Physics 130 General Relativity Seminar Week-8 Fall 2008

Readings: Woodhouse - General Relativity
Chapter 6 - Einstein's Equation
Chapter 7 - Spherical Symmetry
Walecka - General Relativity
Chapter 7 - General Relativity

Present Ideas and Lead Discussion:

Woodhouse Chapter 6	__Erin____
Woodhouse Chapter 7	__Ben P___
Walecka Chapter 7	__Robert__

Present Problems:

Wo7.1 - Time	__Erin____
Wo7.2 - Lie Derivative Wo7.3 - More Lie derivatives	__Ben P___
Wa7.1 - How many independent components? Wa7.2 - Ricci=0 does not imply Riemann=0	__Robert__
Wa7.3 - Manipulating geodesic equations Wa7.4 - Newtonian limit	__Emma____
Wa7.5 - Circular motion Wa7.6 - Radial motion	__Eric____
Wa7.8 - Time difference Wa7.9 - Length of meter stick	__Markus__

EP27. Timelike Geodesics __Chris___

Find the timelike geodesics for the metric

$$ds^2 = \frac{1}{t^2}\left(-dt^2 + dx^2\right)$$

EP28. More Geodesics __Chris___

Consider the 2-dimensional metric

$$ds^2 = a^2\left(d\chi^2 + \sinh^2\chi d\phi^2\right)$$

(a) Compute the connection coefficients Γ^i_{jk}

(b) Compute all components of the Riemann tensor, R^{ij}_{kl}, the Ricci tensor, R^i_j, and the Ricci scalar R.

(c) A geodesic starts at $\chi = b$, $\phi = 0$ with tangent $d\phi/d\lambda = 1$, $d\chi/d\lambda = 0$. Find the trajectory $\chi(\phi)$.

(d) A second geodesic starts at $\chi = b + \xi\ (\xi << 1)$, also initially in the ϕ-direction. How does the separation initially increase or decrease along the two curves.

(e) What is the shape of the geodesic trajectory as $a \to \infty$, $\chi \to 0$ with $r = a\chi$ fixed.

EP29. Parallel Transport on a Sphere (Grade #2) **_Everyone_**

(a) On the surface of a 2-sphere of radius a,

$$ds^2 = a^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Consider the vector $\vec{A}_0 = \vec{e}_\theta$ at $\theta = \theta_0$, $\phi = 0$. The vector is parallel transported all the way around the latitude circle $\theta = \theta_0$ (i.e., over the range $0 \le \phi \le 2\pi$ at $\theta = \theta_0$). What is the resulting vector $\vec{A}$? What is its magnitude $\left(\vec{A}\cdot\vec{A}\right)^{1/2}$?

HINT: derive differential equations for A^θ and A^ϕ as functions of ϕ.

EP30. Curvature on a Sphere __Ben G___

(a) Compute all the nonvanishing components of the Riemann tensor R_{ijkl} $((i,j,k,l) \in (\theta,\phi))$ for the surface of a 2-sphere.

(b) Consider the parallel transport of a tangent vector $\vec{A} = A^\theta \vec{e}_\theta + A^\phi \vec{e}_\phi$ on the sphere around an infinitesimal parallelogram of sides $\vec{e}_\theta d\theta$ and $\vec{e}_\phi d\phi$. Using the results of part (a), show that to first order in $d\Omega = \sin\theta d\theta d\phi$, the length of $\vec{A}$ is unchanged, but its direction rotates through an angle equal to $d\Omega$.

(c) Show that, if $\vec{A}$ is parallel transported around the boundary of any simply connected solid angle Ω, its direction rotates through an angle Ω. ("Simply connected" is a topological term meaning that the boundary of the region could be shrunk to a point; it tells us that there are no holes in the manifold or other pathologies). Using the result of part (b) and intuition from proofs of Stoke's theorem, this should be an easy calculation. Compare with the result of **EP30.**

EP31. Riemann Tensor for 1+1 Spacetimes __Sam_____

(a) Compute all the nonvanishing components of the Riemann tensor for the spacetime with line element

$$ds^2 = -e^{2\phi(x)}dt^2 + e^{2\psi(x)}dx^2$$

(b) For the case $\phi = \psi = \frac{1}{2}\ln\left|g(x - x_0)\right|$ where g and x_0 are constants, show that the spacetime is flat and find a coordinate transformation to globally flat coordinates $(\bar{t},\bar{x})$ such that $ds^2 = -d\bar{t}^2 + d\bar{x}^2$.

EP32. About Vectors Tangent to Geodesics _Margaret_

Let $x^\mu(\tau)$ represent a timelike geodesic curve in spacetime, where τ is the proper time as measured along the curve. Then $u^\mu \equiv dx^\mu / d\tau$ is tangent to the geodesic curve at any point along the curve.

(a) If $g_{\mu\nu}$ is the metric of spacetime, compute the magnitude of the vector u^μ. Do not use units where c = 1, but keep any factors of c explicit. Compare your result with the one obtained in flat Minkowski spacetime. HINT: The magnitude of a timelike vector v^μ is given by $\left(-g_{\mu\nu}v^\mu v^\nu\right)^{1/2}$.

(b) Consider a contravariant timelike vector v^μ at a point P on the geodesic curve. Move the vector v^μ from the point P to an arbitrary point Q on the geodesic curve via parallel transport. Prove that the magnitude of the vector v^μ at the point Q equals the magnitude of the vector v^μ at point P.

(c) Suppose that at the point P on the geodesic curve, $v^\mu = u^\mu$. Now, parallel transport the vector v^μ along the geodesic curve to arbitrary point Q. Show that $v^\mu = u^\mu$ at the point Q.

NOTE: This result implies that a vector tangent to a geodesic at a given point will always remain tangent to the geodesic curve when parallel transported along the geodesic.