

NON-SINGULAR COSMOLOGY IN MODIFIED GRAVITY

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Abstract

A non-singular cosmology is derived in modified gravity (MOG) with a varying gravitational coupling strength $G(t) = G_N \xi(t)$. Assuming that the curvature k , the cosmological constant Λ and ρ vanish at $t = 0$, we obtain a non-singular universe with a negative pressure, $p_G < 0$. The universe expands for $t \rightarrow \infty$ according to the standard radiation and matter dominated solutions. The thermodynamical arrow of time reverses at $t = 0$ always pointing in the direction of increasing entropy \mathcal{S} and the entropy is at a minimum value at $t = 0$, solving the conundrum of the Second Law of Thermodynamics. The Hubble radius $H^{-1}(t)$ is infinite at $t = 0$ removing the curvature and particle horizons. The negative pressure p_G generated by the scalar field ξ at $t \sim 0$ can produce quantum spontaneous creation of particles explaining the origin of matter and radiation.

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1 Introduction

The singularity in standard cosmology at $t = 0$ heralds the breakdown of physics at the big bang. This has been a long-standing problem in cosmology that prevents a rational explanation for the onset of the beginning of the universe. We know from the afterglow of the cosmic microwave background (CMB) radiation with the uniform temperature ~ 2.73 K that, according to the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology, there had to be a hot beginning to the universe. In the following, we shall develop a non-singular cosmology using the effective classical action and field equations based on modified gravity (MOG) [1, 2, 3].

An initial value problem occurs at the beginning of the universe at the big bang $t = 0$, arising from one of the most basic laws of physics: the second law

of thermodynamics. This problem has been addressed by Penrose [4, 5], who has proposed a solution based on making the Weyl curvature tensor vanish at $t = 0$. He also proposed that the universe went through a series of conformally invariant cycles [6].

We do not wish to contemplate a violation of the second law of thermodynamics, which is as cherished as the law of conservation of energy. The problem with the second law of thermodynamics arises because the entropy \mathcal{S} of the universe increases as time increases from $t = 0$, and accordingly the disorder or lack of speciality increases as the universe expands. Therefore, the initial state of the universe was the most special state of all.

In the following, we shall pursue a non-singular cosmological model which can resolve initial value problems such as the horizon problem and the entropy conundrum. We shall study the MOG cosmology using the effective classical scalar-tensor-vector gravity (STVG) [2, 3]. This MOG with its variations of G , the vector field ϕ_μ , the coupling strength ω and its effective mass μ leads to a satisfactory description of galaxy rotation curves, the mass profiles of X-ray clusters of galaxies [7, 8], the bullet cluster 1E0657-56 [9] and to the WMAP data for the CMB, the matter power spectrum and the supernovae data [10] without undetected dark matter and dark energy.

2 MOG Field Equations and Action Principle

The gravitational field equations are given by [2]:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda + Q_{\mu\nu} = 8\pi G_N \xi T_{\mu\nu}, \quad (1)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, $\xi(x)$ denotes the scalar field describing the variation of the gravitational ‘‘constant’’ and $G = G_N \xi(x)$ where G_N denotes Newton’s constant. We have chosen units with $c = 1$. We adopt the metric signature $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ where $\eta_{\mu\nu}$ is the Minkowski spacetime metric, $R = g^{\mu\nu}R_{\mu\nu}$ and Λ denotes the cosmological constant.

We have

$$Q_{\mu\nu} = \xi \left[\nabla^\alpha \nabla_\alpha \left(\frac{1}{\xi} \right) g_{\mu\nu} - \nabla_\mu \nabla_\nu \left(\frac{1}{\xi} \right) \right], \quad (2)$$

where ∇_μ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$. The quantity $Q_{\mu\nu}$ results from a boundary contribution arising from the presence of second derivatives of the metric tensor in R in the action. These boundary contributions are equivalent to those that occur in Brans-Dicke gravity theory [11].

In the original formulation of MOG [2], the vector field ϕ_μ satisfies massive Maxwell-Proca-type field equations. We shall generalize these field equations to massive Yang-Mills field equations of the form:

$$D_\nu B^{a\mu\nu} + \frac{\partial V(\phi)}{\partial \phi_\mu^a} + \frac{1}{\omega} \Delta_\nu \omega B^{a\mu\nu} = -\frac{1}{\omega} J^{a\mu}. \quad (3)$$

Here, $B^{a\mu\nu}$ denotes the Yang-Mills field:

$$B^{a\mu\nu} = \partial_\mu \phi_\nu^a - \partial_\nu \phi_\mu^a + C^{abc} \phi_\mu^b \phi_\nu^c, \quad (4)$$

and we have included contributions from the variation of the effective Yang-Mills coupling strength ω . Moreover, D_μ denotes the Yang-Mills covariant derivative acting on a Dirac field $\Psi(x)$:

$$D_\mu \Psi(x) = [\partial_\mu + \phi_\mu(x)]\Psi(x), \quad (5)$$

where

$$\phi_\mu(x) = -i\omega \phi_\mu^a(x)T^a, \quad B^{\mu\nu}(x) = -i\omega B^{a\mu\nu}(x)T^a. \quad (6)$$

Here, T^a are the matrix representations of the generators of the $U(n)$ symmetry group.

In the absence of contributions from the potential $V(\phi)$ the action for the Yang-Mills field ϕ_μ is invariant under the gauge transformations

$$\phi'_\mu(x) = U(x)\phi_\mu(x)U^{-1}(x) - [\partial_\mu U(x)]U^{-1}(x). \quad (7)$$

However, in general there will be contributions in the potential $V(\phi)$ from a mass term $-(1/2)\mu^2\phi^\mu\phi_\mu$ which will break the gauge symmetry. This mass term can be obtained from a spontaneous symmetry breaking of the action.

Our action takes the form

$$S = S_{\text{Grav}} + S_\phi + S_S + S_M, \quad (8)$$

where

$$S_{\text{Grav}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\frac{1}{G} (R + 2\Lambda) \right], \quad (9)$$

$$S_\phi = - \int d^4x \sqrt{-g} \left[\omega \left(\frac{1}{4} B^{\mu\nu} B_{\mu\nu} + V(\phi) \right) \right], \quad (10)$$

and

$$S_S = \int d^4x \sqrt{-g} \left[\frac{1}{G^3} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu G \nabla_\nu G - V(G) \right) + \frac{1}{G} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \omega \nabla_\nu \omega - V(\omega) \right) + \frac{1}{\mu^2 G} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \mu \nabla_\nu \mu - V(\mu) \right) \right]. \quad (11)$$

Moreover, $V(\phi)$ denotes a potential for the vector field ϕ^μ , while $V(G)$, $V(\omega)$ and $V(\mu)$ denote the three potentials associated with the three scalar fields $G(x)$, $\omega(x)$ and $\mu(x)$, respectively.

The current conservation law is of the form

$$D_\mu J^{a\mu} = 0. \quad (12)$$

The non-abelian charge associated with the fermion current density is no longer a constant of the motion, for the gauge field ϕ_μ is no longer neutral with respect to the

“fifth force” charge and their is a non-linear coupling between the ϕ_μ fields, leading to a non-zero ϕ_μ charge current density. This means that the fermions carrying the non-abelian fifth force charge obey a non-linear relation between the charges. In the Maxwell-Proca version of MOG, which is invariant under $U(1)$ abelian gauge transformations for massless ϕ_μ fields, the fermion fifth force point charges add linearly. The non-linear relation between fermion point charges in the Yang-Mills description of the fifth force can have important implications for the explanation of astrophysical data [7, 8, 9].

The total energy-momentum tensor is given by

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu} + T_{S\mu\nu}, \quad (13)$$

where $T_{M\mu\nu}$, $T_{\phi\mu\nu}$ and $T_{S\mu\nu}$ denote the energy-momentum tensor contributions of ordinary matter, the ϕ_μ field and the scalar fields ξ , ω and μ , respectively. We have

$$T_{\phi\mu\nu} = \omega \left[B_\mu^\alpha B_{\nu\alpha} - g_{\mu\nu} \left(\frac{1}{4} B^{\rho\sigma} B_{\rho\sigma} + V(\phi) \right) + 2 \frac{\partial V(\phi)}{\partial g^{\mu\nu}} \right]. \quad (14)$$

The $\xi(x)$ field yields the energy-momentum tensor:

$$T_{\xi\mu\nu} = -\frac{1}{G_N \xi^3} \left[\nabla_\mu \xi \nabla_\nu \xi - 2 \frac{\partial V(\xi)}{\partial g^{\mu\nu}} - g_{\mu\nu} \left(\frac{1}{2} \nabla_\alpha \xi \nabla^\alpha \xi - V(\xi) \right) \right], \quad (15)$$

where $V(\xi)$ denotes a potential for the scalar field ξ .

From the Bianchi identities

$$\nabla_\nu G^{\mu\nu} = 0, \quad (16)$$

and from the field equations (1), we obtain

$$\nabla_\nu T^{\mu\nu} + \frac{1}{\xi} \nabla_\nu \xi T^{\mu\nu} - \frac{1}{8\pi G_N \xi} \nabla_\nu Q^{\mu\nu} = 0. \quad (17)$$

The scalar field $\xi(x)$ satisfies the field equations

$$\nabla_\alpha \nabla^\alpha \xi + V'(\xi) + N(\xi) = \frac{1}{2} G_N \xi^2 \left(T + \frac{\Lambda}{4\pi G_N \xi} \right), \quad (18)$$

where

$$\begin{aligned} N(\xi) = & -\frac{3}{\xi} \left(\nabla_\alpha \xi \nabla^\alpha \xi + V(\xi) \right) + \frac{1}{16\pi} \xi^2 \nabla_\alpha \nabla^\alpha \left(\frac{1}{\xi} \right) \\ & + \xi \left(\frac{1}{2} \nabla_\alpha \omega \nabla^\alpha \omega - V(\omega) \right) + \frac{\xi}{\mu^2} \left(\frac{1}{2} \nabla_\alpha \mu \nabla^\alpha \mu - V(\mu) \right), \end{aligned} \quad (19)$$

and $T = g^{\mu\nu} T_{\mu\nu}$. Similar field equations hold for the scalar fields $\omega(x)$ and $\mu(x)$ [2].

We observe that our field equations (18) for the variation of G contain a potential $V(\xi)$, which is absent in standard Brans-Dicke gravity [11]. Moreover, the conservation of energy equation (17) is more general than in Brans-Dicke gravity in which $\nabla_\nu T_M^{\mu\nu} = 0$ is imposed from the outset.

The trace free Weyl curvature tensor is defined by

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu} + \frac{1}{3}Rg_{\mu[\sigma}g_{\rho]\nu}. \quad (20)$$

Under a conformal transformation of the metric:

$$\tilde{g}^{\mu\nu} = \Omega^{-2}g^{\mu\nu}, \quad (21)$$

the Weyl tensor is unchanged:

$$\tilde{C}^{\sigma}_{\mu\nu\rho} = C^{\sigma}_{\mu\nu\rho}. \quad (22)$$

The field equations (1) are not conformally invariant due to the non-vanishing trace of the energy-momentum tensor, $T \neq 0$.

3 Non-Singular Cosmology

Let us now consider a cosmological solution to our MOG theory. We adopt a homogeneous and isotropic FLRW background geometry with the line element

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (23)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $k = 0, -1, +1$ for a spatially flat, open and closed universe, respectively. Due to the symmetry of the FLRW background spacetime, we have $\phi_0 \neq 0$, $\phi_i = 0$ and $B_{\mu\nu} = 0$. The metric (23) is conformally flat to a Minkowski spacetime metric.

We define the energy-momentum tensor for a perfect fluid by

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}, \quad (24)$$

where $u^\mu = dx^\mu/ds$ is the 4-velocity of a fluid element and $g_{\mu\nu}u^\mu u^\nu = 1$. Moreover, we have

$$\rho = \rho_M + \rho_\phi + \rho_S, \quad p = p_M + p_\phi + p_S, \quad (25)$$

where ρ_i and p_i denote the components of density and pressure associated with the matter, the ϕ^μ field and the scalar fields ξ , ω and μ , respectively.

The modified Friedmann equations take the form [1, 2, 3]:

$$\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)} = \frac{8\pi G_N \xi(t) \rho(t)}{3} + f(t) + \frac{\Lambda}{3}, \quad (26)$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G_N \xi(t)}{3} [\rho(t) + 3p(t)] + h(t) + \frac{\Lambda}{3}, \quad (27)$$

where $\dot{a} = da/dt$ and

$$f(t) = \frac{\dot{a}(t)}{a(t)} \frac{\dot{\xi}(t)}{\xi(t)}, \quad (28)$$

$$h(t) = \frac{1}{2} \left(\frac{\ddot{\xi}(t)}{\xi(t)} - 2 \frac{\dot{\xi}^2(t)}{\xi^2(t)} + \frac{\dot{a}(t)}{a(t)} \frac{\dot{\xi}(t)}{\xi(t)} \right). \quad (29)$$

The conservation law for matter is given by

$$\dot{\rho} + 3 \frac{d \ln a}{dt} (\rho + p) + \rho \frac{\dot{\xi}}{\xi} + \mathcal{I} = 0, \quad (30)$$

where

$$\mathcal{I} = \frac{3}{8\pi G_N \xi a} (2\dot{a}f + a\dot{f} - 2\dot{a}h). \quad (31)$$

Let us make the simplifying approximation for the equations (18) with $\Lambda = 0$:

$$\ddot{\xi} + 3H\dot{\xi} + V'(\xi) = \frac{1}{2} G_N \xi^2 (\rho - 3p). \quad (32)$$

An approximate solution for ξ in terms of a given potential $V(\xi)$ and for given values of ρ and p can be obtained from (32).

We will assume the following conditions for a non-singular solution to occur at $t = 0$:

1. $a(0) > 0$ (hence, no singularity at $t = 0$),
2. $\dot{a}(0) = 0$ (bounce or static solution),
3. $\xi(0) > 0$,
4. $\ddot{a}(0) > 0$ (bounce solution),
5. $\rho(0) = 0$ and $\dot{\rho}(0) = 0$ (from which follows $\rho(t) > 0$ for $t > 0$ and $t < 0$.)

By setting the cosmological constant and the curvature constant to zero, $\Lambda = k = 0$, we get from the generalized Friedmann equations (26) and (27) for a singularity-free universe:

$$\xi(0)\rho(0) = 0. \quad (33)$$

Because $\xi(0) \neq 0$, otherwise the Friedmann equations become singular, we must have $\rho(0) = 0$.

We require that $\dot{\rho}(0) = 0$ to avoid ρ changing sign at $t = 0$. This means that the strong energy condition, $\rho \geq 0$, is satisfied for all t . Moreover, the conservation law (30) leads to $\mathcal{I}(0) = 0$. From (31) it follows that $a(0)\dot{f}(0) = 0$ and at $t = 0$:

$$\frac{d}{dt} \left(\frac{\dot{a}\dot{\xi}}{a\xi} \right) = \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \frac{\dot{\xi}}{\xi} + \frac{\dot{a}}{a} \left(\frac{\ddot{\xi}}{\xi} - \frac{\dot{\xi}^2}{\xi^2} \right) = 0. \quad (34)$$

Because $\dot{a}(0) = 0$ and $\ddot{a}(0) > 0$, we have $\dot{\xi}(0) = 0$.

Let us now return to the second Friedmann equation (27). We have from (29) at $t = 0$:

$$h = -\frac{1}{2} \xi \ddot{\Theta}, \quad (35)$$

where $\Theta(t) = 1/\xi(t)$. Therefore, from the second Friedmann equation we obtain at $t = 0$:

$$\frac{\ddot{a}}{a} = -4\pi G_N \xi \left(p + \frac{1}{2} \ddot{\Theta} \right). \quad (36)$$

Let us write this equation in the form at $t = 0$:

$$\frac{\ddot{a}}{a} = -4\pi G_N \xi (p_m + p_G), \quad (37)$$

where p_m denotes the matter pressure and we have for $\xi(0) > 0$ at $t = 0$:

$$p_G = \frac{\ddot{\Theta}}{8\pi G_N}. \quad (38)$$

We require that $\ddot{\Theta} < 0$ and $-p_G > p_m$ which gives $\ddot{a}(0) > 0$.

Spacetime at $t \sim 0$ is described by the conformally flat Minkowski metric:

$$ds^2 = dt^2 - a^2(0)(dx^2 + dy^2 + dz^2). \quad (39)$$

The conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad (40)$$

is well-defined for $\Omega = \sqrt{\xi}$, for ξ is finite at $t = 0$. Quantum fluctuations of the ξ field in the neighborhood of $t = 0$ cause the Minkowski spacetime to become unstable for $t \sim 0$ generating an expansion of the universe as t increases in time.

From the first Friedmann equation (26) with $k = \Lambda = 0$, we obtain

$$\frac{\dot{a}(t)}{a(t)} = \left[\frac{8\pi G_N \xi(t) \rho(t)}{3} + f(t) \right]^{1/2}. \quad (41)$$

This equation has the formal solution

$$a(t) = \exp \left\{ \int_0^t dt' \left[\frac{8\pi G_N \xi(t') \rho(t')}{3} + f(t') \right]^{1/2} + A \right\}, \quad (42)$$

where A is a constant of integration and we have $a(0) = \exp(A) = \text{constant}$. We require that a closed solution of (42) becomes the radiation dominant solution $a(t) \propto t^{1/2}$ as t increases from $t = 0$.

The Hawking-Penrose theorems [12, 13, 14] prove that a singularity must occur at $t = 0$ in classical GR when the weak and strong energy conditions are satisfied. The weak and strong energy conditions are satisfied when

$$T_{\mu\nu} U^\mu U^\nu \geq 0, \quad (43)$$

and

$$(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) U^\mu U^\nu \geq 0, \quad (44)$$

where U^μ is a timelike or null vector. The conditions (43) and (44) hold when $\rho \geq 0$ and $\rho + 3p \geq 0$, respectively. We have for $p_G < -p_m$ in the second Friedmann equation (37) that $\rho + 3p$ is negative for $t \sim 0$ violating the strong energy condition. We conclude from this that our non-singular cosmological solution is consistent with the Hawking-Penrose theorems, for the violation of the strong energy condition invalidates the no-go singularity theorems.

4 Evolution of Non-Singular Cosmology

Our cosmological solution derived from our MOG satisfies a non-singular solution in which the universe has $\rho(0) = 0$ and $\dot{a}(0) = \dot{\xi}(0) = 0$ and it satisfies the radiation dominated solution of the FLRW type for $t > 0$. The negative pressure caused by $p_G < -p_m$ at $t \sim 0$ due to the scalar field ξ can lead to a quantum mechanical production of particles. Such a scenario has been investigated by Parker and Fulling [15] and Brout, Englert and Gunzik [16].

The Weyl curvature tensor defined in (20) vanishes identically in the homogeneous and isotropic FLRW universe, $C_{\mu\nu\rho\sigma} = 0$. Therefore, the gravitational degrees of freedom vanish and due to the vanishing of the matter density, $\rho(0) = 0$, the entropy of the universe \mathcal{S} is at a minimum in the neighborhood of $t = 0$. As $\rho(t)$ grows from zero as $|t|$ increases for either $t \rightarrow -\infty$ or $t \rightarrow +\infty$, the entropy will increase and obey the second law of thermodynamics as the universe expands. However, this scenario requires that *the thermodynamical arrow of time reverses* at $t = 0$ and always points in the direction of increasing entropy. Thus, an observer will see a universe with *increasing* entropy as $t \rightarrow -\infty$. The solution for $a(t)$ given by (42) can be time-symmetric or time-asymmetric depending on the particular closed-form solution obtained from $\rho(t)$ and $\xi(t)$. At $t \sim 0$ the gravitational field and the negative pressure $p_G < 0$ can produce matter as pairs of particles are spontaneously created from vacuum fluctuations.

For our non-singular universe the Hubble parameter $H(t) = \dot{a}(t)/a(t)$ obeys $H(0) = 0$ and the universe stops expanding at $t = 0$. The epoch at $t \geq 0$ when the universe begins to expand consists of a hot radiation plasma and due to the red shift of the radiation predicts the observed uniform temperature of the CMB.

The proper particle horizon size is given by

$$d_H(t) = a(t) \int_{-\infty}^t \frac{dt'}{a(t')} = a(t) \int_{-\infty}^t \frac{da(t')}{a^2(t')H(t')}. \quad (45)$$

The horizon size $d_H(t)$ becomes infinite as the universe evolves from $t = -\infty$ through $t = 0$ to $t \rightarrow \infty$. This allows photons to be thermalized through repeated particle collisions and there is no problem with causality, predicting the uniform CMB temperature.

5 Conclusions

We have obtained from MOG a non-singular description of the large scale structure of spacetime. As the universe evolves from $t = -\infty$ there are no cosmological singularities and there is no “big bang” at the putative origin of the universe at $t = 0$. The combined physics of quantum production of matter and MOG explain the origin of matter in the universe shortly after $t = 0$ and the hot radiation plasma and the subsequent expansion of the universe has a similar evolution as in the big

bang model. We have seen that it is possible in the singularity-free MOG cosmology to solve the conundrum of the entropy problem, for the gravitational degrees of freedom are zero due to the vanishing of the Weyl curvature, and, because $\rho(0) = 0$ the entropy \mathcal{S} is at a minimum for $t \sim 0$. The arrow of time for negative and positive t must follow the increase in entropy from the minimum at $t = 0$. This is guaranteed by a reversal of the arrow of time at $t = 0$, which avoids a violation of the second law of thermodynamics. Matter is spontaneously created at $t \sim 0$ due to the gravitational field and quantum fluctuations of the vacuum with negative p_G , explaining the origin of matter and radiation.

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