

Large Scale Cosmological Inhomogeneities, Inflation And Acceleration Without Dark Energy

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Abstract

We describe the universe as a local, inhomogeneous spherical bubble embedded in a flat matter dominated FLRW universe. Generalized exact Friedmann equations describe the expansion of the universe and an early universe inflationary de Sitter solution is obtained. A non-perturbative expression for the deceleration parameter q is derived that can possibly describe the acceleration of the universe without dark energy, due to the effects associated with very long wave length super-horizon inflationary perturbations. The suggestion by Kolbe et al. [9] that long wave length super-horizon inflationary modes can affect a local observable through inhomogeneities is considered in the light of our exact inhomogeneous model.

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1 Introduction

In a recent article, we investigated a cosmology in which a spherically symmetric perturbation enhancement is embedded in an asymptotic FLRW universe [1]. The perturbation enhancement is described by an exact inhomogeneous solution of Einstein's field equations. We found that the large-scale inhomogeneities can lead to a reinterpretation of the luminosity distance d_L of a cosmological source in terms of its red shift z , owing to the observer dependence of these quantities. The time evolution and the expansion rate of the inhomogeneous universe can lead to intrinsic effects such as cosmic variance at large angles and long-wavelength perturbations not described by a FLRW homogeneous and isotropic universe. Therefore, the interpretation of the data using a FLRW model that the accelerating expansion of the universe is caused by dark energy may be misleading. This is important, for it is difficult to explain theoretically the postulated dark energy that causes the acceleration of the universe. The model also leads to an axis pointing towards the center of the spherically symmetric large scale perturbation enhancement with dipole, quadrupole and octopole moments aligned with the axis. It was shown that the luminosity distances and red shifts observed by different observers located at spatially different points of causally disconnected parts of the universe can have varying values. A spatial average of all these observations leads to an intrinsic cosmic variance in e.g. the deceleration parameter q . The distribution of CMB temperature fluctuations can be unevenly distributed in the northern and southern hemispheres.

The popular explanation for the observed large-scale homogeneity of the universe is that the universe underwent an initial inflationary period with more than 60 e-folds [2]. The inflationary cosmic expansion can stretch an initially small, smooth spatial region to a size larger than the horizon size today and explain the present day large-scale homogeneity. The question arises as to whether the initial *local* patch can be sufficiently homogenized to allow inflation to begin [3]. In the following, we shall consider the universe as an expanding bubble with an inhomogeneous

metric and generalized Friedmann equations, including inhomogeneous density and pressure and a cosmological constant. Our main assumptions are spherical symmetry and an inhomogeneous baryotropic fluid that satisfies an equation of state. For the case of a spatially flat inhomogeneous early universe, we obtain a de Sitter inflationary solution.

The acceleration of the expansion of the universe deduced from Type Ia supernovae observations and the CMB WMAP data [4, 5, 6] has been interpreted as due to the cosmological constant (vacuum energy), modifications of Einstein's gravitational field equations at large distances [7], and quintessence fields [8]. The quintessence explanations postulate a new form of matter with negative pressure called dark energy. Recently, it has been suggested that the acceleration is caused by very long wavelength, super-horizon perturbations generated by a period of inflation in the early universe [9, 10]. The backreaction of perturbations on an FLRW background universe has been the subject of investigation by several authors [11]. The predictions based on perturbation theory are limited by the condition $\Phi \ll 1$, where Φ is the gravitational potential.

The inflationary perturbation modes whose wavelengths presently are smaller than $\lambda \leq 10$ Mpc have entered the non-linear regime and have generated galaxies and clusters of galaxies, while longer wavelength modes at super-horizon scales $\geq c/H$ are entering the linear regime today. The effects of the sub-horizon modes are small due to fact that $\delta\rho/\rho \sim 10^{-5}$ at the surface of last scattering. Therefore, these sub-horizon modes produce negligible corrections at second order $\sim 10^{-8}$. However, the super-horizon modes could potentially create a correction to the deceleration parameter q , large enough to remove the need for dark energy. It has been argued recently [12, 13, 14, 15] that second order perturbation effects of the form $\Phi\nabla^2\Phi$ are described by a renormalization of the local spatial curvature and cannot (for a positive energy density) produce a negative deceleration parameter.

One problem with the perturbation calculations is that they ignore all higher gradient terms $\nabla^n\Phi$, and any *non-perturbative effects* that can have a significant influence on the inhomogeneity contributions due to very long wave length modes at super-horizon. These effects will occur for inflationary models in which the number of e-folds of inflation is much larger than the 60 e-folds required to create physically satisfactory fluctuations. In the following, we shall use the exact inhomogeneous model of ref. [1] to derive a formula for the deceleration parameter q that is non-perturbative and whose variance can lead to a negative value for q without dark energy and a cosmological constant Λ .

2 Inhomogeneous Friedmann Equations

Our action takes the form

$$S = S_G + S_M, \quad (1)$$

where

$$S_G = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(R - 2\Lambda). \quad (2)$$

The matter action is given by

$$S_M = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (3)$$

where ϕ is a scalar matter field and $V(\phi)$ is a potential.

For the sake of notational clarity, we write the FLRW line element

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (4)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The general, spherically symmetric inhomogeneous line element is given by [16, 17, 18, 19, 20, 21, 22, 1]:

$$ds^2 = dt^2 - X^2(r, t) dr^2 - R^2(r, t) d\Omega^2. \quad (5)$$

The energy-momentum tensor $T^\mu{}_\nu$ takes the barytropic form

$$T^\mu{}_\nu = (\rho + p)u^\mu u_\nu - p\delta^\mu{}_\nu, \quad (6)$$

where $u^\mu = dx^\mu/ds$ and, in general, the density $\rho = \rho(r, t)$ and the pressure $p = p(r, t)$ depend on both r and t . We have for comoving coordinates $u^0 = 1, u^i = 0, (i = 1, 2, 3)$ and $g^{\mu\nu}u_\mu u_\nu = 1$.

The Einstein gravitational equations are

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}, \quad (7)$$

where $\mathcal{R} = g^{\mu\nu}R_{\mu\nu}$ and Λ is the cosmological constant. Solving the $G_{01} = 0$ equation for the metric (5), we find that

$$X(r, t) = \frac{R'(r, t)}{f(r)}, \quad (8)$$

where $R' = \partial R/\partial r$ and $f(r)$ is an arbitrary function of r .

We obtain the two generalized Friedmann equations [1]:

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} + \frac{1}{R^2}(1 - f^2) - 2\frac{f f'}{R'R} = 8\pi G\rho + \Lambda, \quad (9)$$

$$\frac{\ddot{R}}{R} + \frac{1}{3}\frac{\dot{R}^2}{R^2} + \frac{1}{3}\frac{1}{R^2}(1 - f^2) - \frac{1}{3}\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} + \frac{1}{3}\frac{R''}{R'^2}\frac{f f'}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda, \quad (10)$$

where $\dot{R} = \partial R/\partial t$.

3 De Sitter Inflationary Solution

Let us now consider the very early universe and retain the pressure p and the cosmological constant Λ . We shall picture the early universe as an expanding spherically symmetric bubble with its origin at the big bang. We choose $f(r) = 1$ for all values of r , and obtain the generalized Friedmann equations

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} = 8\pi G\rho + \Lambda, \quad (11)$$

$$\frac{\ddot{R}}{R} + \frac{1}{3}\frac{\dot{R}^2}{R^2} - \frac{1}{3}\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda. \quad (12)$$

By using the notation $H_\perp = \dot{R}/R$ and $H_r = \dot{R}'/R'$, we can write (11) and (12) as

$$H_\perp^2 + 2H_\perp H_r = 8\pi G\rho + \Lambda, \quad (13)$$

$$\frac{\ddot{R}}{R} + \frac{1}{3}H_\perp^2 - \frac{1}{3}H_r H_\perp = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda. \quad (14)$$

For $H_\perp(r, t) = H_r(r, t) = H(t) = \dot{a}(t)/a(t)$ and $R(r, t) = a(t)$, we obtain the Friedmann equations of FLRW for a spatially flat universe:

$$H^2 = \frac{8\pi G\rho}{3} + \frac{1}{3}\Lambda. \quad (15)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda, \quad (16)$$

where $\rho = \rho(t)$ and $p = p(t)$.

From the Bianchi identities $\nabla_\nu G^{\mu\nu} = 0$, we obtain for the inhomogeneous model the conservation law

$$\nabla_\nu T^{\mu\nu} = 0. \quad (17)$$

This becomes

$$\partial_\nu p + \frac{1}{\sqrt{-g}} \partial_\nu [\sqrt{-g}(\rho + p)u^\mu u^\nu] + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0. \quad (18)$$

For $\Gamma^\mu_{00} = 0$ and $\sqrt{-g} = X(r, t)R^2(r, t) \sin \theta$, we obtain

$$\frac{dp}{dt} = \frac{1}{XR^2} \frac{d}{dt} [XR^2(\rho + p)]. \quad (19)$$

From (6) we get

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi), \quad (20)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\vec{\nabla}\phi)^2 - V(\phi). \quad (21)$$

Here, $\phi = \phi(r, t)$ depends on both r and t . From (18) we obtain

$$\dot{\rho}_\phi + \frac{1}{XR^2} \frac{d}{dt} (XR^2)(\rho_\phi + p_\phi) = 0. \quad (22)$$

Differentiating (20) with respect to t we have

$$\ddot{\phi} + \frac{1}{2} \frac{\partial}{\partial \phi} (\vec{\nabla}\phi)^2 + \frac{\partial}{\partial \phi} V(\phi) + \frac{d/dt(XR^2)}{XR^2} [\dot{\phi} + \frac{1}{3}(\vec{\nabla}\phi)^2/\dot{\phi}] = 0. \quad (23)$$

Let us assume an equation of state

$$p(r, t) = w\rho(r, t), \quad (24)$$

where w is a constant. Then, a vacuum equation of state has $w = -1$, while the matter dominated universe has $w = 0$. We choose for simplicity $f(r) = 1$ for all values of r and we assume that the universe is dominated by the cosmological constant with $p(r, t) = -\rho(r, t)$ and $\sqrt{\Lambda/3} = \sqrt{8\pi G\rho_{\text{vac}}/3}$. For the special case of a spatially flat universe, a solution to Eqs.(11) and (12) is given by

$$R(r, t) = R_0 \exp\left[(\sqrt{\Lambda/3})(r + t)\right]. \quad (25)$$

By a change of radial coordinate

$$\tilde{r} = R_0 \exp\left[(\sqrt{\Lambda/3})r\right], \quad (26)$$

we obtain the isotropic and homogeneous de Sitter metric:

$$ds^2 = dt^2 - \exp\left[2(\sqrt{\Lambda/3})t\right] d\tilde{r}^2 - \exp\left[2(\sqrt{\Lambda/3})t\right] \tilde{r}^2 d\Omega^2. \quad (27)$$

Thus, the inhomogeneous expanding bubble can inflate when the cosmological constant (vacuum energy) dominates the expanding bubble. The solution (25) is a special solution and a more general inhomogeneous inflating solution should exist for $f(r) \neq 0$, but it does illustrate that an inflationary solution of Einstein's field equations exists for our more general inhomogeneous metric.

Assuming that the matter scalar field ϕ dominates in the early universe, and substituting for ρ_ϕ and p_ϕ in (11) and (12), we obtain for $\Lambda = 0$:

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R}\frac{\dot{R}'}{R'} = 8\pi G \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2\phi + V(\phi) \right], \quad (28)$$

$$\frac{\ddot{R}}{R} + \frac{1}{3} \frac{\dot{R}^2}{R^2} - \frac{1}{3} \frac{\dot{R}'}{R'} \frac{\dot{R}}{R} = -\frac{8\pi G}{3} [\dot{\phi}^2 - V(\phi)]. \quad (29)$$

We can obtain an inflationary solution of the form (25) by assuming that $V(\phi) \gg \dot{\phi}^2$, $(\vec{\nabla})^2 \phi \sim 0$ and $V(\phi) \sim \text{constant}$. However, we are required to make assumptions about the scalar field ϕ , namely, that in a local patch of inflation it only depends on time t , so that the wave equation (23) can be solved to give the sub-horizon quantum fluctuations that seed the growth of structure. This appears to be an apparently unavoidable fine-tuning of the primordial scalar matter fields that occurs in generic inflationary models.

4 Late-Time Matter Dominated Universe

The late-time matter dominated universe will be pictured as a large scale perturbation enhancement that is described by an exact inhomogeneous spherically symmetric solution of Einstein's field equations. The perturbation enhancement is embedded in a matter dominated universe that approaches asymptotically an Einstein-de Sitter universe as $t \rightarrow \infty$. An observer will be off-center from the origin of coordinates of the spherical perturbation enhancement.

For the matter dominated Lemaitre-Tolman-Bondi (LTB) [16, 17, 18] model with zero pressure $p = 0$ and zero cosmological constant $\Lambda = 0$, the Einstein field equations demand that $R(r, t)$ satisfies

$$2R\dot{R}^2 + 2R(1 - f^2) = F(r), \quad (30)$$

with F being an arbitrary function of class C^2 . There exist three possible solutions depending on whether $f^2 < 1, = 1, > 1$ and they correspond to elliptic (closed), parabolic (flat), and hyperbolic (open) cases, respectively.

The proper density of matter can be expressed as

$$\rho = \frac{F'}{16\pi G R' R^2}. \quad (31)$$

We can solve (31) to obtain

$$\Omega - 1 \equiv \frac{\rho}{\rho_c} - 1 = \frac{1}{H_{\text{eff}}^2} \left(\frac{1 - f^2}{R^2} - 2 \frac{f f'}{R R'} \right), \quad (32)$$

where

$$H_{\text{eff}}^2 = H_{\perp}^2 + 2H_{\perp} H_r, \quad (33)$$

is an effective Hubble parameter and we have

$$8\pi G \rho_c = \frac{\dot{R}^2}{R^2} + 2 \frac{\dot{R}}{R} \frac{\dot{R}'}{R'}. \quad (34)$$

We have the three possibilities for the curvature of spacetime: 1) $f^2 > 1$ open ($\Omega - 1 < 0$), 2) $f^2 = 1$ flat ($\Omega - 1 = 0$), $f^2 < 1$ closed ($\Omega - 1 > 0$).

Since the WMAP data [6] shows that the universe is spatially flat to within a few percent, we shall consider the globally flat case $f^2(r) = 1$. The metric reduces to

$$ds^2 = dt^2 - R'^2(r, t) dr^2 - R^2(r, t) d\Omega^2. \quad (35)$$

A solution for the matter dominated (w=0) universe is

$$R(r, t) = r[t + \beta(r)]^{2/3}, \quad (36)$$

and $\beta(r)$ is an arbitrary function of r of class C^2 [19]. The metric (35) becomes

$$ds^2 = dt^2 - (t + \beta)^{4/3} (Y^2 dr^2 + r^2 d\Omega^2), \quad (37)$$

where

$$Y = 1 + \frac{2r\beta'}{3(t + \beta)}, \quad (38)$$

and

$$\rho = \frac{1}{6\pi G(t + \beta)^2 Y}. \quad (39)$$

The arbitrary function $\beta(r)$ can be specified in terms of a density on some spacelike hypersurface $t = t_0$. The metric and density are singular on the two hypersurfaces $t + \beta = 0$ and $Y = 0$, namely, $t_1 = -\beta$ and $t_2 = -\beta - 2r\beta'/3$, respectively. The model is only valid for $t > \Sigma(r) \equiv \text{Max}[t_1(r), t_2(r)]$, and the hypersurface $t(r) = \Sigma(r)$ defines the big-bang. However, our pressureless model requires that the surface $t(r) = \Sigma(r)$ describes the surface on which the universe becomes matter dominated (in the LFRW model this occurs at $z \sim 10^4$). We observe that even in the spatially flat LTB model, different parts of the universe can enter the matter dominated era at different times. Our expanding inhomogeneous, spherically symmetric bubble is embedded in the matter dominated metric (37). For $\beta = 0$ and in the limit $t \rightarrow \infty$ we obtain the Einstein-de Sitter universe

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2), \quad (40)$$

where $a(t) = t^{2/3}$. Thus, for $\beta = 0$ we obtain the FLRW model. Moreover, the expanding flat LTB model necessarily evolves to the homogeneous and isotropic FLRW model for a non-vanishing density, whatever the initial conditions.

5 The Inhomogeneous Cosmology Deceleration Parameter

Let us expand $R(r, t)$ in a Taylor series

$$\begin{aligned} R(r, t) &= R[r, t_0 - (t_0 - t)] = R(r, t_0) \left[1 - (t_0 - t) \frac{\dot{R}(r, t_0)}{R(r, t_0)} + \frac{1}{2} (t_0 - t)^2 \frac{\ddot{R}(r, t_0)}{R(r, t_0)} - \dots \right] \\ &= R(r, t_0) \left[1 - (t_0 - t) H_{0\perp} - \frac{1}{2} (t_0 - t)^2 q(r, t_0) H_{0\perp}^2 - \dots \right], \end{aligned} \quad (41)$$

where t_0 denotes the present epoch and $H_{0\perp} = \dot{R}(r, t_0)/R(r, t_0)$. Moreover, we have

$$q(r, t_0) = -\frac{\ddot{R}(r, t_0)R(r, t_0)}{\dot{R}^2(r, t_0)}. \quad (42)$$

By substituting for \ddot{R} from Eq.(12), we obtain

$$q(r, t_0) = \frac{1}{3} + \frac{4\pi\rho_0(r, t_0)}{3H_{0\perp}^2(r, t_0)} - \frac{\Lambda}{3H_{0\perp}^2(r, t_0)} - \frac{1}{3} \frac{H_{0r}(r, t_0)}{H_{0\perp}(r, t_0)}, \quad (43)$$

where $H_{0r}(r, t_0) = \dot{R}'(r, t_0)/R'(r, t_0)$.

If we set $H_{0\perp}(r, t_0) = H_{0r}(r, t_0) = H(t_0)$ where $H(t_0) = \dot{a}(t_0)/a(t_0)$, then we obtain the spatially flat FLRW expression for the deceleration parameter:

$$q = \frac{4\pi\rho_0}{3H_0^2} - \frac{\Lambda}{3H_0^2} = \frac{1}{2} - \Omega_\Lambda. \quad (44)$$

We see from (43) that different observers located in different causally disconnected parts of the sky will observe different values for the deceleration parameter q , depending upon their location and distance from the center of the spherically symmetric perturbation enhancement. This can lead to one form of cosmic variance, because the spatial average of all the observed values of local physical quantities, including the deceleration parameter q , will have an intrinsic uncertainty.

Let us rewrite (43) in the form

$$q = \frac{1}{3} + \Omega_{0m\perp} - \Omega_{0\Lambda\perp} - \Omega_{0H\perp}, \quad (45)$$

where

$$\Omega_{0m\perp} = \frac{4\pi\rho_0(r, t_0)}{3H_{0\perp}^2(r, t_0)}, \quad \Omega_{0\Lambda\perp} = \frac{\Lambda}{3H_{0\perp}^2(r, t_0)}, \quad \Omega_{0H\perp} = \frac{1}{3} \frac{H_{0r}(r, t_0)}{H_{0\perp}(r, t_0)}. \quad (46)$$

The variance of q is given by the exact non-perturbative expression:

$$\text{var}(q) \equiv \langle q^2 - \langle q \rangle^2 \rangle^{1/2} = \bar{q}, \quad (47)$$

where

$$\overline{(\dots)} = \frac{\int d^3x (\dots)}{\int d^3x}, \quad (48)$$

denotes the ensemble average.

We now set $\Lambda = 0$ in (45) and obtain

$$q = \frac{1}{3} + \Omega_{0m\perp} - \Omega_{0H\perp}. \quad (49)$$

If we have

$$\Omega_{0H\perp} > \frac{1}{3} + \Omega_{0m\perp}, \quad (50)$$

then the cosmic variance $\text{var}(q)$ can be negative *and cause the universe to accelerate without a cosmological constant or dark energy*. If the super-horizon long wave inflationary modes can generate *non-perturbatively* a sufficiently large effect of $O(1)$, then this can drive $\text{var}(q)$ to negative values.

6 Conditions for Deceleration Parameter

Hirata and Seljak [14] have recently raised some critical questions regarding the Kolbe et al. [9] suggestion that long wave length super-horizon perturbations can cause q to be negative and not violate the strong energy condition $\rho + 3p \geq 0$. They used non-FLRW methods to discuss the deceleration parameter q . They begin by defining a local Hubble value

$$H_1 = \frac{1}{3} \nabla_\mu u^\mu, \quad (51)$$

and a deceleration parameter

$$q_1 = -1 - \frac{1}{H_1^2} u^\mu \nabla_\mu H_1. \quad (52)$$

Here, $u^\mu \nabla_\mu$ is the Lagrangian proper time derivative d/t_{proper} associated with a matter particle. The expansion tensor $\theta_{\mu\nu}$ is related to H_1 by $\theta \equiv \theta^\mu{}_\mu = 3H_1$. The matter particles follow geodesics with $\theta_{\mu\nu} u^\nu = \theta_{\mu\nu} u^\mu = 0$. From the Raychaudhuri equation

$$\frac{d\theta}{dt} = -\frac{\theta^2}{3} - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} u^\mu u^\nu, \quad (53)$$

where $R_{\mu\nu}$ is the Ricci tensor and σ and ω denote the shear and vorticity, respectively, it follows that

$$H_1^2 q_1 = \frac{1}{3} (\sigma^\mu{}_\nu \sigma^\nu{}_\mu - \omega^\mu{}_\nu \omega^\nu{}_\mu) - \frac{1}{3} R_{\mu\nu} u^\mu u^\nu. \quad (54)$$

From Einstein's field equation (7) for $\Lambda = 0$ we obtain

$$H_1^2 q_1 = \frac{1}{3}(\sigma^\mu{}_\nu \sigma^\nu{}_\mu - \omega^\mu{}_\nu \omega^\nu{}_\mu) + \frac{4\pi G}{3}(\rho + 3p). \quad (55)$$

We have $\sigma^\mu{}_\nu \sigma^\nu{}_\mu > 0$, so that if the strong energy condition is satisfied, $\rho + 3p \geq 0$, and the vorticity $\omega^\mu{}_\nu = 0$, it follows that $q_1 \geq 0$. Hirata and Seljak now state that in the synchronous comoving gauge used by Kolbe et al. to calculate the perturbations up to second order, $\omega = 0$, and the deceleration parameter must satisfy $q_1 \geq 0$ when the strong energy condition is satisfied.

In our model the vorticity $\omega \neq 0$ for our exact inhomogeneous solution that describes the large scale perturbation enhancement, so it does not necessarily follow that $q_1 \geq 0$. We do not have to use the synchronous gauge to do calculations of the non-perturbative inhomogeneity and so, in general, $\omega \neq 0$.

Harati and Seljak show that for a single inflaton inflationary model, the long wave length fluctuations at super-horizon cannot give rise to an effect $O(1)$ for $\text{var}(q)$. However, this result is model dependent and a more general hybrid inflation model could lead to infrared modes that can lead to a shifting of $\text{var}(q)$ to negative values, and explain the acceleration of the universe without negative pressure dark energy or a cosmological constant.

If the vorticity $\omega \neq 0$ and the shear $\sigma \sim 0$, then we can consider the possibility that the long wave length super-horizon inflationary modes can lead to a negative q . However, it can be argued on the basis of standard inflationary models that due to the large increase of entropy during re-heating, there will be a large increase of $\rho + 3p$ in (55), while the vorticity is not increased accordingly and, therefore, one may not expect that q can be driven to negative values by a sufficiently large vorticity value. But this depends on the re-heating model and may not be a generic feature of inflation.

Alternatively, one could investigate these issues in a non-inflationary model, such as the variable speed of light bimetric gravity theory [23], which does predict a scale invariant spectrum with the spectral index $n_s \sim 0.97$ in agreement with the WMAP result. The superluminal mechanism that solves the horizon and flatness problems, namely, that the ratio $\gamma = c_\gamma/c_g$, where c_γ and c_g denote the speed of light and the speed of gravitational waves, respectively, becomes unity soon after it reaches a high value in the very early universe, as the effect of a phase transition with $c_g = c = \text{constant}$ (c the currently measured speed of light). The fluctuations that form the seeds of large scale structure are born super-horizon but they do not lead to the same consequences as inflation with a large number of e-folds.

7 Conclusions

We have shown that a special de Sitter inflationary solution exists for a spherically symmetric and inhomogeneous expanding bubble. Further work must be carried out to find more general inhomogeneous de Sitter-like solutions from our generalized Friedmann equations.

We have argued that an exact non-perturbative treatment of inhomogeneities in a cosmology describing a spherically symmetric expanding universe, embedded in an asymptotic Einstein-de Sitter universe, can significantly modify local observables such as the red shift, expansion rate and the luminosity distance when super-horizon, long wave length modes are taken into account. We expect that this will hold true even in the case in which only adiabatic modes are present and spatial gradients and non-perturbative effects are not ignored.

Care must be taken when arguments are made about the sizes of effects associated with the super-horizon inflationary modes in cosmology. According to our results, the various features of the power spectrum derived from the WMAP data could also be subject to significant modifications at the non-perturbative level depending on the model of inflation adopted. The effects of an expansion parameter θ and shear and vorticity parameters should be reinterpreted in our inhomogeneous model together with the effects of very long wave length super-horizon inflationary modes, before conclusions are drawn about their relative importance when compared to an FLRW universe.

The fact that we should consider the effects of super-inflationary modes in cosmology and that they lead to a cosmic variance for various local physical quantities, even though these modes cannot presently be observed, may be inescapable and lead to an intrinsic uncertainty in our interpretation of the universe.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada. I thank Robert Brandenberger, Joel Brownstein, Martin Green and Moshe Rozali for helpful discussions.

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