Physics 130 General Relativity Seminar Week-12 Fall 2008

Readings: Woodhouse - General Relativity
Chapter 12 - Redshift and Horizons
Walecka - General Relativity
Chapter 11 - Cosmology

Present Ideas and Lead Discussion:

Woodhouse Chapter 12	__Ben P___
Walecka Chapter 11	__Robert__

Present Problems:

Wo-Ex12.1 - Unique Value of Proper Time	__Ben P___
Wo-Ex12.2 - Null Geodesics	
Wo-Ex12.3 - Stationary Observer:Kerr Spacetime	
Wo-Ex12.4 - k = -1 metric	_Margaret_
Wo-Ex12.7 - de Sitter metric	
Wa11.1 - The Cosmology in the Chapter	__Robert__
Wa11.2 - Put the Pressure Back	
Wa11.3 - More Properties of the Cosmology	__Eric____
Wa11.4 - Proper Distance	
Wa11.5 - Red-Shift Stuff	__Chris___
Wa11.6 - Horizon Stuff	
Wa11.8 - Cosmic Microwave Background	__Sam_____
Wa11.9 - Temperature versus Scale Factor	

EP56. Robertson-Walker = Minkowski __Emma____

The Robertson-Walker line element for absolutely empty space, $T_i^j = 0$ and $\Lambda = 0$, is

$$ds^2 = dt^2 - a^2(t)\left(\frac{dx^2}{1+x^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

with $a(t) \propto t$. Show that this describes flat space and find the coordinate transformation that brings it to the Minkowski form.

EP57. Red Shift in Model Galaxy __Emma____

Assume that the universe is isotropic and spatially flat. The metric then takes the form

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

where r, θ, and ϕ are co-moving coordinates. By this is meant any galaxy will have constant values of r,θ,ϕ (peculiar motions of galaxies are neglected). The universe is assumed to be matter-dominated with matter density $\rho(t)$ at time t.

(a) Under this circumstance show that the Einstein equations are

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 \quad and \quad \ddot{a} = -\frac{4\pi G}{3}\rho a$$

(b) From the fact that light propagates along null geodesics, show that the cosmological red shift of spectral lines emitted at time t_e and received at time t_0, defined as

$$Z = \frac{\text{wavelength of received line} - \text{wavelength of emitted line}}{\text{wavelength of emitted line}}$$

is

$$Z = \frac{a_0}{a_e} - 1$$

where $a_0 = a(t_0)$, $a_e = a(t_e)$.

(c) In the cosmological model under discussion a given galaxy will decrease in angular size with increasing distance from the observer - up to a critical distance. Beyond this the angular size will increase with distance. What is the red shift Z_{crit} corresponding to the minimum in angular size?

EP58. Expanding Universe _Markus__

The metric of the expanding universe has the form

$$ds^2 = dt^2 - R^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

where the possible curvature of space has been neglected. The detailed form of $R(t)$ depends on the matter content of the universe.

(a) A particle of mass m has energy E_0 and momentum p_0 at time t_0; assume $R(t_0)=R_0$. The particle thereafter propagates freely except for the effects of the above metric. Calculate the energy and momentum as a function of time.

(b) Suppose that the early universe contained a gas of non-interacting massless particle (perhaps photons) subject to gravitational effects only. Show that if at time t0 they were in a thermal distribution at temperature T_0, they remained in a thermal distribution later, but with a temperature that depends on time in a fashion you should determine. HINT: EP55 shows that

photon frequencies change like: $\frac{\nu'}{\nu} = \frac{R(t)}{R(t')}$

volumes change like: $\frac{V(t')}{V(t)} = \frac{R^3(t')}{R^3(t)}$

(c) Show that, instead, a gas of non-interacting massive particles initially in a thermal distribution would not remain in a thermal distribution under the influence of the expansion of the universe.

(d) Suppose that the early universe contained a non-interacting gas of massless photons and also a non-interacting gas of massive particles of mass m (massive neutrinos to be definite). Suppose that at some early time the photons and neutrinos were both in a thermal distribution with a temperature $kT = mc^2$ (m being the neutrino mass) for both photons and neutrinos. It has been observed that in today's universe the photons are in a thermal distribution with kT about $3\text{x}10^{-4}$ eV. In terms of the neutrino mass, what (roughly) would be the typical velocity and kinetic energy of a neutrino today? Assume $m \gg 3\text{x}10^{-4}$ eV.

EP59. Homogeneous, Isotropic Universe __Erin___

Consider a homogeneous, isotropic cosmological model described by the line element

$$ds^2 = -dt^2 + \left(\frac{t}{t_*}\right)\left(dx^2 + dy^2 + dz^2\right)$$

where t_* is a constant

(a) Is the model open, closed or flat?
(b) Is this a matter-dominated universe? Explain.
(c) Assuming the Friedmann equation holds for this universe, find $\rho(t)$.

EP60. Matter-Dominated RW Universe __Erin____

Suppose that a galaxy is observed to have a red-shift z = 1. Assuming a matter-dominated RW cosmology, at what fraction t/t_0 of the present age of the universe did the light leave this galaxy?

EP61. Flat Dust Universe __Ben G___

Consider a flat dust universe with zero cosmological constant.

(a) Solve the cosmological equations and derive the time evolution of the scale parameter a(t).
(b) By considering light emitted at time t, and received at the present time t_0, show that the distance to a star of red-shift z is given by

$$s = 3t_0\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

(c) Explain why a flat universe with zero cosmological constant containing a mixture of dust and radiation will eventually be dominated by the dust.

EP62. Particle Horizon in Flat Dust Universe __Ben G___

The particle horizon is the radius of the sphere of all particles that could be seen by us. It is the maximum straight line distance that could be travelled by a light ray since the beginning of the universe. Obviously, in a static universe this would be t_0. What is it for a k = 0 dust universe?

EP63 The Horizon inside a Collapsing Shell __Markus__

Consider the collapse of a spherical shell of matter of very small thickness and mass M. The shell describes a spherical three-surface in spacetime. Outisde the surface, the geometry is the Schwarzschild geometry with this mass. Inside make the following assumptions:

(1) The worldline of the shell is known as a function $r(\tau)$ going to zero at some finite proper time.
(2) The geometry inside the shell is flat.
(3) The geometry of the three-surface of the collapsing shell is the same inside as outside.

(a) Draw two spacetime diagrams: one an Eddington-Finkelstein diagram and the other corresponding to the spacetime inside in a suitable set of coordinates. Draw the worldline of the shell on both diagrams and indicate how points on the inside and outside correspond. Locate the horizon inside the shell as well as outside.
(b) How does the area of the horizon inside the shell change moving along the light rays which generate it?

EP64 Two Observers on a Kruskal Diagram __Chris___

Two observers in two rockets are hovering above a Schwarzschild black hole of mass M. They hover at fixed radius R such that

$$\left(\frac{R}{2M}-1\right)^{1/2} e^{R/4M} = \frac{1}{2}$$

and fixed angular position. (In fact $R \approx 2.16M$). The first observer leaves this position at t = 0 and travels into the black hole on a straight line in a Kruskal diagram until destroyed in the singularity at the point where the singularity crosses u = 0. The other observer continues to hover at R.

(a) On a Kruskal diagram sketch the worldlines of the two observers.
(b) Is the observer who goes into the black hole following a timelike worldline?
(c) What is the latest Schwarzschild time after the first observer departs that the other observer can send a light signal which will reach the first before being destroyed in the singularity?