Physics 130 General Relativity Seminar Week-4 Fall 2008

Readings: Walecka - General Relativity
Chapter 2 - Particle on a 2-Dimensional Surface
Chapter 3 - Curvilinear Coordinate Systems
Chapter 4 - Particle on a 2-Dimensional Surface - Revisited

Present Ideas and Lead Discussion:

Chapter 2	__Ben G________
Chapter 3	__Margaret_____
Chapter 4	__Eric_________

Present Problems:

2.1 - A unified surface	__Sam__________
3.1 - Kronecker delta	
3.2 - Contraction	
3.3 - Cartesian coordinates	__Emma_________
3.4 - Point transformation	
3.5 - Stress tensor	
3.6 - Affine connection	__Ben P________
3.7 - Gradients	
3.8 - Global basis	
4.1 - Constrained to a surface	
4.2 - Lagrange and geodesics	__Eric_________

EP8. __Markus_______

Consider a tensor T_{ij} in three dimensional Euclidean space. Under an arbitrary rotation of the three-dimensional coordinate space, the tensor is transformed. Show that

(a) If T_{ij} is symmetric and traceless, then the transformed tensor is symmetric and traceless.
(b) If T_{ij} is antisymmetric, then the transformed tensor is symmetric.

State the analogous result for a tensor $T^{\mu\nu}$ in four-dimensional Minkowski space. Define carefully what you mean by a traceless tensor in this case.

EP9. __Markus______

Under a Lorentz transformation, a tensor transforms as follows:

$$F'^{\mu\nu} = \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}F^{\alpha\beta}$$

where Λ is the Lorentz transformation matrix. Consider an inertial frame K at rest, and a second inertial frame K′ moving with velocity v along the x-direction with respect to K. Using the explicit result for Λ corresponding to the transformation between K and K′, determine the electric and magnetic fields in frame K′ in terms of the corresponding fields in frame K.

EP10. __Ben G_______

A Lorentz transformation is the product of a boost with rapidity ζ in direction $\hat{n}_1$, followed by a boost with rapidity ζ in direction $\hat{n}_2$, followed by a boost with rapidity ζ in direction $\hat{n}_3$, where ζ is the same in each case, and the three directions $\hat{n}_1$, $\hat{n}_2$ and $\hat{n}_3$ lie in the same plane separated by 120^{o}. What is the resulting transformation? To lowest order for small ζ, is it a boost or a rotation? At what order does the other (boost or rotation) enter?

EP11. __Chris_______

(a) Observer O at "rest" sees a symmetric tensor $T^{\mu\nu}$ to be diagonal with components (ρ,p,p,p). What are the components of $T_{\mu\nu}$?

(b) Frame O′ moves with speed v in the +x direction with respect to O. What are the components of $T'^{\mu\nu}$ in frame O′? What are the components of $T'_{\mu\nu}$? How can the "rest frame" be identified? Suppose that $p = -\rho$ in the original frame O, what is $T'^{\mu\nu}$ then? Make an insightful observation.

EP12. __Robert______

(a) Show that the sum of any two orthogonal (scalar product is zero) spacelike vectors is spacelike.

(b) Show that a timelike vector and a null vector cannot be orthogonal.

EP13. __Erin_______

Find the Killing vectors for flat space $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$, i.e., write out Killing's equation in flat space, differentiate it once and then solve the resulting differential equation.