Physics 130 General Relativity Seminar Week-11 Fall 2008

Readings: Woodhouse - General Relativity
Chapter 11 - Gravitational Waves
Walecka - General Relativity
Chapter 12 - Gravitational Radiation

Present Ideas and Lead Discussion:

Woodhouse Chapter 11 __Sam_____
Walecka Chapter 12 __Eric____

Present Problems:

Wo11.1 - Transverse traceless gauge __Sam_____
Wo11.2 - Prove general formula _Margaret_
Wo11.3 - Equivalent to plane wave _Robert___

Wa12.3 - Lagrangian features __Eric____
Wa12.4 - Waves from two moving particles __Emma____

Wa12.5 - Gravitational waves source __Erin____
Wa12.6 - Wave properties

Wa12.7 - Power in gravitational waves __Chris___

EP49A. Don't Do Anything! (Graded #5) _Everyone_

An astronaut in command of a spaceship equipped with a powerful rocket motor enters the horizon $r=r_s$ of a Schwarzschild black hole.

(a) Prove that in proper time no larger than $(\pi/2)r_s$, the astronaut reaches the singularity at r=0.

(b) Prove that in order to avoid the singularity for as long as possible, the astronaut ought to move in a purely radial direction. HINT: For purely radial motion, with dr < 0 and $dt = d\phi = d\theta = 0$, show that the increment in proper time is

$$d\tau = -\frac{dr}{\sqrt{\frac{r_s}{r}-1}} \quad , \quad for\ r \le r_s$$

and then integrate this between $r=r_s$ and r=0 to obtain

$$\Delta\tau = (\pi/2)r_s$$

Finally, check that if dt, $d\phi$, $d\theta$ are different from zero, then the increment $d\tau$, for a given value of -dr, is necessarily smaller than the value given above.

(c) Show that in order to achieve the longest proper time the astronaut must use her rocket motor in the following way: outside the horizon, she must brake her fall so as to arrive at $r=r_s$ with nearly zero radial velocity; inside the horizon she must shut off her motor and fall freely. HINT: show that

$\Delta\tau = (\pi/2)r_s$ corresponds to free fall from $r=r_s$.

EP49B Escape from Black Hole by Ejecting Mass __Erin_____

A spaceship whose mission is to study the environment around black holes is hovering at the Schwarzschild radius coordinate R outside a spherical black hole of mass M. To escape back to infinity, the crew must eject part of the rest mass of the ship to propel the remaining fraction to escape velocity. What is the largest fraction f of the rest mass that can escape to infinity? What happens to this fraction as R approaches the Schwarzschild radius of the black hole?

EP50. Gravitational Wave Stuff _Ben P____

(a) Explain briefly why in Einstein's theory of general relativity it is impossible to have monopole or dipole gravitational radiation.

(b) Suppose two compact stars, each of one solar mass, are in circular orbit around each other with a radius of one solar radius. What is the approximate rate of energy loss due to gravitational radiation from this system? What is the time scale for decay for this orbit? Take

$$solar\ mass = 2\times10^{33}\,gm$$

$$solar\ radius = 7\times10^{10}\,cm$$

EP51. More Waves from Masses on a Spring __Emma____

Two equal masses M are at the ends of a massless spring of unstretched length L and spring constant k. The masses started oscillating in line with the spring with an amplitude A so that their center of mass remains fixed.

(a) Calculate the amplitude of gravitational radiation a long distance away from the center of mass of the spring as a function of the angle θ from the axis of the spring to lowest non-vanishing order in A.

(b) Analyze the polarization of the radiation.

(c) Calculate the angular distribution of power radiated in gravitational waves.

EP52. Waves from Accelerating Particle __Ben G___

A particle of mass m moves along the z-axis according to $z(t)=gt^2/2$ (g is a constant) between times t = -T andt = +T and is otherwise moving with constant speed. Calculate the gravitational wave metric perturbations at a large distance L along the positive z-axis.

EP53. Waves from Colliding Battleships __Ben G___

In a desperate attempt to generate gravitational radiation artificially, we take two large battleships of 70,000 tons each, and we make them collide head-on at 40 km/h. Assume that during the collision the battleships decelerate at a constant rate and come to rest in 2.0 sec.

(a) Estimate the gravitational energy radiated during the collision. Treat the battleships as point masses.
(b) Could we detect these waves?

EP54. Waves from Cannon __Markus__

A cannon placed at the origin of coordinates fires a shot of mass 50 kg in a horizontal direction. The barrel of the cannon has a length of 2.0 m; the shot has a uniform acceleration while in the barrel and emerges with a muzzle velocity of 300 m/s. Calculate the gravitational radiation field generated by the shot at a point P on the z-axis at a vertical distance of 20 m above the cannon. What is the maximum value of the wave field? Ignore the gravitational field of the Earth.

EP55. Plane Wave Properties _Markus__

Show that there is a coordinate choice so that the linearized vacuum Einstein equations are

$$\partial^2 h_{ab} = 0$$

where

$$g_{ab} = \eta_{ab} + h_{ab}$$

Find the plane wave solutions to this equation and explain why there are only two polarizations. The transverse trace-free polarization has basis

$$e_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad and \quad e_- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

By taking the e_+ polarization, describe the physical effect of a gravitational plane wave.