Physics 130 General Relativity Seminar Week-7 Fall 2008

Readings: Woodhouse - General Relativity
Chapter 4 - Curved Space-Time
Chapter 5 - Tensor Calculus

Present Ideas and Lead Discussion:

Chapter 4 __Emma________
Chapter 5 __Markus______

Present Problems:

4.1 - Derivatives, derivatives... __Emma________
4.2 - Vector fields
4.3 - Lagrangian solution curve

4.4 - Closed static universe __Chris_______
4.5 - 5-dimensional space-time

5.1 - Gradient covector __Markus______
5.2 - Geodesic triangles
5.3 - Vector field property...

5.4 - Exterior derivative __Eric________
5.5 - Torsion-free connection
5.6 - Transformation rule

5.7 - A bunch of properties.... __Erin________
5.8 - Levi-Civita connection
5.9 - Zero tensor

5.10 - Independent components __Margaret____
5.11 - Symmetries of Riemann Tensor
5.12 - Covector field and geodesic deviation

5.13 - Conditions on Maxwell's equations __Ben P_______
5.14 - Integral curves = geodesics
5.15 - Geodesic equations

5.16 - Tensor field identity __Ben G_______

EP23. __Robert______

Find an expression for

$$\nabla_c \nabla_d T^a_b - \nabla_d \nabla_c T^a_b$$

in terms of the Riemann tensor.

EP24. __Sam_________

(a) Define an isometry

(b) Define the Killing vector and show that it satisfies

$$\nabla_a k_b + \nabla_b k_a = 0$$

(c) Show how the Killing vector defines a constant along geodesics

(d) If k_a and l_a are Killing vectors, show that

$$[k,l]_a = k^b \nabla_b l_a - l^b \nabla_b k_a$$

is also a Killing vector. It is useful to recall the symmetry properties of the Riemann tensor

$$R_{abcd} = -R_{bacd} \quad \text{and} \quad R_{abcd} = R_{cdab}$$

EP25. __Ben G______

A paraboloid in three dimensional Euclidean space

$$ds^2 = dx^2 + dy^2 + dz^2$$

is given by

$$x = u\cos\phi \quad , \quad y = u\sin\phi \quad , \quad z = u^2/2$$

where $u \geq 0$ and $0 \leq \phi \leq 2\pi$.

(a) Show that the metric on the paraboloid is given by

$$ds^2 = (1+u^2)du^2 + u^2 d\phi^2$$

(b) Writing $x^1 = u$ and $x^2 = \phi$ find the Christoffel symbols for this metric.

(c) Solve the equation for parallel transport

$$U^a \nabla_a V^b = 0$$

where

$$U^a = \frac{dx^a}{dt}$$

for the curve $u = u_0$ where u_0 is a positive constant and with initial conditions $V^1 = 1$ and $V^2 = 0$. HINT: the problem is simplified if you take $t = \phi$. The equation of parallel transport will give you two coupled equations for U^1 and U^2, differentiating the $dU^1/d\phi$ equation again allows you to decouple the U^1 equation].

EP26 (Grade #1). __Everyone_____

A certain two-dimensional world is described by the metric

$$ds^2 = \frac{dx^2 + dy^2}{\left[1 + \frac{x^2+y^2}{4a^2}\right]^2}$$

(a) Compute the connection coefficients Γ^i_{jk}.

(b) Let $\vec{\xi} = -y\hat{e}_x + x\hat{e}_y$. Show that $\vec{\xi}$ is a solution of Killing's equation.

(c) What is the conserved quantity that corresponds to this symmetry? Show from the geodesic equation that this quantity is indeed conserved.

(d) Compute the Riemann tensor R^{ij}_{kl}, the Ricci tensor R^i_j, and the Ricci scalar R. What is the shape of this world?