

Lorentz Violation of Quantum Gravity

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Abstract

A quantum gravity theory which becomes renormalizable at short distances due to a spontaneous symmetry breaking of Lorentz invariance and diffeomorphism invariance is studied. A breaking of Lorentz invariance with the breaking patterns $SO(3,1) \rightarrow O(3)$ and $SO(3,1) \rightarrow O(2)$, describing $3+1$ and $2+1$ quantum gravity, respectively, is proposed. A complex time dependent Schrödinger equation (generalized Wheeler-DeWitt equation) for the wave function of the universe exists in the spontaneously broken symmetry phase at Planck energy and in the early universe, uniting quantum mechanics and general relativity. An explanation of the second law of thermodynamics and the spontaneous creation of matter in the early universe can be obtained in the symmetry broken phase of gravity.

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1 Introduction

The quantum theory of gravitation in four dimensions ($D = 4$) quantized on a fixed background such as Minkowski spacetime with the metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is not renormalizable [1, 2]. This has led to much effort to search for a physically consistent and finite quantum gravity theory. Many attempts include string theory [3], loop quantum gravity [4, 5, 6], and a finite non-local, regularized quantum gravity theory [7]. In earlier papers, the local Lorentz and diffeomorphism invariance of gravity was spontaneously broken in a *vierbein* gauge theory [8, 9]. From the symmetry breaking schemes $SO(3,1) \rightarrow O(3)$ and $SO(3,1) \rightarrow O(2)$, quantum gravity theory underwent a reduction to a $3+1$ and $2+1$ theory, respectively. It was assumed that the symmetry broken phase occurred at an energy $E \sim E_P$ where $E_P = 1/M_P \sim 10^{19}$ GeV is the Planck energy. This would correspond to a breaking

of the symmetry at a critical temperature $T \sim T_c \sim E_P$, in the very early universe. The reduction of the quantum gravity to lower-dimensional gravity theories at very short distances can lead to a renormalizable quantum gravity theory [10, 11].

Recently, the idea of reducing quantum gravity to a 3+1 theory has been revived by Horava [12], who based the Lorentz violation on an “anisotropic scaling” of the space and time dimensions. The idea is to introduce a quantity Z , with the physical dimensions: $[Z] = [dx]^z/[dt]$ and for the relativistic gravity theory $Z \rightarrow 1$. We argue that the Lorentz and diffeomorphism violation of gravity has a more intuitive and physical basis in spontaneous symmetry breaking of the gravitational action. The spontaneous symmetry breaking mechanism only breaks the vacuum or ground state of the gravitational system, retaining a “hidden” local gauge invariance symmetry of the action that preserves Takahashi-Ward identities and other attractive properties of the purely gauge invariant formalism.

In the symmetry broken phase and in the 3 + 1 or 2 + 1-dimensional quantum gravity, time becomes “absolute” and is described by the $R \times O(3)$ Lemaître-Friedman-Robertson-Walker (LFRW) cosmology. The breaking of time translational and Lorentz invariance leads to a complex Schrödinger equation (generalized Wheeler-DeWitt equation [13]) for the wave function of the universe, thereby solving the problem of time and uniting quantum mechanics and relativistic gravity [8].

The spontaneous symmetry breaking mechanism in the *vierbein* gauge formalism has 3 massless degrees of freedom associated with the $O(3)$ rotational invariance, and 3 massive degrees of freedom associated with the broken Lorentz “boosts”. The massive quantum gravity in 3 + 1 dimensions can satisfy unitarity and be renormalizable, in contrast to the $D = 4$ quantum gravity which will violate unitarity if renormalizable [14]. Moreover, the 2 + 1 quantum gravity in which both local Lorentz invariance and rotational invariance are broken can for a massive graviton be unitary, ghost-free and renormalizable [11].

The spontaneous symmetry broken phase will induce a violation of conservation of energy and explain the generation of matter in the very early universe. Moreover, in the ordered symmetry broken phase entropy will be at a minimum. After the phase transition as the Universe expands into the disordered phase with $SO(3) \rightarrow SO(3, 1)$ or $O(2) \rightarrow SO(3, 1)$ there will be a large increase in entropy with an arrow of time created by the spontaneous choice of symmetry breaking.

2 Spontaneous Symmetry Breaking of Gravity

Let us define the metric in any non-inertial coordinate system by

$$g_{\mu\nu}(x) = e_{\mu}^a(x)e_{\nu}^b(x)\eta_{ab}, \quad (1)$$

where

$$e_{\mu}^a(X) = \left(\frac{\partial \zeta_X^a(x)}{\partial x^{\mu}} \right)_{x=X}. \quad (2)$$

The ζ_X^a are a set of locally inertial coordinates at X . The vierbeins e_μ^a satisfy the orthogonality relations:

$$e_\mu^a e_b^\mu = \delta_b^a, \quad e_a^\mu e_\nu^a = \delta_\nu^\mu, \quad (3)$$

which allow us to pass from the flat tangent space coordinates (the fibre bundle tangent space) labeled by a, b, c, \dots to the world spacetime coordinates (manifold) labeled by μ, ν, ρ, \dots . The fundamental form (1) is invariant under Lorentz transformations:

$$e'^a_\mu(x) = L_b^a(x) e_\mu^b(x), \quad (4)$$

where $L_b^a(x)$ are the homogeneous $SO(3, 1)$ Lorentz transformation coefficients that can depend on position in spacetime, and which satisfy

$$L_{ac}(x) L_d^a(x) = \eta_{cd}. \quad (5)$$

For a general field $f_n(x)$ the transformation rule will take the form

$$f_n(x) \rightarrow \sum_m [D(L)(x)]_{nm} f_m(x), \quad (6)$$

where $D(L)$ is a matrix representation of the (infinitesimal) Lorentz group.

The e_μ^a will satisfy

$$e_{\mu,\sigma}^a + (\Omega_\sigma)_c^a e_\mu^c - \Gamma_{\sigma\mu}^\rho e_\rho^a = 0, \quad (7)$$

where $e_{\mu,\nu}^a = \partial e_\mu^a / \partial x^\nu$, Ω_μ is the spin connection of gravity and $\Gamma_{\mu\nu}^\lambda$ is the Christoffel connection. Solving for Γ gives

$$\Gamma_{\sigma\lambda\rho} = g_{\delta\rho} \Gamma_{\sigma\lambda}^\delta = \eta_{ab} (D_\sigma e_\lambda^a) e_\rho^b, \quad (8)$$

where

$$D_\sigma e_\mu^a = e_{\mu,\sigma}^a + (\Omega_\sigma)_c^a e_\mu^c \quad (9)$$

is the covariant derivative operator with respect to the gauge connection Ω_μ . By differentiating (1), we get

$$g_{\mu\nu,\sigma} - g_{\rho\nu} \Gamma_{\mu\sigma}^\rho - g_{\mu\rho} \Gamma_{\nu\sigma}^\rho = 0, \quad (10)$$

where we have used $(\Omega_\sigma)_{ca} = -(\Omega_\sigma)_{ac}$.

The (spin) gauge connection Ω_μ remains invariant under the Lorentz transformations provided:

$$(\Omega_\sigma)_b^a \rightarrow [L \Omega_\sigma L^{-1} - (\partial_\sigma L) L^{-1}]_b^a. \quad (11)$$

A curvature tensor can be defined by

$$([D_\mu, D_\nu])_b^a = (R_{\mu\nu})_b^a, \quad (12)$$

where

$$(R_{\mu\nu})_b^a = (\Omega_\nu)_{b,\mu}^a - (\Omega_\mu)_{b,\nu}^a + ([\Omega_\mu, \Omega_\nu])_b^a. \quad (13)$$

The curvature tensor transforms like a gauge field strength:

$$(R_{\mu\nu})_b^a \rightarrow L_c^a (R_{\mu\nu})_d^c (L^{-1})_b^d. \quad (14)$$

In holonomic coordinates, the curvature tensor is

$$R_{\sigma\mu\nu}^\lambda = (R_{\mu\nu})_b^a e_a^\lambda e_\sigma^b \quad (15)$$

and the scalar curvature takes the form

$$R = e^{\mu a} e^{\nu b} (R_{\mu\nu})_{ab}. \quad (16)$$

At the Planck energy E_P the local Lorentz vacuum symmetry is spontaneously broken. We postulate the existence of a field, ϕ , and assume that the vacuum expectation value (vev) of the field, $\langle \phi \rangle_0$, will vanish for $E = E_c < E_P$ or at a temperature $T < T_c \sim M_P$, when the local Lorentz symmetry is restored. At $E \sim E_P$ the non-zero vev will break the symmetry of the ground state of the Universe from $SO(3, 1)$ down to $O(3)$ or $O(2)$. The domain formed by the direction of the vev of the ϕ field will produce a time arrow pointing in the direction of increasing entropy and the expansion of the Universe.

Let us introduce the fields $\phi^a(x)$ which are invariant under Lorentz transformations

$$\phi'^a(x) = L_b^a(x) \phi^b(x). \quad (17)$$

We can use the *vierbein* to convert ϕ^a into a 4-vector in coordinate space: $\phi^\mu = e_a^\mu \phi^a$. The covariant derivative operator acting on ϕ is defined by

$$D_\mu \phi^a = [\partial_\mu \delta_b^a + (\Omega_\mu)_b^a] \phi^b. \quad (18)$$

If we consider infinitesimal Lorentz transformations

$$L_b^a(x) = \delta_b^a + \omega_b^a(x) \quad (19)$$

with

$$\omega_{ab}(x) = -\omega_{ba}(x), \quad (20)$$

then the matrix D in (6) has the form:

$$D(1 + \omega(x)) = 1 + \frac{1}{2} \omega^{ab}(x) \sigma_{ab}, \quad (21)$$

where the σ_{ab} are the six generators of the Lorentz group which satisfy $\sigma_{ab} = -\sigma_{ba}$ and the commutation relations

$$[\sigma_{ab}, \sigma_{cd}] = \eta_{cb} \sigma_{ad} - \eta_{ca} \sigma_{bd} + \eta_{db} \sigma_{ca} - \eta_{da} \sigma_{cb}. \quad (22)$$

The set of fields ϕ transforms as

$$\phi'(x) = \phi(x) + \omega^{ab}(x) \sigma_{ab} \phi(x). \quad (23)$$

The gauge spin connection which satisfies the transformation law (11) is given by

$$\Omega_\mu = \frac{1}{2}\sigma^{ab}e_a^\nu e_{b\nu;\mu}, \quad (24)$$

where ; denotes covariant differentiation with respect to the Christoffel connection. We introduce a spontaneous symmetry breaking sector into the Lagrangian density such that the gravitational vacuum symmetry, which we set equal to the Lagrangian symmetry at low temperatures, will break to a smaller symmetry at high temperature. The breaking of the symmetry at a higher temperature is an example of “anti-restoration” symmetry breaking [8]. The vacuum symmetry breaking leads to the interesting possibility that exact zero temperature conservation laws e.g. electric charge and baryon number are broken in the early Universe. In our case, we shall find that the spontaneous breaking of the Lorentz symmetry of the vacuum leads to a violation of the exact zero temperature conservation of energy in the early Universe [8].

Consider the potential:

$$V(\phi) = \left[\lambda \sum_{a=0}^3 \phi^a \phi_a - \frac{1}{2}\mu^2 \right] \sum_{b=0}^3 \phi^b \phi_b, \quad (25)$$

where $\lambda > 0$ is a coupling constant such that $V(\phi)$ is bounded from below. Our Lagrangian density takes the form [8, 15, 16]:

$$\mathcal{L} = \mathcal{L}_G - \sqrt{-g} \left[\frac{1}{4} B^{ab} B_{ab} + V(\phi) \right], \quad (26)$$

where

$$B_{ab} = D_b \phi_a - D_a \phi_b, \quad (27)$$

and

$$\mathcal{L}_G = -\frac{1}{16\pi G} \int d^4x e [R(\Omega) - 2\Lambda]. \quad (28)$$

Moreover, $e \equiv \sqrt{-g} = \det(e_\mu^a e_{a\nu})^{1/2}$, $R(\Omega)$ denotes the scalar curvature determined by the spin connection and Λ is the cosmological constant.

If V has a minimum at $\phi_a = v_a$, then the spontaneously broken solution is given by $v_a^2 = \mu^2/4\lambda$ and an expansion of V around the minimum yields the mass matrix:

$$(\mu^2)_{ab} = \frac{1}{2} \left(\frac{\partial^2 V}{\partial \phi_a \partial \phi_b} \right)_{\phi_a=v_a}. \quad (29)$$

We can choose ϕ_a to be of the form

$$\phi_a = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix} = \delta_{a0} (\mu^2/4\lambda)^{1/2}. \quad (30)$$

All the other solutions of ϕ_a are related to this one by a Lorentz transformation. Then, the homogeneous Lorentz group $SO(3,1)$ is broken down to the spatial rotation group $O(3)$. The three rotation generators $J_i (i = 1, 2, 3)$ leave the vacuum invariant

$$J_i v_i = 0, \quad (31)$$

while the three Lorentz-boost generators K_i break the vacuum symmetry

$$K_i v_i \neq 0. \quad (32)$$

The J_i and K_i satisfy the commutation relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} K_k. \quad (33)$$

The mass matrix $(\mu^2)_{ab}$ can be calculated from (29):

$$(\mu^2)_{ab} = \left(-\frac{1}{2}\mu^2 + 2\lambda v^2\right)\delta_{ab} + 4\lambda v_a v_b = \mu^2 \delta_{a0} \delta_{b0}, \quad (34)$$

where v denotes the magnitude of v_a . There are three zero-mass Nambu-Goldstone bosons, the same as the number of massive bosons, and there are three massless degrees of freedom corresponding to the unbroken $O(3)$ symmetry. After the spontaneous breaking of the vacuum, one massive physical particle Φ remains. No ghost particles will occur in the unitary gauge. The mass term in the Lagrangian density is given in the unitary gauge by

$$\mathcal{L}_M = \frac{1}{2}\sqrt{-g}v_b v_c (\Omega_\mu)^{ab} (\Omega^\mu)_a^c = \frac{1}{2}\sqrt{-g}(\mu^2/4\lambda) \sum_{i=1}^3 ((\Omega_\mu)^{i0})^2. \quad (35)$$

When Lorentz symmetry is restored for $E < E_c$, then $v = 0$ and $\mathcal{L}_M = 0$ and we obtain the standard GR Lagrangian density with a massless spin-2 graviton, coupled minimally to a spin-1 particle.

We could have extended this symmetry breaking pattern to the case where we have two sets of vector field representations, ϕ_{a1} and ϕ_{a2} . The invariant spin connection can depend on the length of each vector and the angle between them, $|\phi_{a1}|$, $|\phi_{a2}|$, and $\chi = |\phi_{a1}\phi_{a2}^a|$. The solutions for the minimum must be obtained from the conditions imposed on these three quantities. We can choose ϕ_{a1} with only the last component non-zero and ϕ_{a2} with the last two components non-zero in order to satisfy these conditions. The Lorentz $SO(3,1)$ symmetry is then broken down to $O(2)$ (or $U(1)$) symmetry [19].

A phase transition is assumed to occur at the critical temperature T_c , when $v_a \neq 0$ and the Lorentz symmetry is broken and the three gauge fields $(\Omega_\mu)^{i0}$ become massive degrees of freedom. Below T_c the Lorentz symmetry is restored, and we regain the usual classical gravitational field with massless gauge fields Ω_μ . The symmetry breaking will extend to the singularity or the possible singularity-free

initial state at $t = 0$, and since quantum effects associated with gravity do not become important before E_P , we expect that $E_c \sim 10^{19}$ GeV.

A calculation of the effective potential for the symmetry breaking contribution in (26) shows that extra minima in the potential $V(\phi)$ can occur for a noncompact group such as $SO(3,1)$. This fact has been explicitly demonstrated in a model with $O(n) \times O(n)$ symmetric four-dimensional ϕ^4 field theory [20]. This model has two irreducible representations of fields, $\vec{\phi}_1$ and $\vec{\phi}_2$, transforming as $(n,1)$ and $(1,n)$, respectively. The potential is

$$V = \sum_i \frac{1}{2} m_i^2 \vec{\phi}_i^2 + \sum_{i,j} \frac{1}{8} \vec{\phi}_i^2 \lambda_{ij} \vec{\phi}_j^2. \quad (36)$$

The requirement of boundedness from below gives ($\lambda_{12} = \lambda_{21}$):

$$\lambda_{11} > 0, \quad \lambda_{22} > -(\lambda_{11} \lambda_{22})^{1/2}. \quad (37)$$

If we have $\lambda_{12} < -(1 + 2/n)\lambda_{22}$, then the one-loop free energy predicts spontaneous symmetry breaking to $O(n) \times O(n-1)$ at sufficiently high temperatures without symmetry breaking at small temperatures. The standard symmetry breaking restoration theorems can be broken in this case because the dynamical variables $\vec{\phi}_i$ do not form a compact space.

After the symmetry is restored for $E < E_P$, the entropy will rapidly increase provided that no further phase transition occurs which breaks the Lorentz symmetry of the vacuum. Thus, the symmetry breaking mechanism explains in a natural way the low entropy at the initial state at $t \sim 0$ and the large entropy in the present universe.

Since the ordered phase is at a much lower entropy than the disordered phase and due to the existence of a domain determined by the direction of the vev of the ϕ field, a natural explanation is given for the cosmological arrow of time and the origin of the second law of thermodynamics. Thus, the spontaneous symmetry breaking of the gravitational vacuum corresponding to the breaking pattern, $SO(3,1) \rightarrow O(3)$, leads to a manifold with the structure $R \times O(3)$, in which time appears as an absolute external parameter [8]. The vev, $\langle \phi \rangle_0$, points in a chosen direction of time to break the symmetry creating an arrow of time. The evolution from a state of low entropy in the ordered phase to a state of high entropy in the disordered phase explains the second law of thermodynamics.

3 3 + 1 Quantum Gravity

The action in Einstein's gravitational theory for a fixed three-geometry on a boundary is [17, 13]:

$$S_E = \frac{1}{16\pi G} \left[\int_{\partial M} d^3x h^{1/2} 2K + \int_M d^4x (-g)^{1/2} (R + 2\Lambda) \right], \quad (38)$$

where the second term is integrated over spacetime and the first over its boundary, K is the trace of the extrinsic curvature K_{ij} ($i,j=1,2,3$) of the boundary three-surface. We write the metric in the usual 3 + 1 form:

$$ds^2 = (N^2 - N_i N^i) dt^2 - 2N_i dx^i dt - h_{ij} dx^i dx^j, \quad (39)$$

and the action becomes

$$S_E = \frac{1}{16\pi G} \int d^4x h^{1/2} N [-K_{ij} K^{ij} + K^2 - R(h)^{(3)} + 2\Lambda], \quad (40)$$

where

$$K_{ij} = \frac{1}{N} \left[-\frac{1}{2} \frac{\partial h_{ij}}{\partial t} + N_{(i|j)} \right]. \quad (41)$$

$R^{(3)}$ denotes the scalar curvature constructed from the three-metric h_{ij} and a stroke denotes the covariant derivative with respect to the latter quantity. The matter action S_M can also be constructed from the N, N_i, h_{ij} and the matter field.

The super-Hamiltonian density is given by

$$H = NH_0 + N^i H_i = H_0 \sqrt{h} + N^i H_i, \quad (42)$$

where H_0 and H_i are the usual Hamiltonian and momentum constraint functions, defined in terms of the canonically conjugate momenta π^{ij} to the dynamical variables h_{ij} :

$$\pi^{ij} = \frac{\delta L_E}{\delta(\partial h_{ij}/\partial t)}, \quad (43)$$

where L_E is the Einstein-Hilbert Lagrangian density. Classically, the Dirac constraints are

$$H_i = 0, \quad H_0 = 0. \quad (44)$$

These constraints are a direct consequence of the general covariance of Einstein's theory of gravity.

In quantum mechanics, a suitably normalized wave function is defined by the path integral

$$\psi(\vec{x}, t) = - \int [d\vec{x}(t)] \exp[iS(\vec{x}(t))]. \quad (45)$$

We obtain

$$\frac{\partial \psi}{\partial t} = -i \int [d\vec{x}(t)] \frac{\partial S}{\partial t} \exp(iS), \quad (46)$$

which leads to the Schrödinger equation

$$i \frac{\partial \psi}{\partial t} = H \psi. \quad (47)$$

We define the wave function of the universe to be [18, 8]:

$$\Psi[h_{ij}, \phi] = - \int [dg][d\phi] \mu[g, \phi] \exp(iS[g, \phi]), \quad (48)$$

where ϕ denotes a matter field, S is the total action and $\mu[g, \phi]$ is an invariant measure. The integral or sum is over a class of spacetimes with a compact boundary on which the induced metric h_{ij} and field configurations match ϕ on the boundary. A differential equation for the wave function of the Universe, Ψ , can be derived by varying the end conditions on the path integral (48). Since the theory is diffeomorphism invariant the wave function is independent of time and we obtain

$$\frac{\delta\Psi}{\delta N} = -i \int [dg][d\phi] \mu[g, \phi] \left[\frac{\delta S}{\delta N} \right] \exp(iS[g, \phi]) = 0, \quad (49)$$

where we have taken into account the translational invariance of the measure factor $\mu[g, \phi]$. Thus, the value of the integral is left unchanged by an infinitesimal translation of the integration variable N and leads to the operator equation:

$$H_0\Psi = 0. \quad (50)$$

The classical Hamiltonian constraint equation takes the form

$$H_0 = \delta S/\delta N = h^{1/2}(-K^2 + K_{ij}K^{ij} - R^{(3)} + 2\Lambda + 16\pi GT_{nn}) = 0, \quad (51)$$

where T_{nn} is the stress-energy tensor of the matter field projected in the direction normal to the surface. By a suitable factor ordering (ignoring the well-known ‘‘factor ordering’’ problem), the classical equation $\delta S/\delta N = 0$ translates into the operator identity

$$\left\{ -\gamma_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} + h^{1/2} \left[R^{(3)}(h) - 2\Lambda - \frac{16\pi}{M_P^2} T_{nn} \left(-i \frac{\delta}{\delta \phi}, \phi \right) \right] \right\} \Psi[h_{ij}, \phi] = 0, \quad (52)$$

where γ_{ijkl} is the metric on superspace,

$$\gamma_{ijkl} = \frac{1}{2} h^{-1/2} (h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}). \quad (53)$$

This is the familiar Wheeler-DeWitt equation for a closed universe [13].

We would expect that the wave function of the universe should be time dependent and lead to a complex Schrödinger equation or its covariant counterpart– the Tomonaga-Schwinger equation:

$$i \frac{\delta\Psi}{\delta\tau} = \mathcal{H}\Psi, \quad (54)$$

which leads to the ordinary time dependent Schrödinger wave equation for global time variations, with a positive-definite probabilistic interpretation. We therefore propose a new definition of the wave function of the universe which takes the form [8]:

$$\Psi[h_{ij}, \phi] = - \int [dg][d\phi] M[g, \phi] \exp(iS[g, \phi]), \quad (55)$$

where $M[g, \phi]$ is a measure factor that breaks the time translational invariance of the path integral and makes the wave function Ψ explicitly time dependent. We now obtain

$$\frac{\delta\Psi}{\delta N} = - \int [dg][d\phi] \frac{\delta M}{\delta N} \exp(iS) - i \int [dg][d\phi] M[g, \phi] \frac{\delta S}{\delta N} \exp(iS). \quad (56)$$

This leads to the time dependent Schrödinger equation

$$i \frac{\delta\Psi}{\delta N} = \tilde{H}_0 \Psi, \quad (57)$$

where \tilde{H}_0 denotes

$$\tilde{H}_0 = -i \frac{\delta \ln M}{\delta N}. \quad (58)$$

A simple example of a measure factor that brings in an explicit time dependence (or N dependence) is

$$M[g, \phi] = \mu[g, \phi] N^b. \quad (59)$$

This measure factor $M[g, \phi]$ retains the momentum constraint equation $H_i = 0$ as an operator equation:

$$H_i \Psi = 0, \quad (60)$$

while keeping the invariance of the spatial three-geometry at the quantum mechanical level as well as at the classical level. If the measure $M[g, \phi]$ is chosen so that the diffeomorphism group \mathcal{D} is broken down to a sub-group \mathcal{S} , then there will exist a minimal choice of $M[g, \phi]$ which will break time translational invariance. The choice of $M[g, \phi]$ is not unique and some, as yet, unknown physical principle is needed to determine $M[g, \phi]$. At the classical level, we continue to maintain general covariance and the classical constraint equations (44) hold. The Bianchi identities

$$G_{\mu}{}^{\nu}{}_{;\nu} = 0, \quad (61)$$

are valid, where $G_{\mu}{}^{\nu} = R_{\mu}{}^{\nu} - \frac{1}{2} \delta_{\mu}{}^{\nu} R$. It is only the quantum mechanical wave function that breaks the diffeomorphism invariance i.e., N is no longer a free variable for the wave function of the universe. This leads naturally to a cosmic time which can be used to measure time dependent quantum mechanical observables. We find that for any operator O , we get

$$\frac{\delta}{\delta N} \langle O \rangle = i \langle [H, O] \rangle, \quad (62)$$

which constitutes the quantum mechanical version of Hamilton's equation. In contrast to the Wheeler-DeWitt equation, Ehrenfest's theorem follows directly from (62).

4 Conclusions

We have succeeded in arriving at a unification of quantum mechanics and gravity within a conceptually logical picture, since both of these pillars of modern physics are with us to stay. However, to achieve this we have postulated that Poincaré invariance and diffeomorphism invariance are violated at the Planck energy E_P . There exist stringent experimental bounds on violation of Lorentz invariance at lower energies [21], but there is no observational evidence that Lorentz invariance is strictly maintained at the Planck energy.

In $3 + 1$ and $2 + 1$ gravity the power counting of momenta in Feynman loop graphs allows the quantum gravity to be renormalizable [10]. Moreover, the problem of time in general relativity that prevents a logically consistent solution to uniting quantum mechanics and gravity is also resolved. This would lead one to believe that spontaneously breaking Lorentz symmetry at the Planck energy E_P could be a satisfactory solution to quantum gravity. To confirm that this way to resolve the problem of quantum gravity is realized in nature, it is necessary to experimentally detect a violation of Poincaré invariance at the Planck energy.

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