Modified Gravitational Theory as an Alternative to Dark Energy and Dark Matter

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Abstract

The problem of explaining the acceleration of the expansion of the universe and the observational and theoretical difficulties associated with dark matter and dark energy are discussed. The possibility that Einstein gravity does not correctly describe the large-scale structure of the universe is considered and an alternative gravity theory is proposed as a possible resolution to the problems.

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1 Introduction

The surprising observational discovery that the expansion of the universe is accelerating [1, 2, 3] has led to an increasing theoretical effort to understand this phenomenon. Although interpreting the data by postulating a non-zero, positive cosmological constant is the simplest way to understand the data, it is not satisfactory, because it leads to the two serious problems of why the estimates from the standard model and quantum field theory predict preposterously large values of the cosmological constant [4], and the coincidence of dark energy dominance today.

If we simply postulate a repulsive force in the universe associated with a charge density, then we might expect that this force could be responsible for generating the acceleration of the universe. However, for a homogeneous and isotropic universe the net charge density would be zero, although for a finite range force with a small mass there will exist a non-zero charge density [5]. The effect of a Maxwell-type force would be to lower or raise the total energy, leaving the form of the Friedmann equation unchanged. Thus, we would still have to invoke exotic forms of energy with an equation of state, $p = w\rho$, where w is negative and violates the positive energy theorems. For a non-zero cosmological constant $w = -1$.

In addition to the dark energy problem, we are still confronted with the puzzle of dark matter. Any observational detection of a dark matter candidate has eluded us and the fits to galaxy halos using dark matter models are based on several parameters depending on the size of the galaxy being fitted. The possibility that an alternative gravity theory can produce a satisfactory description of galaxy dynamics with little or no dark matter should be studied [6, 7, 8, 9, 10].

Challenging experimental results are often the precursors of a shifting of scientific paradigms. We must now entertain the prospect that the discovery of the mechanism driving the acceleration of the universe can profoundly change our description of the universe. Recent observational data for supernovae (SNe Ia) [2] have produced new conclusive evidence that the universe went from a decelerating phase at $z \sim 0.5$ to an accelerating phase and that the data are consistent with the concordance model with $\Omega_m \sim 0.3, \Omega_\Lambda \sim 0.7$, where Ω_m and Ω_Λ denote the ratios of dark matter density and dark energy density to the critical density $\rho_c = 3H^2/8\pi G$. Moreover, the data give for the equation of state parameter for dark energy, $w_D = -1.02 \pm 0.15$ consistent with a cosmological constant value $w_D = -1$ and $dw_D/dz = 0$ [2].

Given the uneasy tension existing between observational evidence for the acceleration of the universe and the mystery of what constitutes dark matter and dark energy, we shall consider the question of whether Einstein's general relativity (GR) is correct for the large scale structure of the universe. It agrees well with local solar system experimental tests and for the data obtained for observations of the binary pulsar PSR 1913+16 [11]. However, this does not preclude the possibility of a breakdown of the conventional Einstein equations for the large-scale structure of the universe. The standard GR cosmological model agrees well with the abundances of light elements from big bang nucleosynthesis (BBN), and the evolution of the spectrum of primordial density fluctuations, yielding the observed spectrum of temperature anisotropies in the cosmic microwave background (CMB). Also, the age of the universe and the power spectrum of large-scale structure agree reasonably well with the standard cosmological model. However, it could be that additional repulsive gravitational effects from an alternative gravity theory could agree with all of the results in the early universe and yet lead to significant effects in the present universe accounting for its acceleration.

A fundamental change in the predictions of the observational data will presumably only come about from a non-trivial alteration of the mathematical and geometrical formalism that constitutes GR. Recent modifications of Einstein's gravitational theory and developments of cosmological models [12, 13, 14, 15, 16] have led to alterations of the Friedmann equation at large cosmological scales.

In the following, we shall consider the physically non-trivial extension of GR called the nonsymmetric gravitational theory (NGT) [17]. This theory was extensively studied over a period of years, and a version of the theory was discovered that is free of linearized weak field inconsistencies such as ghost poles, tachyons, and other instabilities [18]. The skew sector $g_{[\mu\nu]}$ corresponds to a massive Kalb-Ramond field, and is not to be identified with the electromagnetic field as was done by Einstein [19] in his unified field theory. The skew sector is treated as part of the total gravitational field and together with the symmetric part $g_{(\mu\nu)}$ forms the total nonsymmetric $g_{\mu\nu}$, which generates a non-Riemannian geometry.

A significant modification of the infrared gravitational field and the equations of cosmology can only be obtained by having additional degrees of freedom beyond the two degrees of freedom of Einstein's gravity theory. Such additional degrees of freedom are supplied by NGT, which yields six additional degrees of freedom in the skew sector $g_{[\mu\nu]}$.

As we shall see in the following, and in further work [20], it is possible for NGT to describe the current data on the accelerating universe and the dark matter halos of galaxies, gravitational lensing and cluster behavior, as well as the standard observational results, without invoking the need for dominant, exotic dark matter and a dark energy identified with vacuum energy and the cosmological constant. These results are obtained while retaining the good agreement of Einstein's gravity theory with solar system tests, terrestrial gravitational experiments and the binary pulsar PSR 1913+16.

2 NGT Action and Field Equations

The nonsymmetric $g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ are defined by [17, 18]:

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} \tag{1}$$

and

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{(\mu\nu)} + \Gamma^\lambda_{[\mu\nu]}, \tag{2}$$

where

$$g_{(\mu\nu)} = \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}), \quad g_{[\mu\nu]} = \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}). \tag{3}$$

The contravariant tensor $g^{\mu\nu}$ is defined in terms of the equation

$$g^{\mu\nu} g_{\sigma\nu} = g^{\nu\mu} g_{\nu\sigma} = \delta^\mu{}_\sigma. \tag{4}$$

The NGT action is given by

$$S_{\rm ngt} = S + S_M, \tag{5}$$

where

$$S = \frac{1}{16\pi G}\int d^4x[\mathbf{g}^{\mu\nu}R^*_{\mu\nu}(W) - 2\Lambda\sqrt{-g} - \frac{1}{4}\mu^2\mathbf{g}^{\mu\nu}g_{[\nu\mu]}], \tag{6}$$

and S_M is the matter action satisfying the relation

$$\frac{1}{\sqrt{-g}}\biggl(\frac{\delta S_M}{\delta g^{\mu\nu}}\biggr) = -\frac{1}{2}T_{\mu\nu}. \tag{7}$$

Here, we have chosen units $c=1$, $\mathbf{g}^{\mu\nu}=\sqrt{-g}g^{\mu\nu}$, $g=\mathrm{Det}(g_{\mu\nu})$, Λ is the cosmological constant, μ is a mass associated with the skew field $g_{[\mu\nu]}$. Moreover, $T_{\mu\nu}$ is the nonsymmetric energy-momentum tensor and $R^*_{\mu\nu}(W)$ is the tensor

$$R^*_{\mu\nu}(W)=R_{\mu\nu}(W)-\frac{1}{6}W_\mu W_\nu, \tag{8}$$

where $R_{\mu\nu}(W)$ is the NGT contracted curvature tensor

$$R_{\mu\nu}(W)=W^\beta_{\mu\nu,\beta}-\frac{1}{2}(W^\beta_{\mu\beta,\nu}+W^\beta_{\nu\beta,\mu})-W^\beta_{\alpha\nu}W^\alpha_{\mu\beta}+W^\beta_{\alpha\beta}W^\alpha_{\mu\nu}, \tag{9}$$

defined in terms of the unconstrained nonsymmetric connection:

$$W^\lambda_{\mu\nu}=\Gamma^\lambda_{\mu\nu}-\frac{2}{3}\delta^\lambda{}_\mu W_\nu, \tag{10}$$

where

$$W_\mu=\frac{1}{2}(W^\lambda_{\mu\lambda}-W^\lambda_{\lambda\mu}). \tag{11}$$

Eq.(10) leads to the result

$$\Gamma_\mu=\Gamma^\lambda_{[\mu\lambda]}=0. \tag{12}$$

The contracted tensor $R_{\mu\nu}(W)$ can be written as

$$R_{\mu\nu}(W)=R_{\mu\nu}(\Gamma)+\frac{2}{3}W_{[\mu,\nu]}, \tag{13}$$

where

$$R_{\mu\nu}(\Gamma)=\Gamma^\beta_{\mu\nu,\beta}-\frac{1}{2}\left(\Gamma^\beta_{(\mu\beta),\nu}+\Gamma^\beta_{(\nu\beta),\mu}\right)-\Gamma^\beta_{\alpha\nu}\Gamma^\alpha_{\mu\beta}+\Gamma^\beta_{(\alpha\beta)}\Gamma^\alpha_{\mu\nu}. \tag{14}$$

A variation of the action $S_{\rm ngt}$ yields the field equations in the presence of matter sources

$$G^*_{\mu\nu}(W)+\Lambda g_{\mu\nu}+S_{\mu\nu}=8\pi GT_{\mu\nu}, \tag{15}$$

$$\mathbf{g}^{[\mu\nu]}{}_{,\nu}=-\frac{1}{2}\mathbf{g}^{(\mu\alpha)}W_\alpha, \tag{16}$$

$$\mathbf{g}^{\mu\nu}{}_{,\sigma}+\mathbf{g}^{\rho\nu}W^\mu_{\rho\sigma}+\mathbf{g}^{\mu\rho}W^\nu_{\sigma\rho}-\mathbf{g}^{\mu\nu}W^\rho_{\sigma\rho}$$
$$+\frac{2}{3}\delta^\nu_\sigma\mathbf{g}^{\mu\rho}W^\beta_{[\rho\beta]}+\frac{1}{6}(\mathbf{g}^{(\mu\beta)}W_\beta\delta^\nu_\sigma-\mathbf{g}^{(\nu\beta)}W_\beta\delta^\mu_\sigma)=0. \tag{17}$$

Here, we have $G^*_{\mu\nu}(W)=R^*_{\mu\nu}(W)-\frac{1}{2}g_{\mu\nu}\mathcal{R}^*(W)$, where $\mathcal{R}^*(W)=g^{\mu\nu}R^*_{\mu\nu}(W)$, and

$$S_{\mu\nu}=\frac{1}{4}\mu^2(g_{[\mu\nu]}+\frac{1}{2}g_{\mu\nu}g^{[\sigma\rho]}g_{[\rho\sigma]}+g^{[\sigma\rho]}g_{\mu\sigma}g_{\rho\nu}). \tag{18}$$

The generalized Bianchi identities

$$[\mathbf{g}^{\alpha\nu}G_{\rho\nu}(\Gamma)+\mathbf{g}^{\nu\alpha}G_{\nu\rho}(\Gamma)]_{,\alpha}+g^{\mu\nu}{}_{,\rho}\mathbf{G}_{\mu\nu}=0, \tag{19}$$

give rise to the matter response equations

$$g_{\mu\rho}\mathbf{T}^{\mu\nu}{}_{,\nu} + g_{\rho\mu}\mathbf{T}^{\nu\mu}{}_{,\nu} + (g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho})\mathbf{T}^{\mu\nu} = 0. \tag{20}$$

It has been proved that the present version of NGT described above does not possess any ghost poles or tachyons in the linear weak field approximation [18], either as an expansion about Minkowski spacetime or about a generic GR background. This cures the inconsistencies discovered by Damour, Deser and McCarthy in an earlier version of NGT [21]. In NGT there are three distinct possible metric tensors with three different local light cone structures. The definition of a spacelike surface is consequently dependent on the chosen coupling of matter to geometry and it is not possible to unambiguously apply a $(3+1)$ decomposition of field variables in order to perform a Hamiltonian constraint analysis using the standard methods. Further studies of the non-linear and non-perturbative solutions of massive NGT and its Cauchy development have to be undertaken. In the following, we shall identify $g_{(\mu\nu)}$ with the metric tensor of spacetime.

3 Cosmological Solutions

For the case of a spherically symmetric field, the form of $g_{\mu\nu}$ in NGT is given by

$$g_{\mu\nu} = \begin{pmatrix} -\alpha & 0 & 0 & w \\ 0 & -\beta & f\sin\theta & 0 \\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\ -w & 0 & 0 & \gamma \end{pmatrix}, \tag{21}$$

where α, β, γ and w are functions of r and t. The tensor $g^{\mu\nu}$ has the components:

$$g^{\mu\nu} = \begin{pmatrix} \frac{\gamma}{w^2-\alpha\gamma} & 0 & 0 & \frac{w}{w^2-\alpha\gamma} \\ 0 & -\frac{\beta}{\beta^2+f^2} & \frac{f\csc\theta}{\beta^2+f^2} & 0 \\ 0 & -\frac{f\csc\theta}{\beta^2+f^2} & -\frac{\beta\csc^2\theta}{\beta^2+f^2} & 0 \\ -\frac{w}{w^2-\alpha\gamma} & 0 & 0 & -\frac{\alpha}{w^2-\alpha\gamma} \end{pmatrix}. \tag{22}$$

We have

$$\sqrt{-g} = \sin\theta[(\alpha\gamma - w^2)(\beta^2 + f^2)]^{1/2}. \tag{23}$$

For a comoving coordinate system, we obtain for the velocity vector $u^\mu = dx^\mu/ds$, which satisfies the normalization condition $g_{(\mu\nu)}u^\mu u^\nu = 1$:

$$u^0 = \frac{1}{\sqrt{\gamma}}, \quad u^r = u^\theta = u^\phi = 0. \tag{24}$$

We set $w = 0$ so that only the $g_{[23]}$ component of $g_{[\mu\nu]}$ is different from zero. This corresponds to setting the magnetic monopole charge and static magnetic field in Maxwell's theory to zero, if we define

$$g^{*[\mu\nu]} = \epsilon^{\mu\nu\sigma\rho}g_{[\sigma\rho]}, \tag{25}$$

where $\epsilon^{\mu\nu\sigma\rho}$ is the Levi-Civita tensor density and we identify $g^{*[0i]}$ and $g^{*[jk]}$ as the static "magnetic" and "electric" potentials, respectively, associated with the massive Kalb-Ramond potential field.

The vector W_μ can be determined from

$$W_\mu = -\frac{2}{\sqrt{-g}} s_{(\mu\rho)} \mathbf{g}^{[\rho\sigma]}{}_{,\sigma}, \tag{26}$$

where $s_{(\mu\alpha)} g^{(\alpha\nu)} = \delta^\nu{}_\mu$. For the skew symmetric field with $w = 0$, it follows from (16) and (26) that $W_\mu = 0$.

The energy-momentum tensor for a fluid is

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu} + K^{[\mu\nu]}, \tag{27}$$

where $K^{[\mu\nu]}$ is a skew symmetric source tensor identified with the intrinsic spin or fluid vorticity. The fluid vorticity is defined by

$$\omega_{\mu\nu} = u_{[\mu,\nu]} + a_{[\mu} u_{\nu]}, \tag{28}$$

where a_μ is the fluid's four-acceleration. We only consider couplings to $u_{[\mu,\nu]}$ to avoid derivative couplings. A rotational action is given by

$$S_R = \int d^4x \mathcal{L}_R = \int d^4x \kappa \rho g_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} u_{[\alpha,\beta]}, \tag{29}$$

where κ is a coupling constant. By varying this action with respect to $g_{\mu\nu}$ we get

$$K^{[\mu\nu]} = \kappa\rho \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} u_{[\alpha,\beta]}. \tag{30}$$

We define

$$T_{\mu\nu} = g_{\mu\beta} g_{\alpha\nu} T^{\alpha\beta}, \tag{31}$$

and from (4) and (27), we get

$$T = \rho - 3p + g_{[\alpha\beta]} K^{[\alpha\beta]} = \rho - 3p + 2fK, \tag{32}$$

where $K^{[23]} = K/\sin\theta$.

We can write the field equations (15) for $W_\mu = 0$ in the form

$$R_{\mu\nu}(\Gamma) - g_{\mu\nu}\Lambda + S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{S} = 8\pi G (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T), \tag{33}$$

where $\mathcal{S} = g^{\mu\nu} S_{\mu\nu}$. The metric takes the canonical Gaussian form for comoving polar coordinates

$$ds^2 = dt^2 - \alpha(r,t) dr^2 - \beta(r,t)[d\theta^2 + \sin^2\theta d\phi^2]. \tag{34}$$

We shall assume that $\beta(r,t) \gg f(r,t)$ and that a solution can be found by a separation of variables

$$\alpha(r,t) = h(r)R^2(t), \quad \beta(r,t) = r^2 S^2(t). \tag{35}$$

From the field equations (88)(see Appendix A), we get

$$\frac{\dot{R}}{R} - \frac{\dot{S}}{S} = \frac{1}{2}Zr, \tag{36}$$

where $\dot{R} = \partial R/\partial t$ and Z is given by

$$Z = \frac{\dot{\beta}' f^2}{\beta^3} - \frac{5\dot{\beta}\beta' f^2}{2\beta^4} - \frac{\dot{\alpha}\beta' f^2}{2\alpha\beta^3} + \frac{2\dot{\beta} f f'}{\beta^3} - \frac{f\dot{f}'}{\beta^2} - \frac{3f'\dot{f}}{2\beta^2}$$
$$+\frac{\dot{\alpha} f f'}{2\alpha\beta^2} + \frac{2\beta' f\dot{f}}{\beta^3}. \tag{37}$$

We shall assume that $Z \approx 0$ which from (36) gives $R(t) \approx S(t)$. This leads to a metric of the form

$$ds^2 = dt^2 - R^2(t)\Big[h(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big]. \tag{38}$$

We cannot impose exact homogeneity and isotropy on the skew sector $g_{[\mu\nu]}$, for this would lead to $f(r,t) = w(r,t) = 0$.

We shall further simplify our calculations by assuming that the mass parameter $\mu \approx 0$, that the cosmological constant $\Lambda = 0$ and that we can neglect any effects due to the antisymmetric source tensor $K^{[\mu\nu]}$ associated with vorticity of the matter fluid.

With the assumption that $\beta \gg f$, the equations of motion become (see, Appendix A):

$$2b(r) + \ddot{R}(t)R(t) + 2\dot{R}^2(t) - R^2(t)W(r,t) = 4\pi G R^2(t)[\rho(r,t) - p(r,t)], \tag{39}$$

$$-\ddot{R}(t)R(t) + \frac{1}{3}R^2(t)Y(r,t) = \frac{4\pi G}{3}R^2(t)[\rho(r,t) + 3p(r,t)], \tag{40}$$

where

$$2b(r) = \frac{h'(r)}{rh^2(r)}. \tag{41}$$

The functions W and Y are given by

$$W(r,t) = \frac{\alpha'\beta' f^2}{2\alpha^2\beta^3} - \frac{\beta'' f^2}{\alpha\beta^3} + \frac{\dot{\alpha}\dot{\beta} f^2}{2\alpha\beta^3} + \frac{5\beta'^2 f^2}{2\alpha\beta^4} - \frac{\dot{\alpha} f\dot{f}}{2\alpha\beta^2}$$
$$-\frac{\alpha' f f'}{2\alpha^2\beta^2} - \frac{f f''}{\alpha\beta^2} - \frac{4 f f'\beta'}{\alpha\beta^3} + \frac{3f'^2}{2\alpha\beta^2}, \tag{42}$$

$$Y(r,t) = \frac{\ddot{\beta} f^2}{\beta^3} - \frac{5\dot{\beta}^2 f^2}{2\beta^4} - \frac{3\dot{f}^2}{2\beta^2} + \frac{4\dot{\beta} f\dot{f}}{\beta^3} - \frac{f\ddot{f}}{\beta^2}. \tag{43}$$

Within our approximation scheme, W and Y can be expressed in the form

$$W(r,t) = \frac{h' f^2}{h^2 r^5 R^6} - \frac{2f^2}{hr^6 R^6} + \frac{2\dot{R}^2 f^2}{r^4 R^6} + \frac{10 f^2}{hr^6 R^6} - \frac{\dot{R} f\dot{f}}{r^4 R^5} - \frac{h' f f'}{2h^2 r^4 R^6}$$

$$-\frac{f f''}{h4^4 R^6} - \frac{8 f f'}{hr^5 R^6} + \frac{3f'^2}{2hr^4 R^6}, \tag{44}$$

$$Y(r,t) = \frac{2(\dot{R}^2 + R\ddot{R}) f^2}{r^4 R^6} - \frac{10\dot{R}^2 f^2}{r^4 R^6} - \frac{3\dot{f}^2}{2r^4 R^4} + \frac{8\dot{R} f\dot{f}}{r^4 R^5} - \frac{f\ddot{f}}{r^4 R^4}. \tag{45}$$

Eliminating $\ddot{R}$ by adding (39) and (40), we get

$$\dot{R}^2(t) + b(r) = \frac{8\pi G}{3}\rho(r,t) R^2(t) + Q(r,t) R^2(t), \tag{46}$$

where

$$Q = \frac{1}{2} W - \frac{1}{6} Y. \tag{47}$$

From (40) we obtain

$$\ddot{R}(t) = -\frac{4\pi G}{3} R(t)[\rho(r,t) + 3p(r,t)] + \frac{1}{3} R(t) Y(r,t). \tag{48}$$

Let us consider an expansion of $f(r,t)$ about a background $f_0(r,t)$:

$$f(r,t) = f_0(r,t) + \delta f(r,t) + ... \tag{49}$$

We shall identify the fluctuations $\delta f(r,t)$ about the background $f_0(r,t)$ as any matter content additional to the visible baryon matter of the universe, $\rho_m(r,t) = \rho(\delta f(r,t))$. Thus, ρ_m replaces the exotic cold dark matter (CDM) of the standard cosmological model. We also have the expansions

$$W = W_0 + \delta W + ...; \quad Y = Y_0 + \delta Y + ...;$$

$$Q = Q_0 + \delta Q + ... \tag{50}$$

We define the matter density to be

$$\rho_M = \rho_b + \rho_m, \tag{51}$$

where ρ_b denotes the baryon density. The background field f_0 will describe the source of dark energy density and we shall consider slowly varying solutions of f_0, which rise towards the present epoch with red shift $z \sim 0$ and a solution of δf that yields a $\rho_m = \rho(\delta f)$ that decreases with increasing time as $1/R^3$.

We can now write in place of (46) and (48):

$$\dot{R}^2(t) + b(r) = \frac{8\pi G}{3}\rho_M(r,t)R^2(t) + Q_0(r,t)R^2(t), \tag{52}$$

and

$$\ddot{R}(t) = -\frac{4\pi G}{3}R(t)[\rho_M(r,t) + 3p_M(r,t)] + \frac{1}{3}R(t)Y_0(r,t), \tag{53}$$

where $Q_0(r,t) = Q(f_0(r,t))$ and $Y_0(r,t) = Y(f_0(r,t))$.

We can write Eq.(52) as

$$H^2 + \frac{b}{R^2} = \Omega H^2, \tag{54}$$

where $H = \dot{R}/R$,

$$\Omega = \Omega_M + \Omega_{f_0}, \tag{55}$$

and

$$\Omega_M = \frac{8\pi G\rho_M}{3H^2}, \quad \Omega_{f_0} = \frac{Q_0}{H^2}. \tag{56}$$

From Eq.(41) for $h = 1$, we obtain $b = 0$ and $\Omega = 1$ and

$$H^2 = \frac{8\pi G}{3}\rho_M + Q_0, \tag{57}$$

which describes a spatially flat universe. The line element takes the approximate form of a flat, homogeneous and isotropic FRW universe

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{58}$$

4 Accelerating Expansion of the Universe

Let us assume that there exists a solution of the field equations such that asymptotically for large r and t:

$$Q_0'(r,t) \sim 0, \quad Y_0'(r,t) \sim 0, \tag{59}$$

and

$$\rho_M(r,t) \to \rho_M(t), \quad p_M(r,t) \to p_M(t). \tag{60}$$

Here, we choose the big bang to begin on a hypersurface $\Sigma(r,t)$ with $r = t = 0$, so that asymptotically for large r and t we approach the present universe.

It follows from (53) that $\ddot{R} > 0$ when $Y_0 > 4\pi G(\rho_M + 3p_M)$. If we assume that there exist solutions for $Q_0(r,t)$ and $Y_0(r,t)$, such that they are sufficiently small in the early universe, then we will retain the good agreement of Einstein gravity with the BBN era with $\rho_{\rm rad} \propto 1/R^4$. As the universe expands beyond the BBN era at the temperatures, $T \sim 60$ kev-1 MeV, and $\rho_M \propto 1/R^3$, then we must seek solutions such that Q_0 and Y_0 begin to increase and reach slowly varying values with

$\Omega^0_{f_0} \sim 0.7$ and $\Omega^0_M \sim 0.3$, where Ω^0_M and $\Omega^0_{f_0}$ denote the present values of Ω_M and Ω_{f_0}, respectively. Provided we can find solutions of the field equations that satisfy these conditions, then it should be possible to fit the combined supernovae, cluster and CMB data [2].

We observe from (44) and (45) that Q_0 and Y_0 are functions of the behavior of R and f and their derivatives. If f_0 grows sufficiently with R as t increases, then Q_0 and Y_0 can dominate the matter contribution ρ_M as the universe evolves towards the current epoch with $p_M \approx 0$. A detailed solution of the field equations is required to determine the dynamical behavior of R, f, Q and Y.

In the present epoch, $p_M \approx 0$ and (53) gives

$$\frac{\ddot{R}}{R} = -\frac{1}{3}[4\pi G\rho_M - Y_0]. \tag{61}$$

We can now define an effective equation of state parameter for the universe:

$$w_{\rm eff} = \frac{1}{3}(4\pi G\rho_M - Y_0). \tag{62}$$

We see that $Y_0 > 4\pi G\rho_M$ corresponds to the usual condition for acceleration $w_D < -1/3$ with the dark energy equation of state $p_D = w_D\rho_D$.

We can explain the evolution of Hubble expansion acceleration within NGT, without violating the positive energy conditions for matter and radiation. We satisfy the strong energy condition for matter $\rho_M + 3p_M > 0$ throughout the evolution of the universe and *there is no need for a cosmological constant*. The Q_0 and Y_0 contributions to the expansion of the universe increase at a slow rate up to values today with $Y_0 > 4\pi G\rho_M$ ($p_M \approx 0$). The cosmological constant $\Lambda = 0$ during this evolution.

We must also guarantee that the influence of the Y and Q contributions do not conflict with galaxy formation, i.e. the additional NGT contributions to the Friedmann equation must not couple too strongly with the attractive gravitational effects predicted by the field equations in the galaxy formation epoch. This issue and the other required evolutionary effects of Q and Y must be determined by a numerical computation of the NGT field equations.

5 Conclusions

We have proposed that NGT may explain, as gravitational phenomena, the accelerating universe without unknown dark energy, and the observed flat rotation curves of galaxies without the undetected exotic dark matter [20]. We do expect that there is some dark matter in the universe in the form of dark baryons and neutrinos with non-vanishing mass. Such a theory can be falsified with data, whereas it is difficult to falsify the dark energy and dark matter hypotheses.

NGT would be required to explain the formation of galaxy structure without CDM. We know that the standard ΛCDM model of structure formation and the description of the CMB power spectrum is remarkably successful, so that solutions of the NGT equations must produce an equally successful description of the data. To investigate in detail whether NGT can succeed in producing a successful account of the large-scale structure of the universe will require solving the field equations. These are issues that require further investigation.

6 Appendix A: The Γ-Connections and Field Equations

The NGT compatibility equation is given by

$$g_{\lambda\nu,\eta} - g_{\rho\nu}\Gamma^{\rho}_{\lambda\eta} - g_{\lambda\rho}\Gamma^{\rho}_{\eta\nu} = \frac{1}{6}g^{(\mu\rho)}(g_{\rho\nu}g_{\lambda\eta} - g_{\eta\nu}g_{\lambda\rho} - g_{\lambda\nu}g_{[\rho\eta]})W_\mu, \qquad (63)$$

where W_μ is determined from (26). When $w(r,t) = g_{[01]}(r,t) = 0$, it follows that $W_\mu = 0$ and the compatibility equation reads

$$g_{\lambda\nu,\eta} - g_{\rho\nu}\Gamma^{\rho}_{\lambda\eta} - g_{\lambda\rho}\Gamma^{\rho}_{\eta\nu} = 0. \qquad (64)$$

The non-vanishing components of the Γ-connections are:

$$\Gamma^1_{11} = \frac{\alpha'}{2\alpha}, \qquad (65)$$

$$\Gamma^1_{(10)} = \frac{\dot\alpha}{2\alpha}, \qquad (66)$$

$$\Gamma^1_{22} = \Gamma^1_{33}\mathrm{cosec}^2\theta = \frac{1}{2\alpha}\left(fB - \frac{1}{2}\beta A'\right), \qquad (67)$$

$$\Gamma^1_{00} = \frac{\gamma'}{2\alpha}, \qquad (68)$$

$$\Gamma^2_{(12)} = \Gamma^3_{(13)} = \frac{1}{4}A', \qquad (69)$$

$$\Gamma^2_{(20)} = \Gamma^3_{(30)} = \frac{1}{4}\dot A, \qquad (70)$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta, \qquad (71)$$

$$\Gamma^3_{(23)} = \cot\theta, \qquad (72)$$

$$\Gamma^0_{(11)} = \frac{\dot\alpha}{2\gamma}, \qquad (73)$$

$$\Gamma^0_{(10)} = \frac{\gamma'}{2\gamma}, \qquad (74)$$

$$\Gamma^0_{22} = \Gamma^0_{33}\mathrm{cosec}^2\theta = -\frac{1}{2\gamma}\left(fD - \frac{1}{2}\beta\dot A\right), \qquad (75)$$

$$\Gamma^0_{00} = \frac{\dot\gamma}{2\gamma}, \qquad (76)$$

$$\Gamma^1_{[23]} = \frac{\sin\theta}{2\alpha}\Big(\frac{1}{2}fA' + \beta B\Big), \tag{77}$$

$$\Gamma^2_{[13]} = -\Gamma^3_{[12]}\sin^2\theta = \frac{1}{2}B\sin\theta, \tag{78}$$

$$\Gamma^2_{[30]} = -\Gamma^3_{[20]}\sin^2\theta = -\frac{1}{2}D\sin\theta, \tag{79}$$

$$\Gamma^0_{[23]} = -\frac{\sin\theta}{2\gamma}\Big(\frac{1}{2}f\dot{A} + \beta D\Big), \tag{80}$$

where A, B and D are given by

$$A = \ln(\beta^2 + f^2), \tag{81}$$

$$B = \frac{f\beta' - \beta f'}{\beta^2 + f^2}, \tag{82}$$

and

$$D = \frac{\dot{\beta}f - \dot{f}\beta}{\beta^2 + f^2}. \tag{83}$$

The NGT field equations in the presence of sources are given by

$$R_{11}(\Gamma) = -\frac{1}{2}A'' - \frac{1}{8}[(A')^2 + 4B^2] + \frac{\alpha' A'}{4\alpha} + \frac{\gamma'}{2\gamma}\Big(\frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma}\Big)$$
$$-\Big(\frac{\gamma'}{2\gamma}\Big)' + \frac{\partial}{\partial t}\Big(\frac{\dot{\alpha}}{2\gamma}\Big) + \frac{\dot{\alpha}}{2\gamma}\Big(\frac{\dot{\gamma}}{2\gamma} - \frac{\dot{\alpha}}{2\alpha} + \frac{1}{2}\dot{A}\Big) + \Lambda\alpha - \frac{1}{4}\mu^2\frac{\alpha f^2}{\beta^2 + f^2}$$
$$= 4\pi G\alpha(\rho - p + 2fK), \tag{84}$$

$$R_{22}(\Gamma) = R_{33}(\Gamma)\mathrm{cosec}^2\theta = 1 + \Big(\frac{2fB - \beta A'}{4\alpha}\Big)' + \Big(\frac{2fB - \beta A'}{8\alpha^2\gamma}\Big)(\alpha'\gamma + \gamma'\alpha)$$
$$+\frac{B(fA' + 2\beta B)}{4\alpha} - \frac{\partial}{\partial t}\Big(\frac{2fD - \beta\dot{A}}{4\gamma}\Big) - \frac{2fD - \beta\dot{A}}{8\alpha\gamma^2}(\dot{\alpha}\gamma + \dot{\gamma}\alpha)$$
$$-\frac{D}{4\gamma}(f\dot{A} + 2\beta D) + \Lambda\beta + \frac{1}{4}\mu^2\frac{\beta f^2}{\beta^2 + f^2} = 4\pi G\beta(\rho - p + 2fK), \tag{85}$$

$$R_{00}(\Gamma) = -\frac{1}{2}\ddot{A} - \frac{1}{8}(\dot{A}^2 + 4D^2) + \frac{\dot{\gamma}}{4\gamma}\dot{A} + \frac{\dot{\alpha}}{2\alpha}\Big(\frac{\dot{\gamma}}{2\gamma} - \frac{\dot{\alpha}}{2\alpha}\Big)$$
$$-\frac{\partial}{\partial t}\Big(\frac{\dot{\alpha}}{2\alpha}\Big) + \Big(\frac{\gamma'}{2\alpha}\Big)' + \frac{\gamma'}{2\alpha}\Big(\frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma} + \frac{1}{2}A'\Big)$$
$$-\Lambda\gamma + \frac{1}{4}\mu^2\frac{\gamma f^2}{\beta^2 + f^2} = 4\pi G\gamma(\rho + 3p - 2fK), \tag{86}$$

$$R_{[10]}(\Gamma) = 8\pi GK_{[10]} = 0, \tag{87}$$

$$R_{(10)}(\Gamma) = -\frac{1}{2}\dot{A}' + \frac{1}{4}A'\Big(\frac{\dot{\alpha}}{\alpha} - \frac{1}{2}\dot{A}\Big) + \frac{1}{4}\frac{\gamma'\dot{A}}{\gamma} - \frac{1}{2}BD = 0, \tag{88}$$

$$
\begin{aligned}
R_{[23]}(\Gamma) = \sin\theta\Big[\Big(\frac{fA' + 2\beta B}{4\alpha}\Big)' + \frac{1}{8\alpha}(fA' + 2\beta B)\Big(\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma}\Big) \\
-\frac{B}{4\alpha}(2fB - \beta A') - \frac{1}{8\gamma}(f\dot{A} + 2\beta D)\Big(\frac{\dot{\gamma}}{\gamma} + \frac{\dot{\alpha}}{\alpha}\Big) \\
-\frac{\partial}{\partial t}\Big(\frac{f\dot{A} + 2\beta D}{4\gamma}\Big) + \frac{D}{4\gamma}(2fD - \beta\dot{A})\Big] \\
-\Big[\Lambda f - \frac{1}{4}\mu^2 f\Big(1 + \frac{\beta^2}{\beta^2 + f^2}\Big)\Big]\sin\theta = -4\pi G f \sin\theta(\rho - p). \qquad (89)
\end{aligned}
$$

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