination, *and we fit the acoustical power spectrum data without a cold dark matter fluid component.* For $t < t_{\mathrm{dec}}$, where t_{dec} denotes the time of matter-radiation decoupling, luminous baryons and photons are tightly coupled and for photons the dominant collision mechanism is scattering by non-relativistic electrons due to Thompson scattering. It follows that luminous baryons are dragged along with photons and perturbations at wavelength $\lambda_w < \ell_s$ will be partly erased where ℓ_s is the proper Silk length given by $\ell_s \sim 3.5\,\mathrm{Mpc}\,\Omega_{b\mathrm{eff}}^{-1/2}$.[30] We have $\ell_s \sim 6\,\mathrm{Mpc}$ for $\Omega_{b\mathrm{eff}} \sim 0.3$ compared to $\ell_s \sim 18\,\mathrm{Mpc}$ for $\Omega_{bN} \sim 0.04$. The Silk

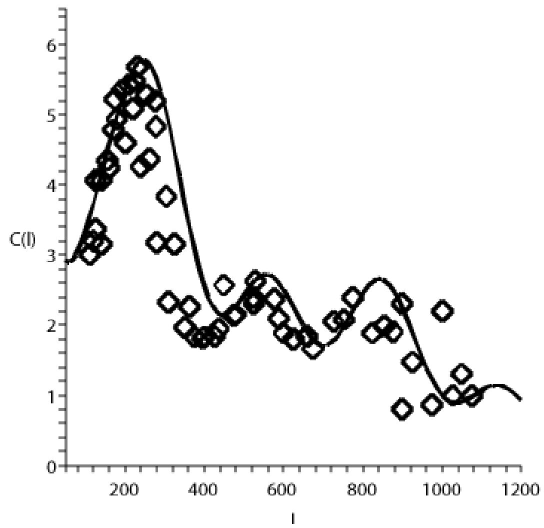

Fig. 1. The solid line shows the result of the calculation of the power spectrum acoustical oscillations: $C(l)$, and the $\diamond s$ correspond to the WMAP, Archeops and Boomerang data in units $\mu K^2 \times 10^{-3}$ as presented in Refs. 27-30.

mass is reduced by more than an order of magnitude[c]. Thus, sufficient baryonic perturbations should survive before $t \sim t_{\rm dec}$ to explain the power spectrum without collisionless dark matter.

Our predictions for the CMB power spectrum for large angular scales corresponding to $l < 100$ will involve the integrated Sachs-Wolfe contributions obtained from the modified gravitational potential.

9. Conclusions

We have demonstrated that a modified gravity theory[1] can lead to a satisfactory fit to the galaxy rotation curve data, mass profiles of x-ray cluster data, the solar system and the binary pulsar PSR 1913+16 data. Moreover, we can provide an explanation for the Pioneer 10/11 anomalous acceleration data, given that the anomaly is caused by gravity. We can fit satisfactorily the acoustical oscillation spectrum obtained in the cosmic microwave background data by employing the analytical formula for the fluctuation spectrum derived by Mukhanov.[25]

Λ_G obtained from the varying gravitational constant in our MOG replaces the standard cosmological constant Λ in the concordance model. Thus, the accelerating expansion of the universe is obtained from the MOG scenario.

An important problem to investigate is whether an N-body simulation calculation based on our MOG scenario can predict the observed large scale galaxy surveys. The formation of proto-galaxy structure before and after the epoch of recombination and the growth of galaxies and clusters of galaxies at later times in the expansion of the universe has to be explained.

[c]Note that there will be a fraction of dark baryonic matter before decoupling.

We have succeeded in fitting in a unified picture a large amount of data over 16 orders of magnitude in distance scale from Earth to the surface of last scattering some 13.7 billion years ago, using our modified gravitational theory without exotic dark matter. The data fitting ranges over four distance scales: the solar system, galaxies, clusters of galaxies and the CMB power spectrum data at the surface of last scattering.

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