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TESTING MODIFIED GRAVITY WITH GLOBULAR CLUSTER VELOCITY DISPERSIONS

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ABSTRACT

Globular clusters (GCs) in the Milky Way have characteristic velocity dispersions that are consistent with the predictions of Newtonian gravity, and may be at odds with Modified Newtonian Dynamics (MOND). We discuss a modified gravity (MOG) theory that successfully predicts galaxy rotation curves, galaxy cluster masses and velocity dispersions, lensing, and cosmological observations, yet produces predictions consistent with Newtonian theory for smaller systems, such as GCs. MOG produces velocity dispersion predictions for GCs that are independent of the distance from the galactic center, which may not be the case for MOND. New observations of distant GCs may produce strong criteria that can be used to distinguish between competing gravitational theories.

Subject headings: gravity: theory — dark matter — globular clusters

1. INTRODUCTION

Modified Gravity (MOG, Moffat (2005, 2006a,b); Moffat & Toth (2007b)) is a fully covariant theory of gravity that is based on postulating the existence of a massive vector field, ϕ_μ. The choice of a massive vector field is motivated by our desire to introduce a *repulsive* modification of the law of gravitation at short range. The vector field is coupled universally to matter. The theory yields a Yukawa-like modification of gravity with three constants: in addition to the gravitational constant G, we must also consider the coupling constant ω that determines the coupling strength between the ϕ_μ field and matter, and a further constant μ that arises as a result of considering a vector field of non-zero mass, and controls the coupling range. In the most general case, these constants must be allowed to run. An approximate solution of the MOG field equations (Moffat & Toth 2007b) allows us to compute the values of μ and ω as functions of the source mass.

MOG was used successfully to describe observational phenomena on astrophysical and cosmological scales without resorting to dark matter. The theory correctly predicts galaxy rotation curves (Brownstein & Moffat 2006a; Moffat & Toth 2007b), the mass and thermal profiles of clusters of galaxies (Brownstein & Moffat 2006b; Moffat & Toth 2007b), the merging of the two clusters (Bullet Cluster, Brownstein & Moffat (2007)), the acoustical peaks of the cosmic microwave background (Moffat 2006b; Moffat & Toth 2007a), the velocity dispersion of satellite galaxies (Moffat & Toth 2007), the mass power spectrum and the luminosity-distance relationship of distant Type Ia supernovae (Moffat & Toth 2007a).

Globular clusters (GCs) in the Milky Way provide a particularly interesting case for testing alternate gravity theories (Baumgardt et al. 2005; Scarpa et al. 2007), such as MOG or Milgrom's Modified Newtonian Dynamics (MOND, Milgrom (1983); Bekenstein (2004)).

GCs at different distances from the galactic center experience galactic gravity at varying strengths. The internal gravitational field of a GC also varies depending on the mass (typically, 10^4–10^6 $M_\odot$) and size (typically, a few pc to a few ten pc in diameter) of the GC in question. Using the characteristic acceleration ($a_0 = 1.2 \times 10^{-10}$ m/s^2) of MOND, for example, we find GCs that experience either galactic or internal acceleration above, or below this value. MOND predicts velocity dispersions different from the Newtonian prediction for a GC whose stars experience a combined acceleration less than a_0.

On the other hand, MOG predicts identical velocity dispersions for GCs of the same size and composition, regardless of their distance from the galactic center. For this reason, GCs provide a unique test by which different gravitational theories can be compared.

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2. THEORY

The bulk properties of GCs, with the possible exception of their innermost regions, can be modeled using the collisionless Boltzmann equation (Binney & Tremaine 1987), from which the statistical properties of the velocity distribution of stars can be derived. In particular, one can derive a formulation for the velocity dispersion tensor that, in the isotropic, non-rotating case, reduces to a scalar quantity. This quantity can be determined using the appropriate Jeans equation. For this calculation, one requires knowledge of the distribution function (DF) that determines the number of stars in a given region of space, and the gravitational potential. We begin our discussion with the latter.

2.1. *Modified Gravity*

Our modified gravity (MOG) theory predicts a Yukawa-like modification of Newton's law of acceleration (Moffat 2006a; Moffat & Toth 2007b), in the form

$$a_{\rm MOG} = -\frac{G_N M}{r^2}(1 + \alpha(1 - (1 + \mu r)e^{-\mu r})), \quad (1)$$

where G_N is Newton's gravitational constant.

In accordance with our recent results (Moffat & Toth 2007b), the parameters α and μ can now be *predicted*:

$$\alpha = \frac{M}{(\sqrt{M} + C'_1)^2}\left(\frac{G_\infty}{G_N} - 1\right), \quad (2)$$

$$\mu = \frac{C_2'}{\sqrt{M}}, \tag{3}$$

where

$$G_\infty \simeq 20 G_N, \tag{4}$$

$$C_1' \simeq 25000\ M_\odot^{1/2}, \tag{5}$$

$$C_2' \simeq 6250\ M_\odot^{1/2}\text{kpc}^{-1}. \tag{6}$$

For even a large GC, with mass exceeding $10^6\ M_\odot$, the predicted values are

$$\alpha \simeq 0.03, \tag{7}$$

$$\mu \simeq (160\ \text{pc})^{-1}. \tag{8}$$

Given the smallness of α and the fact that μ^{-1} is much larger than the GC radius, it is clear that our theory predicts Newtonian behavior for such GCs:

$$a_{\rm MOG} \simeq a_{\rm Newton} = -\frac{G_N M}{r^2}. \tag{9}$$

For smaller GCs, the predictions are even closer to Newtonian values.

In contrast, the MOND acceleration $a_{\rm MOND}$ is given by the solution of the non-linear equation

$$a_{\rm MOND}\mu\left(\frac{|a_{\rm MOND}|}{a_0}\right) = -\frac{G_N M}{r^2}, \tag{10}$$

where $a_0 = 1.2 \times 10^{-8}\text{cm sec}^{-2}$. The form of the function $\mu(x)$ originally proposed by Milgrom (1983) is given by $\mu(x) = x/\sqrt{1+x^2}$; however, better fits and better asymptotic behavior are achieved using $\mu(x) = x/(1+x)$ (Famaey & Binney 2005), which yields the acceleration function

$$a_{\rm MOND} = -\frac{G_N M}{r^2}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a_0 r^2}{G_N M}}\right). \tag{11}$$

Regardless of the form of $\mu(x)$ chosen, when the combined acceleration experienced by stars in a GC is below a_0, MOND predicts dynamical behavior that is markedly different from the Newtonian prediction.

2.2. *The Jeans equation*

In the spherically symmetric, non-rotating case the Jeans equation for the velocity dispersion $\sigma(r)$ takes the following form (see Eq. 4-64a in Binney & Tremaine (1987)):

$$\frac{\partial(\nu\sigma^2)}{\partial r} + \nu\frac{\partial\Phi}{\partial r} = 0, \tag{12}$$

where r is the radial distance from the GC center, $\nu(r)$ is the number density distribution function, and $\Phi(r)$ is the gravitational potential. If $\nu(r)$ and $\Phi(r)$ are known, the velocity dispersion can be obtained by direct integration. Using $a(r) = \partial\Phi/\partial r$ and utilizing the fact that $\lim_{r\to\infty}\sigma^2(r) = 0$, we get

$$\sigma^2(r) = \frac{1}{\nu}\int_r^\infty \nu a(r')\ dr'. \tag{13}$$

The observed velocity dispersion of GCs is a function not of the actual radial distance r but the projected distance R between the GC center and the star being observed. The velocity dispersion $\sigma_{\rm LOS}(R)$ of stars observed along the line-of-sight (LOS) at projected distance R from the GC center is related to $\sigma(r)$ as

TABLE 1
PROPERTIES OF GCS STUDIED BY SCARPA ET AL. (2007). DATA FOR AM 1 AND PAL 14 ARE ALSO INCLUDED. THE DISTANCE R_g FROM THE GALACTIC CENTER, LUMINOSITY $L_\odot$ IN UNITS OF SOLAR LUMINOSITY, AND THE HALF-LIGHT RADIUS r_h (PC) ARE SHOWN (HARRIS 1996). MASS-TO-LIGHT RATIOS ARE ESTIMATED BY FITTING THE VELOCITY DISPERSION USING THE HERNQUIST MODEL, EXCEPT FOR AM 1 AND PAL 14, FOR WHICH $M/L = 2$ WAS FIXED.

Name	R_g (kpc)	$L_\odot$	r_h (pc)	$M_\odot/L_\odot$
NGC288	7.4	3.94×10^4	2.9	4.38
NGC5139	6.4	1.04×10^6	6.4	2.79
NGC6171	3.3	5.65×10^4	5.0	2.54
NGC6341	9.6	1.51×10^5	2.6	1.50
NGC7078	10.4	3.70×10^5	3.2	0.85
NGC7099	7.1	7.45×10^4	2.7	1.51
AM 1	123.2	6.08×10^3	17.7	2
Pal 14	69.0	6.19×10^3	24.7	2

$$\sigma^2_{\rm LOS}(R) = \frac{\int_0^\infty \sigma^2(y)\nu(y)\ dy}{\int_0^\infty \nu(y)\ dy} \tag{14}$$

where

$$y^2 = r^2 - R^2, \tag{15}$$

as can be verified by simple geometric reasoning. Eliminating y, we can rewrite (14) as

$$\sigma^2_{\rm LOS}(R) = \frac{\int_R^\infty r\sigma^2(r)\nu(r)/\sqrt{r^2-R^2}\ dr}{\int_R^\infty r\nu(r)/\sqrt{r^2-R^2}\ dr}. \tag{16}$$

2.3. *Density Distribution*

Several models exist that can mimic the density distribution of a spherically symmetric set of stars. One particularly simple model is that of Hernquist (1990), which models the number density of stars as a function of radius as

$$\nu_{\rm Hernquist}(r) = \frac{N r_0}{2\pi r(r+r_0)^3}, \tag{17}$$

where N is the total number of stars, and r_0 is a characteristic radius.

Another, similar model is that of Jaffe (1983):

$$\nu_{\rm Jaffe}(r) = \frac{N r_0}{4\pi r^2(r+r_0)^2}. \tag{18}$$

Without benefiting from a photometric profile of the globular cluster under investigation, there are no *a priori* reasons to prefer one model over another. We found that the choice of model does not have a significant impact on the conclusions we present; hereinafter, we shall be using Hernquist's model consistently, but we note that similar results are obtained using alternate number density distribution functions.

3. OBSERVATIONS AND PREDICTIONS

Velocity dispersion data for several GCs were recently published by Scarpa et al. (2007). We read velocity dispersion values and their standard deviations from Figures 1–2 and 4 of Scarpa et al. (2007), for NGC 288,

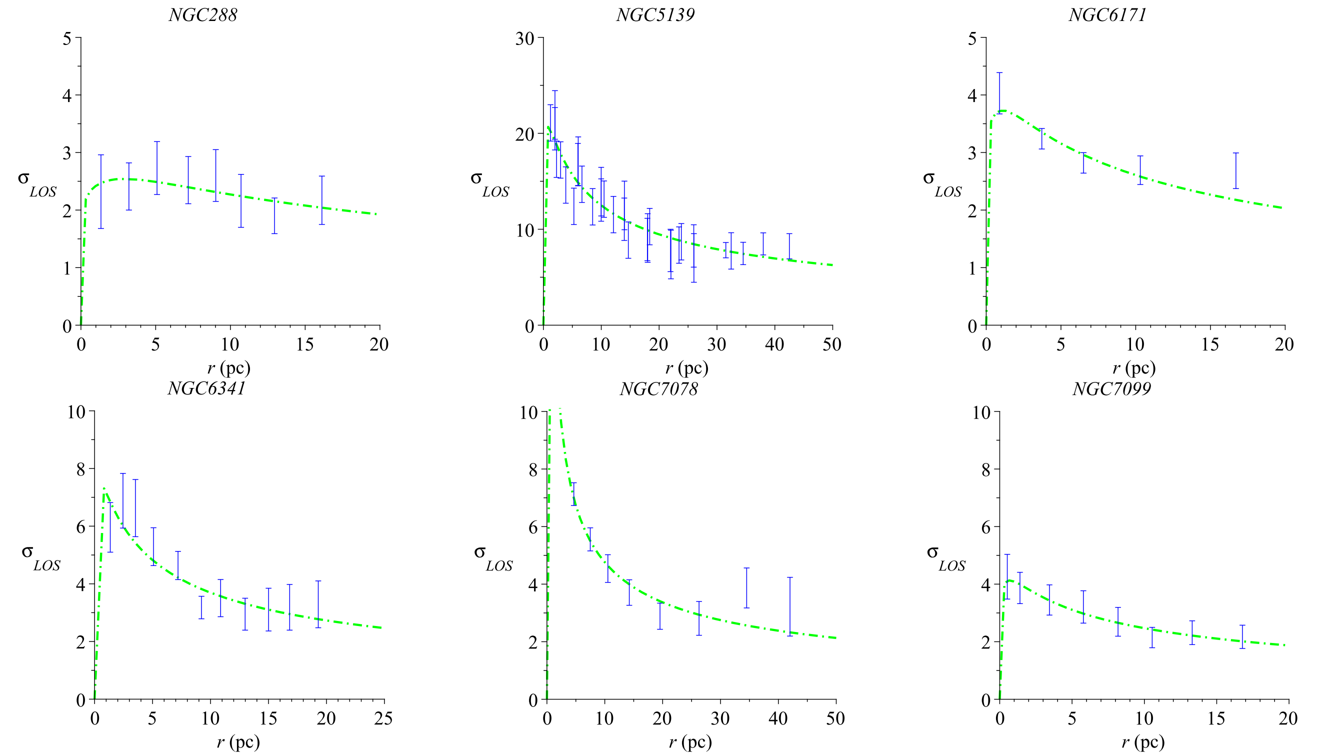

FIG. 1.— Fitting velocity dispersions obtained from the Jeans equation to globular cluster data (blue error bars from Scarpa et al. (2007)), using the Hernquist model and MOG or Newtonian gravity (dash-dot green line).

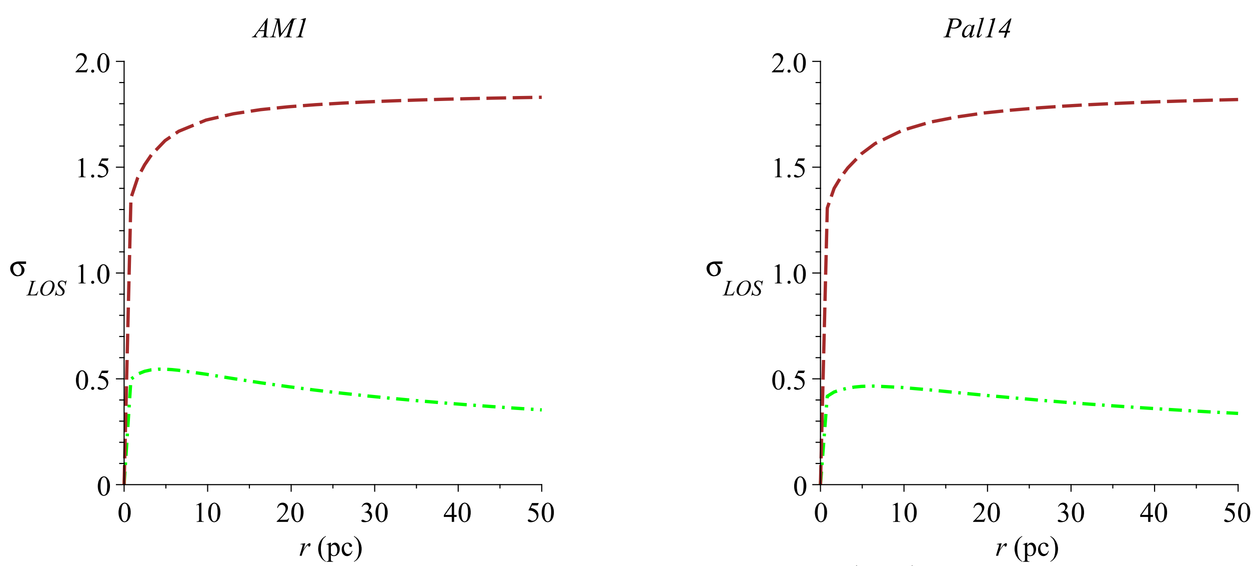

FIG. 2.— Predicted velocity dispersion curves for two distant GCs. Dash-dot line (green) is the prediction obtained using MOG or Newtonian gravity; dashed (brown) curve is the MOND prediction. In both cases, we used $M/L = 2$ and we equated the parameter r_0 of the Hernquist model with the half-light radius.

NGC 5139 (ω Centauri), NGC 6171 (M107), NGC 6341 (M92), NGC 7078 (M15), and NGC 7099 (M30). Some of the basic characteristics of these GCs are summarized in Table 1.

Using the Hernquist distribution as the number density distribution function for a spherically symmetric cluster of stars with isotropic velocity dispersion, we obtained very good fits to the velocity dispersion data (Figure 1). These results also yield mass-to-light ratios ranging between $0.8 < M/L < 4.4$ (Table 1), which are typical for globular clusters.

For these results, we used the Newtonian gravitational potential. These calculations are consistent with Newtonian theory, MOG (given the smallness of the predicted value of the MOG α parameter and the large size of the parameter μ^{-1}), and also MOND, as the GCs in question are located relatively near the galactic center, and the galactic acceleration is always greater than a_0.

The possible presence of dark matter does not appreciable alter these results either. A typical dark matter density for the galactic halo is $\sim 7.8 \times 10^{-3}\ M_\odot/\mathrm{pc}^3$ ($\simeq$0.3 GeV/cm^3; see Sumner (2002)), a density that is much smaller than the globular cluster's stellar mass density.

The situation is different, however, in the case of MOND and globular clusters that are located a long distance away from the galactic center. To quote Milgrom (1983): "We are then compelled to conclude that the internal dynamics of the open clusters embedded in the field of the Galaxy is different from that of a similar but isolated cluster." For instance, Pal 14, located at 69 kpc from the galactic center, would experience a galactic acceleration of $\sim 2.3 \times 10^{-11}$ m/s^2, which is well within the MOND regime. As this is a low mass cluster of stars, its internal accelerations are also significantly below MOND's a_0, except perhaps in the innermost regions of the cluster.

Two distant clusters, AM 1 and Pal 14, are presently the subject of an observational project by Kroupa et al. As the absolute luminosity of these GCs is known, using a (typical) value of $M/L \simeq 2$ we can obtain a crude estimate of their mass, allowing us to apply the Jeans equation and derive a velocity dispersion profile using both Newtonian and MOND gravity. These predictions are shown in Figure 2.

4. DISCUSSION

For globular clusters with a mass of a few times $10^6\ M_\odot$ or less, MOG predicts little or no observable deviation from Newtonian gravity. This is verified by our demonstration that a simple model, using a spherically symmetric distribution and no velocity anisotropy, can easily reproduce the velocity dispersion profiles of several diverse globular clusters with varying mass.

The predictions of neither Newtonian gravity nor MOG depend on the distance from the galactic center. The same remains true when dark matter is considered; although the density of dark matter may be a function of distance from the galactic center, at predicted dark matter densities, the amount of dark matter contained within a GC does not noticeably alter the dynamics of the cluster.

The situation is different for MOND; for a low-mass cluster, internal accelerations are below the MOND threshold of $a_0 \simeq 1.2 \times 10^{-10}$ m/s^2, and if the cluster is far enough from the galactic center, its galactic acceleration is also below this value. For this reason, distant GCs may offer a unique method to distinguish observationally between MOG and MOND.

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