1 Cosmology

When Ymir lived long ago
Was no sand or sea, no surging waves.
Nowhere was there earth nor heaven above.
But a grinning gap and grass nowhere.
Voluspa,[1] about 1000 AD

1.1 General principles

Cosmology is a science about the Universe as a whole. In a wider meaning, especially in the historical context, there are included into the cosmology also the philosophical problems of the relations between human beings and the world. However, here we will restrict the meaning of cosmology to the physical cosmology, which is based on astronomical observations and physical experiments and theories. This does not mean that we can ignore in the cosmology either the existence of life or the problems of the mind and the process of the recognition of the Universe. The later indicate the limitations of the contemporary cosmological theories and the former gives some restrictions to physically plausible models via the so-called *anthropic principle*, according to which from the variety of all physically possible models of the Universe there are acceptable only those, which enable the rise of human beings.[2] In this sense, the cosmology studies on the most general level the conditions for the cosmobiology, which, in turn, can be treated as one of its observational tests.

The definition of the subject of cosmology indicates its peculiarity between the natural sciences: it deals with (by definition) a single object; it is observational and theoretical but by no means experimental science; in principle, it comprises all other sciences. Another feature of cosmology (which is to some extent common to other empirical sciences also) is the interlink between the observation and the theory: the theory should be based on the results of observations, however, the correct interpretation of the observations requires the proper theory.

The range of the space studied by the cosmology was growing throughout its historical evolution proportionally to the size of the region reachable by the contemporary observational techniques. The experience from these observations is then usually extrapolated to the space (and time) beyond these limits. In this sense, it may seem to be exaggerated to take the results of cosmology at any stage as a given model of the whole Universe. However, modern physics yields the insight into the laws of nature which allows us to treat the results as more reliable than a mere assumption. Especially the general theory of relativity, which gives a close connection between the geometry of the space-time and its matter content, allows us to judge (with limitations which are obvious) that our present observations are approaching the limits of the part of the Universe, which in principle is observable.

To be able to extrapolate to the whole Universe the results obtained from the astronomically observable part of it (the so-called *Meta-galaxy*) there is used an assumption named the *Copernican principle*. This can be formulated as: "The Earth does not have a privileged position in the Universe." This principle is often replaced by a more restrictive *cosmological principle*, according to which: "The Universe is (when averaging over a sufficiently large area) homogeneous and isotropic."

The above formulation (as well as the name) of the Copernican principle reflects the negation of the former anthropocentric views on the Universe and should be understood in a more general sense that there is no special position (body) in the Universe at all, but for each one there exists another one which is more or less equivalent.[3] The cosmological principle could be (with many obstacles)

[1] Translated by W.H. Auden and P.B. Taylor.

[2] This condition of existence of an observer is thus a selection effect. Intuitively, the assumption of its applicability is the existence of many universes, only some of which satisfy the conditions required for the rise of life. A possibility of spontaneous creation of such separated space-time regions as a quantum fluctuation in a larger physical space is investigated in the framework of the quantum field theory.

[3] In the discussion of the conditions for life we shall see that the position of the Earth satisfies several special, but not exceptional conditions.

tested observationally. The results of such studies are not conclusive, so we have to take it as a conjecture which implies the simplest cosmological models.

Together with the assumption of the homogeneity of the Universe in time,[4] the cosmological principle leads to inner contradictions, the so-called *cosmological paradoxes.* The best known is the *Kepler – Olbers* paradox (named also *the dark sky paradox*), according to which the whole sky should have the mean brightness of the surface of a star.[5] This conclusion can be understood from the viewpoint of thermodynamics as the thermodynamic equilibrium of surfaces of stars with the interstellar radiation.[6] The Universe should approach this equilibrium (the so-called *thermal death of the Universe*) also in the case of violation of the homogeneity in time, like it would be in the case of rise of stars in a finite past.[7]

One possibility to solve the Kepler – Olbers paradox is by the so-called *hierarchical cosmologies* suggested by I. Kant. According to such theories, the matter is concentrated into spatially isolated clusters of different orders in such a way, that each cluster of a given order is an element of a higher order cluster.[8] An example is the hierarchy atom – star – galaxy – cluster of galaxies. The mean density of mass is decreasing with the increasing order of the cluster. Hierarchical models satisfy the Copernican principle, but violate the cosmological principle. In nature, there may exist an even more general fractal structure of clustering, which includes both the homogeneous distribution as well as the clustering, not only on integer but a continuous set of orders. Moreover, this generalized hierarchical structure need not be fixed, but it can be subjected to an evolution in time.

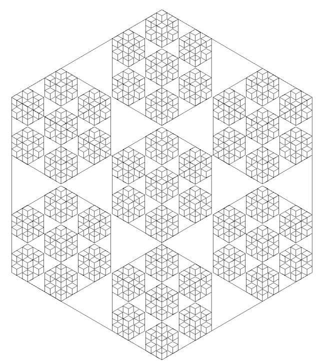

Figure 1: Four orders of 3-dim. Cantor discontinuum (the empty central part of each interval is slightly bigger than $\frac{1}{3}$ in the figure).

[4]If a violation of basic physical principles of conservation is allowed, the static state of the Universe must be assumed as an additional condition to its time homogeneity.

[5]The reason is that the intensity of light is constant along the ray and each ray should intersect the disc of some star, because the solid angle spanned by a typical star decreases with the square of its distance, while the number of stars in a spherical shell of constant thickness increases in the same rate. This argument was first used by J. Kepler in his *Disertatio cum Sidereus nuncius* against Galileo's speculations about the infinite Universe of stars concluded from his discovery of stellar consistence of the Milky Way. Just Kepler formulated some time before the law of decrease of the radiative flux with the square of distance. This law was an inspiration for Newton's gravitation law, from which, however, an analogous gravitational paradox can be constructed. Olbers rediscovered the optical paradox as an analogue to the gravitational one.

[6]The obscuration of distant stars, which is really observed, seemed to be the solution of the dark sky paradox. However, the thermodynamic arguments show that the cold clouds should be heated soon and start glowing.

[7]Such an explanation is sometimes claimed to be an alternative explanation to the Big-bang model. However, without the Big-bang, the synchronous ignition of stars in a finite past is unphysical and only temporarily helping, while in the Big-bang models the thermodynamic equilibrium was originally valid with the radiative temperature gradually decreasing, and only later a rise of stars contributed to a reheating of the intervening matter.

[8]A mathematical model of such a configuration is the set of points, the coordinates of which can be written in the form $x = \sum_{i=-\infty}^{\infty} c_i 10^i$, where $c_i = +1, 0, -1$. The same configuration of the points around the interval $\langle -1, 1\rangle$ is then repeated aground $\langle 9, 11\rangle$, but there is an empty space in $\langle 2, 8\rangle$. The set is self-similar on all decimal scales. Another example is the Cantor discontinuum, which is self-similar on the scales of integer powers of 3, and on each scale the central third of the characteristic interval is empty.

1.2 Relativistic cosmology

A qualitatively new insight into cosmology has been opened by general relativity which replaced the study of distribution and motion of matter in a fixed Euclidean space by a non-Euclidean geometry of the space-time. The matter (either particles or fields) moves in a curved space-time like in a flat Minkowskian space-time which is its locally tangent space (e.g. free particles move on geodetics) but at the same time the stress-energy tenzor of the matter produces the curvature of the space-time according to the Einstein field equations[9]

$$\mathcal{R}_{\iota\kappa} - \frac{1}{2} g_{\iota\kappa} \mathcal{R} - \Lambda g_{\iota\kappa} = 8\pi G T_{\iota\kappa} \ . \tag{1}$$

This so-called geometrodynamics thus mutually causally connects the space-time geometry with the distribution and motion of the mass and enables thus the cosmology to study the whole space of the Universe as an attribute of the matter.[10] A.A. Friedmann and W. Robertson showed that following the cosmological principle, the Universe must have in a fixed time the geometry of space with constant curvature (the so-called *Friedmann – Robertson – Walker models* of the Universe).[11] The curvature of the space can be either positive or zero or negative, and can be characterized by the index of curvature $k = +1, 0, -1$, accordingly. The case $k = 0$ corresponds to the Euclidean geometry of flat space (the circumference of any circle with radius r reads $o = 2\pi r$; the so called *Einstein – de Sitter model*). The case $k = -1$ corresponds also to an infinite (open) universe, however, the geometry of the space is that of a saddle surface ($o > 2\pi r$). Finally, in the case $k = +1$ the space has the geometry of a supersphere ($o < 2\pi r$) and has a finite volume (it is closed; both cases $k = \pm 1$ are called *Friedmann – Lemaître models*) – see Fig. 2. If we introduce generalized spherical coordinates

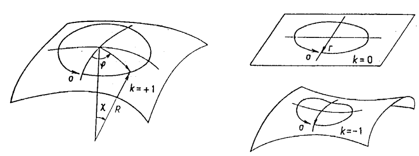

Figure 2: The geometry of homogeneous isotropic spaces.

$\{\chi, \vartheta, \varphi\}$ in the space, the metric of the space-time will be given by the space-time interval[12]

$$ds^2 = c^2 dt^2 - R^2(t)[d\chi^2 + \Sigma^2(\chi)(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] \ , \tag{2}$$

where

$$\Sigma(\chi) = \begin{cases} \sin\chi \\ \chi \\ \sinh\chi \end{cases} , \quad \text{for} \quad k = \begin{cases} +1 \\ 0 \\ -1 \end{cases} , \tag{3}$$

and R is a time-dependent radius of curvature of space (or a scaling factor in the case $k = 0$). From the Einstein field equations, there follow equations of motion for the geometry of space (characterized by R) and for the density of mass

$$\frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} - \Lambda\frac{c^2}{3} = \frac{8\pi G}{3}\rho \ , \tag{4}$$

[9] In units $c = 1$, $\mathcal{R}_{\iota\kappa}$ are the components of Ricci tenzor – for details see the lectures on general relativity.

[10] In this respect the general relativity satisfies the *Mach's principle*, according to which the inertial systems are priviledged owing to their non-accelerated motion with respect to the distribution of mass in the Universe.

[11] It means that the four-dimensional space-time, which according to the special relativity is locally invariant to the Lorentz transformation, can be sliced by a special choice of the proper time t of cosmic matter into a set of three-dimensional spaces, which are homogeneous and isotropic, and their geometry has the same curvature everywhere in space and is dependent on t only.

[12] The same space-time geometry is used to be alternatively expressed also in other coordinate systems – cf. exercise 1.

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} - \Lambda c^2 = -\frac{8\pi G}{c^2} P \ , \tag{5}$$

where G is the gravitational constant and c is the speed of light. The components of the stress-energy tensor of the matter in the universe, the mass density ρ, and the pressure P, are functions of time t only, and there follows for them from Eqs. (4) and (5) the equation of continuity of mass-energy,[13]

$$\frac{d}{dt}(\rho c^2 R^3) + P\frac{dR^3}{dt} = 0 \ . \tag{6}$$

The first term can be interpreted as the change in unit time of the total energy of the mass contained in the space, while the second one is the work exerted by the pressure due to the change of the volume of the Universe. The term proportional to the *cosmological constant* Λ was introduced by Einstein into his gravitational law to enable the existence of a stationary solution ($\dot{R} = 0$) of equations (4) and (5). The value of the cosmological constant $\Lambda > 0$ represents, in fact, a repulsive force which could hold the Universe apart against the attractive gravitational force. However, just the Friedmann dynamical models enable us to explain the cosmological paradoxes and they give several theoretical predictions, which are confirmed by observations.

Exercise 1 *Find the form of Eq. (2) in the case that the radial space coordinate χ is replaced by another coordinate ρ by substitution $\chi = \chi(\rho)$. Find the special cases of this substitution and the corresponding resulting metrics which lead to*

$$ds^2 = c^2dt^2 - R^2(t)[f(\rho)d\rho^2 + \rho^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] \ , \tag{7}$$

or to the conformly flat coordinates

$$ds^2 = c^2dt^2 - R^2(t)f(\rho)[d\rho^2 + \rho^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] \ . \tag{8}$$

Exercise 2 *Prove that Eq. (6) follows from Eqs. (4) and (5).*

[13] This equation is a special case of the general equation of continuity, $T^{\iota\kappa}{}_{;\kappa}$, which follows from the Einstein field equations (1).

1.3 Newtonian cosmology

The expansion of the Universe and some of its properties can be understood in the framework of non-relativistic mechanics – the so-called *Newtonian cosmology*. On the surface of a homogeneous sphere with radius r and density ρ (and, consequently, the mass $M = \frac{4}{3}\pi r^3 \rho$) is acting the gravity acceleration

$$g = \frac{GM}{r^2} = \frac{4}{3}\pi G \rho r \qquad (9)$$

towards the centre of the sphere. Any thought sphere of radius r_0 in an infinite homogeneous medium

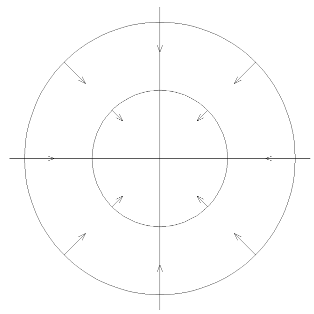

Figure 3: Gravitational collapse of infinite homogeneous medium.

(which is initially at time t_0 in rest and which is not held by any repulsive force) must thus collapse ($r = r(t, r_0)$) with the acceleration of the surface $\ddot{r} = -g \sim r_0$. Consequently, also the velocity $v \equiv \dot{r}$ must be proportional to r_0 in any time, i.e. it satisfies the Hubble's relation

$$v = Hr \; , \qquad (10)$$

where the so-called *Hubble constant* H is independent on the distance r, but generally it is a function of time. Obviously, the relative motion of any two elements of the medium is independent on the choice of the center of the spheres.

Because the mass of the sphere is constant, its density is increasing with the time, $\rho = \rho(t) \sim r^{-3}(t)$, but it remains homogeneous. The kinetic energy per unit mass of the relative motion of the surface with respect to the centre will be

$$E_{\mathrm{kin}} = \frac{1}{2}\dot{r}^2 = E - V = E + \frac{4}{3}\pi G \rho r^2 \; , \qquad (11)$$

where E is the total energy which is constant and $V = -GM/r$ is the potential energy, which is getting more and more negative in the course of the collapse. It is obvious from the analogy with the vertical motion in a gravitational field that for $E < 0$, there must precede an expansion (from $r = 0$ to $r = r_{\mathrm{max}}$) to the contraction (back to $r = 0$ in finite time), and for $E \geq 0$ there are two possible solutions: a contraction from $r = \infty$ at $t = -\infty$ to $r = 0$ in finite t, or an expansion from $r = 0$ in finite t to $r = \infty$ in $t = \infty$ – cf. Tab. 1 and Fig. 4. The critical case $E = 0$ gives a boundary between the solutions of infinite expansion (or contraction from infinity) and the expansion deccelerated and turned back into a collapse. These types of dynamics can be distinguished by comparing the relation between the instantaneous value of Hubble constant and the density. Alternatively, the density can be determined by the rate of the gravitational deceleration, which is usually expressed in terms of the dimension-less so-called *deceleration parameter*

$$q = -\frac{\ddot{r}r}{\dot{r}^2} \; , \qquad (12)$$

Comparing equations (11) and (4) for $\Lambda = 0$, one can see that the radius r of any sphere in the Newtonian cosmology satisfies the same equation of energy conservation as the radius R of the

Universe in the Friedmann model where the term $-\frac{1}{2}c^2 k$ plays the role of the energy E. In the case of dust with $P = 0$ (or at least $P \ll \rho c^2$), the Newtonian cosmology gives the same dynamics of the radius $R = R(t)$ of the Universe as the Friedmann models.

Obviously, for $r(t) \to 0$ the potential energy of the sphere $V \to -\infty$ and the constant energy of the motion is neglegible ($E \ll |V|$) on the right-hand side of Eq. (11), and consequently $\dot{r} \sim r^{-1/2}$ and $r^{3/2} \sim t$. This is valid precisely for all r in the critical case, corresponding to the relativistic case with $k = 0$, $P = 0$ and $\Lambda = 0$.

Exercise 3 *Find the relation between the type of the dynamics of the Newtonian model and the instantaneous values of* H *and* ρ *or* q*.*

Exercise 4 *Find the age of Universe in terms of the instantaneous value of* H *for the critical case* ($E = 0$) *and for the extreme case of zero density.*

1.4 Dynamics of the Universe

From the non-relativistic point of view, the dynamics of the Universe, i.e. the function $R = R(t)$ in the metric (2) should not be influenced by any value of the pressure P, because, due to the homogeneity of the space, there is a zero gradient of P, and it thus cannot exert any force on the motion of the mass. However, from the point of view of general relativity, the pressure as a component of the stress-energy tensor is also a source of a gravitational field which according to the difference between Eqs. (4) and (5)

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + \frac{3}{c^2}P) + \Lambda\frac{c^2}{3} , \tag{13}$$

will contribute to the acceleration of $R(t)$.[14] The dynamics of the Universe is thus influenced by the matter (and energy) content of the Universe and its corresponding equation of state.

For example in the case of a coherent dust[15] which agrees with the above given Newtonian cosmology

$$P_{\text{mat}} = 0 \qquad \Rightarrow \qquad \rho_{\text{mat}} \sim R^{-3} \qquad (R \sim t^{\frac{2}{3}}) . \tag{14}$$

However, if the Universe was filled by a photon gas (i.e. radiation) only, or by any other ultrarelativistic gas, then

$$P_{\text{rad}} = \frac{1}{3}c^2\rho_{\text{rad}} \qquad \Rightarrow \qquad \rho_{\text{rad}} \sim R^{-4} \qquad (R \sim t^{\frac{1}{2}}) . \tag{15}$$

In both cases ρ can be integrated from Eq. (6) in the form of a power dependence on R. The terms with k and Λ are thus negligible for $R \to 0$ compared to the right hand side of Eq. (4) and the solution $R(t)$ can then be asymptotically approximated by the simple power-law expressions given in the parentheses. For a mixture of both gases, the Universe is dominated by the radiation ($\rho_{\text{rad}} > \rho_{\text{mat}}$) for a sufficiently small R, and its dynamics thus obeys Eq. (15). For larger values of R at a later epoch, the Universe is dominated by mass ($\rho_{\text{mat}} > \rho_{\text{rad}}$), and its dynamics thus satisfies (14). At even later epochs the term with the curvature becomes important, and the dynamics of the Universe depends on the value of k as can be seen in Tab. 1 and Fig. 4. These solutions start from a singularity ($R = 0$, $\rho = \infty$) and either re-collapse again in the case of a closed Universe or expand forever in the case of an opened Universe.

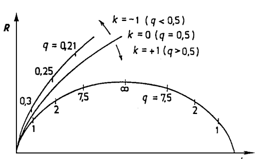

Figure 4: The dynamics of Friedmann models with zero pressure and cosmological constant.

It is important to note that the presence of a relativistic matter in the Universe dominates the equation of state and also the dynamics of the Universe near the singularity. Following the Stefan-Boltzmann law, the temperature for the radiation is determined by the energy density, which varies with the radius of the Universe,

$$\rho_{\text{rad}}c^2 = \sigma T^4 \sim R^{-4} . \tag{16}$$

It thus means that the temperature of the radiation $T \sim R^{-1}$ and consequently $T \to \infty$ near the singularity. This is why the singularity is called the *hot big bang*.

[14] The influence of the pressure on the dynamics of the Universe can also be understood as a result of the conservation laws (6), which are a consequence of the Einstein field equations, because, due to the equivalence of mass and energy, the work exerted by the pressure will change the dependence $\rho = \rho(R)$.

[15] And approximately also for any classical non-relativistic gas because then $P \ll \rho c^2$.

$k=+1$	$R=R_0(1-\cos\eta)$ $t=t_0(\eta-\sin\eta)$
$k=0$	$R=R_0(t/t_0)^{2/3}$
$k=-1$	$R=R_0(\cosh\eta-1)$ $t=t_0(\sinh\eta-\eta)$

Table 1: The dynamics of the Friedmann models with zero pressure and cosmological constant.

Owing to the strangeness of the physical conception of the initial singularity, it became temporarily popular to apply the stationary cosmological models (the so-called steady state models by H. Bondi, T. Gold and F. Hoyle), according to which equation (6) of continuity of mass is violated by a spontaneous creation of mass everywhere in the Universe, with the rate just needed to balance the decreasing mass density due to the cosmological expansion. However, these models which are homogeneous also in time, do not agree with the results of the statistics of distribution of galaxies at large distances (i.e., with their density in the past), and they were completely abandoned after the discovery of the *cosmic background radiation* which also confirms a high temperature in the earlier epochs of the evolution of the Universe.

Following the Grand-unification theories (GUT), the equation of state $P=-\rho c^2$ could be valid during the very early epoch (the so-called *inflationary epoch*) of the evolution of the Universe. Consequently, following Eq. (6),

$$P=-\rho c^2 \qquad \Rightarrow \qquad \rho = constant \qquad (R\sim \mathrm{e}^{Ht})\ . \tag{17}$$

The work exerted by the expansion of the Universe against the negative pressure thus could in these circumstances really produce a mass balancing the decrease of its density due to the expansion, and it thus could temporarily mimic the effect of the cosmological constant.

Exercise 5 *Verify that the solutions given in Tab. 1 satisfy Eqs. (4) and (5) and find the proper values of the free parameters.*

1.5 Observational cosmology

The contemporary physical theories admit a large variety of theoretical models of the Universe. It is thus necessary to verify them and to choose between them by comparison of the observed properties of the Universe with theoretical predictions of different models.

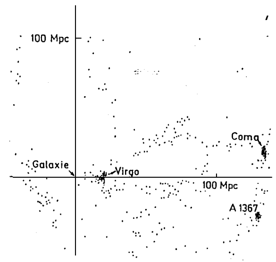

Figure 5: Large-scale structure of the Universe.

The basic observational test is the study of the distribution of discrete objects (e.g., galaxies and quasars). The statistics of the angular distribution (which is complicated due to partial screening by our Galaxy) as well as the distribution of objects at different distances r does not, generally, contradict the assumption of the homogeneity of the Universe. However, there are observed local condensations on different scales. Their existence imposes an additional constraint on the theoretical models. The largest formations presently known form the so-called *large-scale structure* resembling a foam with *large voids* with the size of the order of 100 Mpc – see Fig. 5. The luminous matter is concentrated into some walls (the so-called *pancakes*) between the voids and their cross-sections (threads) and their knots. These structures can be statistically described in terms of fractal dimensions

$$D = \frac{d\ln N}{d\ln r} = \frac{r}{N}\frac{dN}{dr} \ , \tag{18}$$

where r is the radius of a sphere around a chosen origin (the position of the Earth), and $N = N(r)$ is the number of the discrete sources contained in this sphere. Obviously, for the homogeneous distribution of sources in flat space $D = 3$, while in a planar structure $D = 2$, in a linear structure $D = 1$ and for an isolated condensation $D = 0$.[16]

The statistics of the distribution in r excludes the stationary models (both the static as well as the steady state). However, the non-stationarity of the Universe complicates the determination of the radial distances. For a relatively small neighbourhood of our observational site, the distance of a cosmic object can be found in the standard way of the Newtonian physics, e.g., from its angular size or its apparent brightness, provided that the proper size or intrinsic luminosity of the object are known (e.g., if they can be supposed to be equal in average for objects of the same kind – the objects are the so-called standard rods or standard candelas).[17]

At cosmological distances the relation of the magnitude of the observed quantities to the distance of the source is influenced by the geometry and dynamics of the Universe in a way which can be calculated depending on the cosmological model. It is obvious from Eq. (2) that the photons comming to the observer at $\chi = 0$ have $d\vartheta = 0 = d\varphi$ and hence the proper length D of a standard rod (oriented perpendicularly to the line of sight e.g. in direction of the coordinate ϑ) at event $\{t, \chi\}$ seen under the angle $\Delta\vartheta$ is

$$D = R(t)\Sigma(\chi)\Delta\vartheta \ , \tag{19}$$

[16] The results of most statistical studies indicate that D approaches the value of 3 at large scales, some observers argue that it is rather close to 2.

[17] In the static Euclidean space-time it would be the simple relation of inverse proportionality or inverse square dependence, respectively.

from where the distance can be found.[18]

In a simillar way the solid angle spanned by the source dicreases with the square of the angular distance. In addition, the energy of each photon radiated by the object is redshifted by the cosmological expansion by the factor $(1+z)$, as will be shown later, and in the same portion is increased the time-interval, during which the photons were emited. Consequently, the luminosity distance d_L defined by the ratio of the apparent luminosity L_{app} of the object and its intrinsic luminosity L_{int} (as seen from an unit distance) by $L_{app} = L_{int}/d_L^2$ is given by

$$d_L = d_A(1+z) = R(t)\Sigma(\chi)(1+z) \ . \tag{20}$$

Owing to the non-stationarity of the Universe there arises in practice a problem to find at different distances (and consequently in different ages of the Universe) mutually comparable sources and to exclude their evolutionary effects. More tricky physical processes must thus be used to set and verify the scale of radial distances (e.g. the gravitational lensing).

The most important confirmation of the dynamics of the Universe was done by Hubble's discovery of the redshift of spectral lines in the light from distant galaxies. It gives the evidence of the radial motions of objects with velocity v (approximately) proportional to their distance r, according Eq. (10) where the Hubble constant is determined by the function $R(t)$,

$$H = \frac{\dot{R}(t)}{R(t)} \ , \tag{21}$$

and it has observationally found value $H \simeq 75\text{km s}^{-1}\ \text{Mpc}^{-1}$. It means that, in fact, it is also a function of time. The redshift z of the wavelengths of spectral lines,

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} \ , \tag{22}$$

where $\lambda_{\rm em}$ and $\lambda_{\rm obs}$ are the emitted and observed wavelengths, resp., is then the result of subsequent Doppler increase of the wavelength of a photon during its travel from the source to the observer. The final value of the redshift depends on the ratio of how much the radius R of the Universe has increased in the course of the travel time of the photon,[19]

$$1 + z = \frac{R(t_{\rm obs})}{R(t_{\rm em})} \ . \tag{23}$$

The redshift z is used as a measure of the distance (and thus also the measure of the 'antiquity') of the distant sources. Its relation to the real actual distance r or the radial coordinate χ is determined by the function $R(t)$,

$$r = R(t_{\rm obs})\chi = R(t_{\rm obs}) \int_{t_{\rm em}}^{t_{\rm obs}} R^{-1}(t) c dt \ . \tag{24}$$

The actual velocity Hr of the recession (which, however, does not have any direct physical meaning) can be greater than c for objects with large z (e.g., in the Einstein – de Sitter universe for $z > 3$), and cannot be calculated from z by the special-relativistic expression for the Doppler effect.

The limit $r_{\rm h}$ of the expression (24) for $t_{\rm em} \to 0$, i.e. $z \to \infty$ is called *particle horizon.* This defines the most distant point ($\chi = \chi_{\rm h}$) from which any information could reach us (i.e. to reach the event $\chi = 0, t = t_{\rm obs}$) in the course of time from the beginning $t = 0$ of the Universe. The horizon $r_{\rm h}$

[18] Note that the so-called angular distance $d_A \equiv D/\Delta\vartheta = R(t)\Sigma(\chi) = R(t)\rho$, where rho is the radial coordinate used in the expression (7) for the metric, gives the circumference of a circle subtending the rod at the epoch of emission of the observed photons. The present size of the circle is rescaled due to the expansion factor $R(t)$. Moreover, the distance measured radially differs from d_A due to the curvature of the space (e.g. for $k = +1$ and $\chi > \pi/2$ the angular size of the rod is increasing with χ, what can be understood as a focussing of the radiation by the curvature of space).

[19] This can be easily seen from the equation $d\eta^2 = d\chi^2$ of a radially moving photon, where η is the so-called conformal time, in which Eq. (2) has the form $ds^2 = R^2(\eta)[d\eta^2 - d\chi^2 - \Sigma^2(\chi)(d\vartheta^2 + \sin^2\vartheta d\varphi^2)]$. Two world-lines of photons (or maxima of an electromagnetic wave) have constant diference in η and thus diference in time $\Delta t \sim R(t)$.

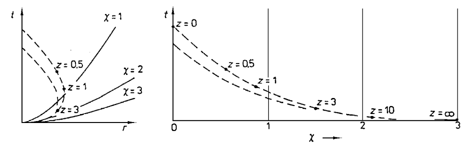

Figure 6: Past light-cone in physical and co-moving coordinates.

increases with the increasing time $t_{\rm obs}$. However, in some models exists a finite limit $\lim_{t_{\rm obs}\to\infty} r_{\rm h}$ (the so-called *event horizon*), hence the more distant regions of the Universe are causally separated forever. E.g. for a power-law expansion $R(t) = kt^\kappa$ (i.e. for both the matter- and radiation-dominated universe the comoving coordinate of the horizon reads[20]

$$\chi_{\rm h} = \frac{c}{k}\int t^{-\kappa}dt = \frac{ct}{(1-\kappa)R} , \tag{25}$$

while during an exponential expansion $R(t) = k\exp(Ht)$ between the epochs t_1 and t_2 the horizon increases for[21]

$$\Delta\chi_{\rm h} = \frac{c}{k}\int \exp(-Ht)dt = \frac{c}{H}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) . \tag{26}$$

Consequently, if $R_2 \gg R_1$, the final horizon after an initial power-law and subsequent exponential expansion

$$\chi_{\rm h} = \frac{ct_2}{(1-\kappa)R_2}\left[\frac{R_2t_1}{R_1t_2} + \frac{1-\kappa}{\log(R_2/R_1)}\left(1-\frac{t_1}{t_2}\right)\left(\frac{R_2}{R_1}-1\right)\right] , \tag{27}$$

is much larger than it would be for the power-law only.

In principle, the deceleration parameter q, which is in agreement with Eq. (12) expressed in terms of $R(t)$ as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} , \tag{28}$$

can be determined by measuring the non-linearities of the Hubble-relation $z = z(r)$. Neglecting in Eq. (5) the terms with P and Λ (which apparently really are negligible at the present epoch of the evolution of the Universe) we arrive at the relation

$$kc^2 = R^2H^2(2q-1) . \tag{29}$$

The geometry of the whole Universe can thus be determined from the value of q. The critical value ($k = 0$) of the curvature corresponds to $q_{\rm crit} = \frac{1}{2}$. The observed value of q coincides – within quite large observational errors – with $q_{\rm crit}$.

Similarly, according to Eq. (4), k could be determined from the present mean density of the Universe comparing it with the critical value

$$\rho_{\rm crit} = \frac{3H^2}{8\pi G} \simeq 10^{-26}{\rm kg.m}^{-3} . \tag{30}$$

The observed luminous matter of the galaxies gives a density approximately one or two orders of magnitude lower. However, it follows from the inner dynamics of galaxies as well as from the dynamics

[20]The horizon $R\chi$ is thus of the order of Gpc.

[21]Note that in the case of exponential expansion lasting forever the above defined horizon as the limit of the past light-cone is at infinity, because the model was asymptotically static. However, the distance c/H defines the most distant observers which can be reached by a future light-cone. It means that any two particles will be causally disconnected after they separate behind this limit.

of clusters of galaxies that there must be present also a great deal of the so called *dark matter* (or the hidden matter) which holds the galaxies bounded by its gravity. Up to now, the substance of this dark matter is unknown. It could consist of neutrinos if their rest mass would be of the order of a few eV or it could consist of some still unknown particles. From the point of view of observability of this matter can not be excluded some forms of non-luminous bodies from the normal baryonic matter either. The estimates of the total amount of the dark matter in the Universe are so uncertain, that we cannot determine the value of k.

The value of the deceleration parameter can be restricted by comparison of the observed age t of the Universe with the Friedmann models according to which it should be

$$t = \frac{1}{(2q-1)H}\left[\frac{q}{\sqrt{|2q-1|}} f\left(\frac{1-q}{q}\right) - 1\right], \quad \text{where } f(x) = \begin{cases} \arccos x & \text{for } x \leq 1 \\ \operatorname{arccosh} x & \text{for } x > 1 \end{cases} . \tag{31}$$

In the case $q = \frac{1}{2}$, i.e. $k = 0$, $t = \frac{2}{3}H^{-1} \simeq 8.7 \times 10^9$years (for $H = 75$km s^{-1} Mpc^{-1}), for $q > \frac{1}{2}$ the age t is even smaller. Following the radiometric chronology of meteorites and Earth and Moon rocks the lower limit of the age of the Solar system is about 4 or 5×10^9 years. Comparing the theoretical models of the stellar evolution with the observed HR-diagrams the age of some stellar clusters can be estimated to be up to 10 or 15×10^9years. It is an evidence in favour of either $k = -1$ or a smaller value of H.

Exercise 6 *Find the mean fractal dimension of the 3-dimensional Cantor discontinuum (cf. Fig 1).*

Exercise 7 *Find the distance to SMC by comparing the observed periods P (in days) and the magnitudes m of cepheids in the SMC, given in the following table:*

star	P	m	star	P	m	star	P	m
HV 2019	1.62	16.8	HV 1825	4.27	15.6	HV 847	27.5	13.8
HV 2035	2.00	16.7	HV 1903	5.13	15.6	HV 840	33.1	13.4
HV 844	2.24	16.3	HV 1945	6.46	15.2	HV 11182	39.8	13.6
HV 2046	2.57	16.0	HV 2060	10.2	14.3	HV 1837	42.7	13.1
HV 1809	2.82	16.1	HV 1873	12.9	14.7	HV 1877	50.1	13.1
HV 1987	3.16	16.0	HV 1954	16.6	13.8			

with the periods and absolute magnitudes M of cepheids in the Galaxy:

star	logP	M	star	logP	M	star	logP	M
SU Cas	0.29	-1.7	V 350 Sgr	0.71	-3.0	S Nor	0.99	-3.7
EV Sct	0.49	-2.4	CV Mon	0.73	-3.0	Z Lac	1.04	-4.1
SS Sct	0.56	-2.4	RR Lac	0.81	-3.4	RW Cas	1.17	-4.5
SU Cyg	0.58	-2.8	U Sgr	0.83	-3.5	Y Oph	1.23	-5.3
Y Lac	0.64	-2.8	η Aql	0.86	-3.5	T Mon	1.34	-5.6
FF Aql	0.65	-3.1	RX Cam	0.90	-3.7	SV Vul	1.65	-6.4
CF Cas	0.69	-3.4	DL Cas	0.90	-3.7			

Cf. Passachoff J.M., Goebel R.V. 1979: Sky and Tel. (March) 241

Exercise 8 *Find the value of the Hubble constant using the magnitudes m and redshifts z of galaxies in the following table, assuming that the absolute magnitudes of these galaxies are $M = -22$.*

m	z	m	z	m	z	m	z
7.00	0.00086	13.20	0.0188	14.50	0.0590	16.00	0.1343
8.90	0.00577	13.62	0.0241	14.80	0.0216	16.70	0.0497
8.91	0.00344	13.95	0.0302	14.86	0.0326	17.00	0.1300
11.87	0.0176	14.00	0.0163	15.00	0.0209	17.54	0.1959
12.14	0.0169	14.00	0.0203	15.00	0.0677	17.94	0.2106
12.28	0.0177	14.07	0.0450	15.19	0.0600	18.00	0.0728
12.50	0.0181	14.20	0.030	15.50	0.0437	18.02	0.2201
12.84	0.0289	14.30	0.0145	15.80	0.0518	19.00	0.18
12.90	0.0215	14.50	0.0374	16.00	0.0718	19.00	0.2561

Cf. Evans A. 1978: Sky and Tel. 55, 299

Exercise 9 *Due to gravitational lensing by a galaxy* G *at* $z_G = 0.39$*, the quasar PKC 0957+561 at* $z_Q = 1.4136$ *is imaged as a multiple source with the brightest components* A *and* B *on the opposite sides of* G *at angular distances from it equal to* $4.95''$ *and* $0.75''$*, resp.*

(a) Calculate the distance of the quasar and the galaxy from the Earth, the apparent linear separation of components A*,* B*, and the distances between the galaxy and the rays from* A *and* B*.*

(b) Using the simplified model of a point-like mass M *of the galaxy, according to which the deflecting angle of the light reads* $\varphi = 4Gc^{-2}M/r$*, find* M *and the true angular separation between the quasar and the galaxy.*

(c) Estimate the Hubble constant using the time delay $\Delta\tau_{AB} \simeq 417$ *days in the variability of the images* A *and* B*.*

(d) Find the theoretical ratio of the brightness of the images A *and* B*, and compare it with its value* $\simeq 1.7$ *observed in the radio wavelengths.*

1.6 Cosmic background radiation

In the thermodynamic equilibrium of the electromagnetic field with its emitters/absorbers, the space is filled by radiation, the density and spectrum of which is described for each thermodynamic temperature T by the Planck law for the black-body radiation

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \, . \tag{32}$$

In an expanding universe, the thermodynamic equilibrium is generally violated due to the different dependence of the temperature T of a relativistic gas (e.g. photons) and a non-relativistic gas (e.g., standard baryonic matter) on the volume in the course of their adiabatic expansion.[22] However, as it was shown in 1946 by G. Gamow, the conditions in the early stages of the cosmological expansion were similar to those in the stellar nuclei, i.e., there was not only a large density but also a high temperature, which described both the radiations as well as the baryonic matter, because the relaxation time in which the thermodynamic equilibrium can be achieved was much shorter than the age of the Universe at that time. In the course of the cosmological expansion, the relaxation time increased (due to the decreasing density and energy of the particles) more than the age of the Universe, and starting from the moment of the so-called *decoupling of radiation from matter* when the hydrogen recombined, the radiation almost ceased to interact with baryonic matter. The photon density decreases as $n \sim R^{-3}$ during the subsequent expansion of the Universe, and due to the redshift (i.e., the Doppler effect) also the frequency ν of each photon decreases as $\nu \sim R^{-1}$. The radiation thus preserves the Planckian distribution with the temperature T inversely proportional to the radius of the Universe ($T \sim R^{-1}$; the total energy of the radiation field is decreasing due to the work exerted by its pressure – cf. Eqs. (6) and (15)) despite it is no more in thermodynamic equilibrium with the matter.[23] This so-called *cosmic background radiation* (CBR) is observed at present time as a homogeneous and isotropic radiation with the temperature $T \simeq 2.73$K (Gamow predicted $T \simeq 10$K). The density of energy of this radiation field is now smaller (in ratio $\simeq 1 : 10^4$) than that of the rest-mass energy of baryonic matter. However, in the past during the so called radiation-dominated epoch, the density of the mass-energy of radiation was larger than the density of the baryonic matter. The existence of CBR is, in principle, a confirmation of the hot big-bang cosmological model. The present observational and theoretical investigation of the deviations of CBR from the exact isotropy, homogeneity and the Planckian spectrum, enables us to improve the scenario of the early evolution of the Universe and of the formation of galaxies.

CBR was discovered in 1965 by A.A. Penzias and R.W. Wilson during tests of a microwave radiometer at the wavelength $\simeq 7$cm. The direct measurements are experimentally difficult and they require the use of cryogenic technique. The main source of the noise is thermal radiation from the Earth and its atmosphere. This is why the measurements are often performed from airplanes, balloons and satellites (Relikt, COBE). Next, it is necessary to subtract in the data-processing the radiation of local sources like planets, stars, the Galaxy (i.e., interstellar matter) or discrete extragalactic sources. As an indirect method of the measurement of CBR can be used determination of the excitation temperature of interstellar molecules (e.g., CN) from the observation of strengths of absorption lines in the optical spectrum of stars. Due to the irradiation of molecules by nearby stars, this method yields an upper limit on the intensity of CBR only. However, it is the only test of the homogeneity of the CBR in space on a relatively large scale.

The largest observed deflection of CBR from the exactly isotropic equilibrium distribution is the dipole component of the dependence of the intensity or the temperature T

$$\Delta T \sim \frac{vT}{c} \cos\vartheta \, , \tag{33}$$

[22] This different behavior of relativistic and non-relativistic gas is caused by their different equations of state. The thermodynamic equilibrium in the Universe is also violated by the existence of local inhomogeneities like galaxies and stars, which have higher densities and temperatures and which thus emit hot radiation into their cooler environment.

[23] The mean temperature of baryonic matter, which is already nonrelativistic at this stage, starts to decrease much faster from this moment. However, the decoupling enables clustering of matter, and the temperature of local condensations (e.g., stars) can increase and their high-energy radiation can cause a re-ionization of the surroundings.

which arise by the Doppler effect (and the aberration) in the case of the motion of the observer (with the velocity v in the direction containing the angle ϑ with the line of sight) with respect to the inertial system in which the radiation is isotropic. CBR thus defines a "local rest system" of the Universe (which is for its privilege sometimes mentioned as a "new ether"). The measured velocity of the Sun with respect to CBR is about 380 km/s in the direction with galactic coordinates $l = 253°$, $b = 47°$, the velocity of the Galaxy with respect to CBR is thus about 600 km/s.

A quadrupole deflection (for which the direction characteristics has the form of a three-axial elipsoid) could arise if the cosmological expansion (and hence also the Hubble constant) would not be the same in all directions. Fluctuations with smaller angular dimensions could arise at the epoch of the decoupling of the matter from the radiation on possible inhomogeneities in the spatial distribution of the density and the velocity field, which were the seeds of the later formed galaxies (the so-called *Sachs – Wolfe effect*). In the case of *adiabatic perturbations* any local enhancement of the density $\Delta\rho > 0$ (which produces some local depression in the gravity potential $\Delta\Phi < 0$) is accompanied also by an increase of temperature $\Delta T > 0$. The photons emitted in these regions are thus hotter, however, before reaching the observer they are gravitationally redshifted (in the matter-dominated universe $\Delta T = -\frac{1}{3}c^{-2}T\Delta\Phi$). These perturbations of isotropy of CBR have been observed by COBE and some other instruments on angular scale $\sim 1°$ and amplitude $\frac{\Delta T}{T} \sim 10^{-5}$.

The observed smallness of the amplitude of such perturbations is in fact also a cosmological paradox (the so-called *paradox of the horizon*) because it means that there was the same temperature and density, e.g. in the events at the opposite directions on the sky in the same time, despite these events were still causally disconnected, i.e., they could not exchange any information about the values of these quantities. A possibility to solve this paradox is offered by the inflation cosmologies, in which the true horizon is many orders larger than the apparent one corresponding to the epoch of the decoupling – cf. Eq. (27). An additional distortion of CBR could take place also in later epochs of the evolution of the Universe, e.g., after the re-ionization of the intergalactic matter by the high-energy radiation from the first formed quasars and stars, or by the inverse Compton scattering[24] on high-energy particles in the clusters of galaxies (the so-called *Zel'dovitch – Sunyaev effect*), or due to the gravitational interaction with the density perturbations (*Rees – Sciama effect*). The variety of these effects can influence not only the intensity but also the spectrum and polarization of CBR.

[24] This particular effect tends to establish a more general Bose–Einstein distribution with non-zero chemical potential instead of the Planckian distribution for the CBR- photons.

1.7 Evolution of inhomogeneities

The perturbations of homogeneity of the Universe observed in the cosmic background radiation are of relative order 10^{-5}, while at the present epoch they are highly non-linear, i.e., of an order > 1. To understand the history of their evolution we can first compare it with the collapse of a protostellar cloud. In a gas with density ρ and pressure P, any local contraction δr of a volume with radius r leads to an increase of its density and pressure, and the gradient of the pressure intends to smear out this perturbation. However, for a sufficiently large r, the potential energy of the self-gravitation of the volume released by this contraction (which is of the order $\delta(GM^2/r) \sim G\rho^2 r^4 \delta r$) can prevail over the work ($Pr^2\delta r$) needed to compress the gas and the contraction thus continues by gravitational collapse of volumes with radii larger than the *Jeans length*

$$r_{\rm J} \sim \sqrt{\frac{P}{G\rho^2}} \ . \tag{34}$$

In the epoch dominated by the radiation, $P \sim \rho c^2$ and the Jeans length was of the order of the horizon, hence all inhomogeneities on smaller scales were efficiently smoothed out by the radiation pressure. The highly non-linear fluctuations of density like the galaxies and stars could thus start to develop after the decoupling of the matter from the radiation only, when $P \simeq \frac{kT}{m}\rho$ and the Jeans mass $M_{\rm J} \sim \rho r^3 \sim (\frac{kT}{Gm})^{3/2}\rho^{-1/2}$.

Because the pressure was non-relativistic in the subsequent matter dominated epoch of the Universe evolution, the evolution of the inhomogeneities can be calculated as a perturbation of the Newtonian cosmological model, i.e., by solution of hydrodynamic equations (continuity of mass, momentum, and the Poisson eq.)

$$\dot\rho + \nabla(\rho v) = 0 \ , \tag{35}$$

$$\dot v + (v\nabla)v = -\nabla\Phi - \frac{1}{\rho}\nabla P \ , \tag{36}$$

$$\nabla^2\Phi = 4\pi G\rho \ . \tag{37}$$

Assuming their solution to be in the form of the Newtonian solution plus a small perturbation

$$\rho(t,r) = \rho_0 R^{-3}(1+\delta) \ , \tag{38}$$

$$v(t,r) = \frac{\dot R}{R}r + u \ , \tag{39}$$

$$\Phi(t,r) = \frac{2}{3}\pi G\rho_0 R^{-3} r^2 + \varphi \ , \tag{40}$$

we get for the perturbations linear equations

$$\dot\delta + \frac{\dot R}{R}(r\nabla)\delta + \nabla u = 0 \ , \tag{41}$$

$$\dot u + \frac{\dot R}{R}(r\nabla)u + \frac{\dot R}{R}u = -\nabla\varphi - \left[\frac{dP}{d\rho}\right]\nabla\delta \ , \tag{42}$$

$$\nabla^2\varphi = 4\pi G\rho_0 R^{-3}\delta \ . \tag{43}$$

If we express this perturbation in comoving coordinates[25] and perform the Fourier transform, i.e., we suppose $\delta(t,r) = \tilde\delta(t,k)\exp(i\frac{kr}{R})$, etc., we get the equations in the form

$$\dot{\tilde\delta} + \frac{i}{R}k\tilde u = 0 \ , \tag{44}$$

$$\dot{\tilde u} + \frac{\dot R}{R}\tilde u = -\frac{i}{R}k\tilde\varphi - \left[\frac{dP}{d\rho}\right]\frac{i}{R}k\tilde\delta \ , \tag{45}$$

$$-\frac{1}{R^2}k^2\tilde\varphi = 4\pi G\rho_0 R^{-3}\tilde\delta \ , \tag{46}$$

[25] It means that we transform from t, r to $\tau = t, \chi = r/R(t)$ so that $\partial_r = \frac{1}{R}\partial_\chi$ and $\partial_t = \partial_\tau - \frac{\dot R}{R}r\partial_\chi$.

from where we finally get the differential equation

$$\ddot{\tilde{\delta}} + 2\frac{\dot{R}}{R}\dot{\tilde{\delta}} + \left(\left[\frac{dP}{d\rho} \right] \frac{k^2}{R^2} - 4\pi G \rho_0 R^{-3} \right) \tilde{\delta} = 0 \tag{47}$$

for the density perturbation. It is obvious from this that the modes with wavelengths smaller than the Jeans one are dumped (because the term in the parentheses is positive and the time derivatives must thus be negative) and the modes with the longest wavelengths ($k = 0$) behave in time as

$$\delta \simeq \delta_+ t^{2/3} + \delta_- t^{-1} \ . \tag{48}$$

There are several hypotheses about the nature of the initial perturbations. In the case of the so-called *adiabatic perturbations* for which $\delta n_\gamma / n_\gamma = \delta n_B / n_B$ (so that the entropy per baryon is constant), the characteristic mass of inhomogeneities is determined by the dimension of the particle horizon at that time and it is of the order of 10^{12} up to $10^{15} M_\odot$ which corresponds to the mass of large clusters of galaxies. According to this scenario the large-scale structure arose first, and then gradually fragmented so that the galaxies appeared at $z \simeq 10$ or 3 only. According to the *isothermic scenario*, $\delta n_\gamma = 0$, while δn_B could form already at the radiation-dominated epoch on the scale of Jeans' gravitational instability of the baryon gas, i.e. $\simeq 10^5 M_\odot$, which corresponds to the globular clusters. Inhomogeneities of this size could exist already at $z \simeq 1000$, and the more massive systems (rather of a spherical shape) were gradually built from them later. The formation of the structure of the Universe could be influenced by the blast waves arisen by the explosions of the primordial supermassive stars.

It follows from the observation of the cosmic background radiation that the amplitude of adiabatic perturbations was small at $z \simeq 1000$, despite the opposite regions were not causally connected in a case of power-law form of $R(t)$. This observed homogeneity of the Universe is involved in the initial conditions in Friedmann models. However, it is a natural desire to explain it by some relaxation processes in the very early epochs of the evolution of any initially inhomogeneous universe. One such possibility is offered by the inflation epoch of the evolution of the Universe, which is expressed by Eq. (17). If R increases for many orders (it is supposed to be up to 10^{30}) in the course of the inflation epoch, then the real horizon is larger than the observed sphere corresponding to $z = 1000$ in the same ratio. In addition to the isotropy of the Universe, the inflationary models explain also its proximity to the critical model (i.e., $q = 0.5$) and some other problems resulting from the theories of grand unification (e.g. the problem of unobservable density of monopoles). This fine tuning of conditions which are satisfied by the observed Universe, despite they are not trivial from the point of view of physics, is necessary for the origin of stars, the nucleosynthesis in their interiors, and thus also for the rise of life. It can thus be explained also by the anthropic principle.

1.8 Nucleosynthesis

It follows from the equation of continuity (6) that the mass contained in the Universe had a large density and hence probably also a large temperature when the radius R of the Universe was small. The conditions in the whole Universe could then be similar to those in stellar interiors, and there could take place some thermonuclear reactions. For each reaction (either chemical, thermonuclear or a reaction of elementary particles, or for a ionization process, a radiative transition, etc.) there exists also the inverse process. In the thermodynamic equilibrium, the ratio of abundances of the products of these reactions is determined by the temperature and the density. Because the relaxation time $\tau_{\rm r}$ for reaching this equilibrium usually decreases fast with increasing density, it was much smaller than the age t of the Universe at that time when R was small, and hence the chemical composition of the Universe corresponded to the equilibrium one. At the moment when the density decreased due to the cosmological expansion and $\tau_{\rm r}$ became larger than t, the detailed equilibrium was broken. In the first approximation, the nowadays chemical composition of the Universe is a frozen picture of the situation at that instant. For a more accurate calculation, it is necessary to solve the appropriate kinetic equations. However, in principle, the chemical composition of the Universe is determined by the value of the temperature in the epoch with given density. An ionized matter is in thermodynamic equilibrium with the radiation, hence the temperature is described by the density of photons. The resulting chemical composition is thus dependent on the ratio of the photon density n_γ and the baryon density $n_{\rm B}$, i.e., on the entropy per baryon, because the entropy of the equilibrium photon gas, which is proportional to the density of photons, is dominant. The resulting abundance of He^4 (which is $\simeq 0.25$ by mass in a good agreement with the observations) is poorly sensitive to $n_{\rm B}/n_\gamma$ because it is determined mainly by the relative abundance of neutrons and protons ($\simeq 0.2$) at the moment when their mutual transmutations finished due to the drop of the temperature. After the plasma recombination, the relaxation time $\tau_{\rm r}$ of the radiative processes becomes larger than the age t of the Universe and the radiation decouples from the mass. It means that it is freely cooling with the temperature $T \sim R^{-1}$ (while the non-relativistic particles of the mass are cooling faster with $T \sim R^{-2}$). This radiation is now observed as the cosmic background radiation, the density (temperature) of which is well measured. The mean density of baryons in the Universe can thus be calculated from the observed chemical composition of the mass in the Universe. This method determines the density of the baryonic matter including the contribution of dark matter also. However, it is influenced by the subsequent enrichment of the primordial chemical composition by heavy elements produced in the nucleosynthesis in stellar interiors. The determination of the mean chemical composition is also hampered by the increase of its local inhomogeneities. The best agreement with the observations gives the baryon density $n_{\rm B} \simeq 10^{-9} n_\gamma$, which corresponds to $\rho \simeq 0.1 \rho_{\rm crit}$.

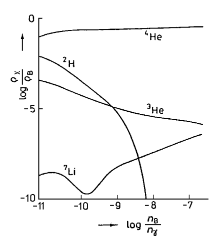

Figure 7: Relative abundance of elements as a function of entropy per baryon.

A mutual transmutation took place between the photons and the pairs of particles and antiparticles in the early epochs of the evolution of the Universe, when the mean energy of the γ-photons was higher than the corresponding threshold value. In the equilibrium of creation and annihilation processes, the numbers of photons and particles with $mc^2 < h\nu$ were approximately equal (nonetheless the dynamics of the Universe was still ‘dominated by radiation’ because the particles were highly relativistic like the photons). In the course of the cooling, the baryons annihilated first and then the leptons also. Because we now observe a non-zero density of particles and practically no antiparticles, there must have been a small excess of particles in comparison with antiparticles (in ratio approximately $(1+10^{-9})$:1) – the so-called *baryon asymmetry*.