Nonsymmetric Gravitational Theory

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Abstract

A new version of nonsymmetric gravitational theory is presented. The field equations are expanded about the Minkowski metric, giving in lowest order the linear Einstein field equations and massive Proca field equations for the antisymmetric field $g_{[\mu\nu]}$. An expansion about an arbitrary Einstein background metric yields massive Proca field equations with couplings to only physical modes. It follows that the new version of NGT is free of ghost poles, tachyons and higher-order poles and there are no problems with asymptotic boundary conditions. A static spherically symmetric solution of the field equations in the short-range approximation is everywhere regular and does not contain a black hole event horizon.

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1. Introduction

A nonsymmetric gravitational theory (NGT), based on the decompositions of the metric $g_{\mu\nu}$ and the connection $\Gamma^\lambda_{\mu\nu}$:

$$g_{(\mu\nu)} = \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}), \quad g_{[\mu\nu]} = \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}), \tag{1.1}$$

and

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{(\mu\nu)} + \Gamma^\lambda_{[\mu\nu]}, \tag{1.2}$$

is presented[1]. The theory is free of ghosts, tachyons and higher-order poles in the propagator in the linear approximation. An expansion of $g_{\mu\nu}$ about an arbitrary Einstein background metric also yields field equations to first order in $g_{[\mu\nu]}$, which are free of couplings to unphysical (negative energy) modes; the solutions of the field equations have consistent asymptotic boundary conditions.

In view of the difficulty in obtaining physically consistent geometrical generalizations of Einstein gravitational theory (EGT), it is interesting that such a consistent theory can be formulated[2].

A static spherically symmetric solution of the NGT field equations in the short-range approximation is everywhere regular and does not contain a black hole event horizon[3].

2. Nonsymmetric Gravitational Theory

The non-Riemannian geometry is based on the nonsymmetric field structure with a nonsymmetric $g_{\mu\nu}$ and affine connection $\Gamma^\lambda_{\mu\nu}$, defined in Eqs.(1.1) and (1.2)[4]. The contravariant tensor $g^{\mu\nu}$ is defined in terms of the equation:

$$g^{\mu\nu}g_{\sigma\nu} = g^{\nu\mu}g_{\nu\sigma} = \delta^\mu_\sigma. \tag{2.1}$$

The Lagrangian density is given by

$$\mathcal{L}_{NGT} = \mathcal{L}_R + \mathcal{L}_M, \tag{2.2}$$

where

$$\mathcal{L}_R = \mathbf{g}^{\mu\nu}R_{\mu\nu}(W) - 2\lambda\sqrt{-g} - \frac{1}{4}\mu^2\mathbf{g}^{\mu\nu}g_{[\nu\mu]} - \frac{1}{6}g^{\mu\nu}W_\mu W_\nu, \tag{2.3}$$

where λ is the cosmological constant and μ^2 is an additional cosmological constant associated with $g_{[\mu\nu]}$. Moreover, $\mathcal{L}_M$ is the matter Lagrangian density ($G = c = 1$):

$$\mathcal{L}_M = -8\pi g^{\mu\nu}\mathbf{T}_{\mu\nu}. \tag{2.4}$$

Here, $\mathbf{g}^{\mu\nu}=\sqrt{-g}g^{\mu\nu}$ and $R_{\mu\nu}(W)$ is the NGT contracted curvature tensor:

$$R_{\mu\nu}(W)=W^\beta_{\mu\nu,\beta}-\frac{1}{2}(W^\beta_{\mu\beta,\nu}+W^\beta_{\nu\beta,\mu})-W^\beta_{\alpha\nu}W^\alpha_{\mu\beta}+W^\beta_{\alpha\beta}W^\alpha_{\mu\nu}, \tag{2.5}$$

defined in terms of the unconstrained nonsymmetric connection:

$$W^\lambda_{\mu\nu}=\Gamma^\lambda_{\mu\nu}-\frac{2}{3}\delta^\lambda_\mu W_\nu, \tag{2.6}$$

where

$$W_\mu=\frac{1}{2}(W^\lambda_{\mu\lambda}-W^\lambda_{\lambda\mu}). \tag{2.7}$$

Eq.(2.6) leads to the result:

$$\Gamma_\mu=\Gamma^\lambda_{[\mu\lambda]}=0. \tag{2.8}$$

The NGT contracted curvature tensor can be written as

$$R_{\mu\nu}(W)=R_{\mu\nu}(\Gamma)+\frac{2}{3}W_{[\mu,\nu]}, \tag{2.9}$$

where $R_{\mu\nu}(\Gamma)$ is defined by

$$R_{\mu\nu}(\Gamma)=\Gamma^\beta_{\mu\nu,\beta}-\frac{1}{2}\left(\Gamma^\beta_{(\mu\beta),\nu}+\Gamma^\beta_{(\nu\beta),\mu}\right)-\Gamma^\beta_{\alpha\nu}\Gamma^\alpha_{\mu\beta}+\Gamma^\beta_{(\alpha\beta)}\Gamma^\alpha_{\mu\nu}. \tag{2.10}$$

The field equations in the presence of matter sources are given by:

$$G_{\mu\nu}(W)+\lambda g_{\mu\nu}+\frac{1}{4}\mu^2C_{\mu\nu}-\frac{1}{6}(P_{\mu\nu}-\frac{1}{2}g_{\mu\nu}P)=8\pi T_{\mu\nu}, \tag{2.11}$$

$$\mathbf{g}^{[\mu\nu]}{}_{,\nu}=-\frac{1}{2}\mathbf{g}^{(\mu\beta)}W_\beta, \tag{2.12}$$

$$\mathbf{g}^{\mu\nu}{}_{,\sigma}+\mathbf{g}^{\rho\nu}W^\mu_{\rho\sigma}+\mathbf{g}^{\mu\rho}W^\nu_{\sigma\rho}-\mathbf{g}^{\mu\nu}W^\rho_{\sigma\rho}+\frac{2}{3}\delta^\nu_\sigma\mathbf{g}^{\mu\rho}W^\beta_{[\rho\beta]}$$
$$+\frac{1}{6}(\mathbf{g}^{(\mu\beta)}W_\beta\delta^\nu_\sigma-\mathbf{g}^{(\nu\beta)}W_\beta\delta^\mu_\sigma)=0. \tag{2.13}$$

Here, we have

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R, \tag{2.14}$$

$$C_{\mu\nu}=g_{[\mu\nu]}+\frac{1}{2}g_{\mu\nu}g^{[\sigma\rho]}g_{[\rho\sigma]}+g^{[\sigma\rho]}g_{\mu\sigma}g_{\rho\nu}, \tag{2.15}$$

$$P_{\mu\nu} = W_\mu W_\nu, \tag{2.16}$$

and $P = g^{\mu\nu} P_{\mu\nu} = g^{(\mu\nu)} W_\mu W_\nu$.

In empty space, the field equations (2.11) become:

$$R_{\mu\nu}(\Gamma) = \frac{2}{3} W_{[\nu,\mu]} + \lambda g_{\mu\nu} - \frac{1}{4}\mu^2 (C_{\mu\nu} - \frac{1}{2} g_{\mu\nu} C) + \frac{1}{6} P_{\mu\nu}. \tag{2.17}$$

The generalized Bianchi identities:

$$[\mathbf{g}^{\alpha\nu} G_{\rho\nu}(\Gamma) + \mathbf{g}^{\nu\alpha} G_{\nu\rho}(\Gamma)]_{,\alpha} + g^{\mu\nu}{}_{,\rho} \mathbf{G}_{\mu\nu} = 0, \tag{2.18}$$

give rise to the matter response equations:

$$g_{\mu\rho} \mathbf{T}^{\mu\nu}{}_{,\nu} + g_{\rho\mu} \mathbf{T}^{\nu\mu}{}_{,\nu} + (g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}) \mathbf{T}^{\mu\nu} = 0. \tag{2.19}$$

In general, the last term in Eq.(2.3) is of the form, $\frac{1}{2}\sigma g^{\mu\nu} W_\mu W_\nu$. The coupling constant σ is chosen to be $\sigma = -\frac{1}{3}$, so that the theory yields consistent ghost and tachyon free perturbative solutions to the field equations.

3. Linear Approximation

Let us assume that $\lambda = 0$ and expand $g_{\mu\nu}$ about Minkowski spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + {}^{(1)}h_{\mu\nu} + ..., \tag{3.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor: $\eta_{\mu\nu} = \mathrm{diag}(-1, -1, -1, +1)$. We also expand $\Gamma^\lambda_{\mu\nu}$ and $W^\lambda_{\mu\nu}$:

$$\Gamma^\lambda_{\mu\nu} = {}^{(1)}\Gamma^\lambda_{\mu\nu} + {}^{(2)}\Gamma^\lambda_{\mu\nu} + ...,$$

$$W_\mu = {}^{(1)}W_\mu + {}^{(2)}W_\mu + ... \tag{3.2}$$

Let us adopt the notation: $\psi_{\mu\nu} = {}^{(1)}h_{[\mu\nu]}$. To first order of approximation, Eq.(2.12) gives

$$\psi_\mu = -\frac{1}{2} W_\mu, \tag{3.3}$$

where for convenience W_μ denotes ${}^{(1)}W_\mu$. Moreover,

$$\psi_\mu = \psi_{\mu\beta}{}^{,\beta} = \eta^{\beta\sigma} \psi_{\mu\beta,\sigma}. \tag{3.4}$$

From Eq.(2.13), we obtain the result to first order:

$$ {}^{(1)}\Gamma^\lambda_{\mu\nu} = \frac{1}{2}\eta^{\lambda\sigma}({}^{(1)}h_{\sigma\nu,\mu} + {}^{(1)}h_{\mu\sigma,\nu} - {}^{(1)}h_{\nu\mu,\sigma}) + \frac{1}{6}(\delta^\lambda_\nu W_\mu - \delta^\lambda_\mu W_\nu). \tag{3.5} $$

The antisymmetric and symmetric field equations derived from Eq.(2.11) decouple to lowest order; the symmetric equations are the usual Einstein field equations in the linear approximation. The skew equations are given by

$$ (\Box + \mu^2)\psi_{\mu\nu} = J_{\mu\nu}, \tag{3.6} $$

where

$$ J_{\mu\nu} = 16\pi(T_{[\mu\nu]} + \frac{2}{\mu^2}T_{[[\mu\sigma],\nu]}{}^{,\sigma}). \tag{3.7} $$

We have

$$ W_\mu = -\frac{32\pi}{\mu^2}T_{[\mu\nu]}{}^{,\nu}, \tag{3.8} $$

and from Eq.(3.3) we get

$$ \psi_\mu = \frac{16\pi}{\mu^2}T_{[\mu\nu]}{}^{,\nu}. \tag{3.9} $$

In the wave-zone, $T_{\mu\nu} = 0$, and Eqs.(3.6) and (3.9) become

$$ (\Box + \mu^2)\psi_{\mu\nu} = 0, \tag{3.10} $$

$$ \psi_\mu = 0. \tag{3.11} $$

These equations can be obtained from the Lagrangian:

$$ \mathcal{L}_\psi = \frac{1}{4}\psi^2_{\mu\nu,\lambda} - \frac{1}{2}\psi^2_\mu - \frac{1}{4}\mu^2\psi^2_{\mu\nu}. \tag{3.12} $$

The $\psi_{\mu\nu}$ has the spin decomposition:

$$ \psi_{\mu\nu} = 1_b \oplus 1_e, \tag{3.13} $$

where 1_b and 1_e denote the magnetic and electric vectors, respectively. Only the magnetic vector propagates corresponding to a massive spin 1^+ pseudovector field with the propagator:

$$ \Pi = \frac{P^1_b}{k^2}, \tag{3.14} $$

where P_b^1 is the magnetic projection operator defined by

$$P_b^1 = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} - \theta_{\mu\sigma}\theta_{\nu\rho}), \tag{3.15}$$

$$\theta_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \tag{3.16}$$

and k_μ denotes the momentum four vector. The Lagrangian (3.12) is free of ghosts, tachyons and higher-order poles[5] and the Hamiltonian is positive and bounded from below.

The physical requirement that the weak field linear approximation of the NGT field equations is free of ghosts, tachyons and higher-order poles in the propagator is satisfied by the version of NGT presented here. This overcomes the problem of obtaining a consistent geometrical NGT, in which the physical vacuum in the theory is stable.

As shown by van Nieuwenhuizen[5], in flat Minkowski spacetime there exist only two physically consistent Lagrangians for the antisymmetric potential field $\psi_{\mu\nu}$, which are free of ghosts, tachyons and higher-order poles in the propagator, namely, the massless gauge invariant theory, and the massive Proca-type theory with the Lagrangian (3.12). Since NGT does not possess a massless gauge invariance, then the only other possibility is that it should reduce to the physical, massive Proca-type theory. We have, in fact, now discovered a fully geometrical NGT scheme that fulfils this requirement.

4. Expansion of the Field Equations Around a Curved Background

Let us now consider the expansion of the field equations around an arbitrary Einstein background metric. We shall introduce the notation: $g_{[\mu\nu]} = a_{\mu\nu}$. We have

$$g_{\mu\nu} = g_{S\mu\nu} + {}^{(1)}g_{\mu\nu} + ..., \quad \Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{S\mu\nu} + {}^{(1)}\Gamma^\lambda_{\mu\nu} + ..., \quad W_\mu = {}^{(1)}W_\mu + {}^{(2)}W_\mu + ..., \tag{4.1}$$

where $g_{S\mu\nu}$ and $\Gamma^\lambda_{S\mu\nu}$ denote the Einstein background metric tensor and connection, respectively, and $g^{\lambda\alpha}_S g_{S\beta\lambda} = \delta^\alpha_\beta$. We also introduce the notation for the Riemann tensor:

$$B^\sigma{}_{\mu\nu\rho} = \Gamma^\sigma_{S\mu\nu,\rho} - \Gamma^\sigma_{S\mu\rho,\nu} - \Gamma^\sigma_{S\alpha\nu}\Gamma^\alpha_{S\mu\rho} + \Gamma^\sigma_{S\alpha\rho}\Gamma^\alpha_{S\mu\nu}. \tag{4.2}$$

By performing the contraction on the suffixes σ and ρ, we get the Ricci tensor, $B_{\mu\nu} = B^\alpha{}_{\mu\nu\alpha}$. We get to first order in $a_{\mu\nu}$ and W_μ (we denote ${}^{(1)}W_\mu$ by W_μ and ${}^{(1)}a_{\mu\nu}$ by $a_{\mu\nu}$) the field equations:

$$B_{\mu\nu} = 0, \tag{4.3}$$

$$\nabla^\sigma a_{\mu\sigma} = \frac{1}{2\mu^2}\left[\nabla^\nu\left(4g^{\lambda\sigma}_S g^{\alpha\beta}_S B_{\alpha\nu\lambda\mu}a_{\sigma\beta} - 2(Ba)_{\mu\nu}\right)\right]. \tag{4.4}$$

$$(\Box_S + \mu^2)a_{\mu\nu} = M_{\mu\nu}, \tag{4.5}$$

where

$$M_{\mu\nu} = 2g^{\lambda\sigma}_S g^{\alpha\beta}_S B_{\alpha\nu\lambda\mu}a_{\sigma\beta} + \frac{1}{\mu^2}\nabla_{[\nu}\nabla^\rho\left[4g^{\lambda\sigma}_S g^{\alpha\beta}_S B_{\alpha\rho\lambda\mu]}a_{\sigma\beta} - 2(Ba)_{\mu]\rho}\right], \tag{4.6}$$

and $\Box_S = \nabla^\sigma\nabla_\sigma, \nabla^\sigma = g^{\sigma\alpha}_S\nabla_\alpha$. Also, $(Ba)_{\mu\nu}$ denotes additional terms involving products of the Riemann tensor and $a_{\mu\nu}$.

As before, W_μ does not propagate and there is no coupling to unphysical modes through the effective source tensor formed from the Riemann tensor and $a_{\mu\nu}$.

The energy associated with the flux of gravitational waves, calculated in the wavezone for $r \to \infty$, is positive definite.

5. Static Spherically Symmetric Solution

In the case of a static spherically symmetric field, Papapetrou has derived the canonical form of $g_{\mu\nu}$ in NGT[6]:

$$g_{\mu\nu} = \begin{pmatrix} -\alpha & 0 & 0 & w \\ 0 & -\beta & f\sin\theta & 0 \\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\ -w & 0 & 0 & \gamma \end{pmatrix}, \tag{5.1}$$

where α, β, γ and w are functions of r. For the theory in which there is no NGT magnetic monopole charge, we have $w = 0$ and only the $g_{[23]}$ component of $g_{[\mu\nu]}$ survives.

The line element for a spherically symmetric body is given by

$$ds^2 = \gamma(r)dt^2 - \alpha(r)dr^2 - \beta(r)(d\theta^2 + \sin^2\theta d\phi^2). \tag{5.2}$$

We have

$$\sqrt{-g} = \sin\theta[\alpha\gamma(\beta^2 + f^2)]^{1/2}. \tag{5.3}$$

The vector W_μ can be determined from Eq.(2.12):

$$W_\mu = -2k_{\mu\rho}\mathbf{g}^{[\rho\sigma]}{}_{,\sigma}, \tag{5.4}$$

where $k_{\mu\nu}$ is defined by $k_{\mu\alpha}g^{(\mu\beta)} = \delta^\beta_\alpha$. For the static spherically symmetric field with $w = 0$ it follows from (5.4) that $W_\mu = 0$.

We choose $\lambda = 0, \beta = r^2$ and demand the boundary conditions:

$$\alpha \to 1, \quad \gamma \to 1, \quad f \to 0, \tag{5.5}$$

as $r \to \infty$. We also assume the short-range approximation for which the μ^2 contributions in the vacuum field equations can be neglected. If $\mu^{-1} >> 2m$, then we can use the static, spherically symmetric Wyman solution[7]:

$$\gamma = \exp(\nu), \tag{5.6}$$

$$\alpha = m^2(\nu')^2\exp(-\nu)(1 + s^2)(\cosh(a\nu) - \cos(b\nu))^{-2}, \tag{5.7}$$

$$f = [2m^2\exp(-\nu)(\sinh(a\nu)\sin(b\nu)$$

$$+s(1 - \cosh(a\nu)\cos(b\nu))](\cosh(a\nu) - \cos(b\nu))^{-2}, \tag{5.8}$$

where ν is implicitly determined by the equation:

$$\exp(\nu)(\cosh(a\nu) - \cos(b\nu))^2\frac{r^2}{2m^2} = \cosh(a\nu)\cos(b\nu) - 1 + s\sinh(a\nu)\sin(b\nu). \tag{5.9}$$

Here, s is a dimensionless constant of integration.

We find for $2m/r < 1$ and $0 < sm^2/r^2 < 1$ that α and γ take the Schwarzschild form:

$$\gamma = \alpha^{-1} = 1 - \frac{2m}{r}. \tag{5.10}$$

Near $r = 0$ we can develop expansions where $r/m < 1$ and $0 < |s| < 1$. The leading terms are

$$\gamma = \gamma_0 + \frac{\gamma_0(1+\mathcal{O}(s^2))}{2|s|}\left(\frac{r}{m}\right)^2 + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right), \tag{5.11}$$

$$\alpha = \frac{4\gamma_0(1+\mathcal{O}(s^2))}{s^2}\left(\frac{r}{m}\right)^2 + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right), \tag{5.12}$$

$$f = m^2\left(4 - \frac{|s|\pi}{2} + s|s| + \mathcal{O}(s^3)\right) + \frac{|s| + s^2\pi/8 + \mathcal{O}(s^3)}{4}r^2 + \mathcal{O}(r^4), \tag{5.13}$$

$$\gamma_0 = \exp\left(-\frac{\pi + 2s}{|s|} + \mathcal{O}(s)\right).... \tag{5.14}$$

These solutions clearly illustrate the non-analytic nature of the limit $s \to 0$ in the strong gravitational field regime.

The singularity caused by the vanishing of $\alpha(r)$ at $r = 0$ is a *coordinate* singularity, which can be removed by transforming to another coordinate frame of reference[3]. The curvature invariants do not, of course, contain any coordinate singularities.

The NGT curvature invariants such as the generalized Kretschmann scalar:

$$K = R^{\lambda\mu\nu\rho}R_{\lambda\mu\nu\rho} \tag{5.15}$$

are finite.

The solution is everywhere non-singular and there is no event horizon at $r = 2m$. A black hole is replaced in the theory by a superdense object which can be stable for an arbitrarily large mass[3,8].

6. Conclusions

We have succeeded in deriving a geometrical version of NGT which yields a stable, ghost and tachyon free linear approximation, when the field equations are expanded about Minkowski spacetime or about an arbitrary Einstein background metric.

In the short-range approximation, in which μ^{-1} is large compared to $2m$, a static spherically symmetric solution can be obtained from the field equations which is regular everywhere in spacetime and which does not contain an event horizon at $r = 2m$. The black hole that is predicted by the collapse of a sufficiently massive star in EGT is replaced by a superdense object that is stable for an arbitrarily large mass.

An important result of this study is that a classical theory of gravity can be formulated, which has a physically consistent perturbative expansion for weak fields, and does not have singularities and black holes. It satisfies the standard gravitational experimental tests. Recently, a solution has been published for inhomogeneous gravitational collapse of a matter cloud with a general form of matter, which leads to a naked singularity[9]. The collapse is related to the choice of initial data for the Einstein field equations, and the naked singularity would occur in generic situations based on regular initial conditions satisfying the weak energy condition. This result would lead to the demise of EGT, for it represents a local violation of the Cauchy data for collapse; it would provide a strong motivation for seriously considering a classical gravity theory such as NGT with everywhere regular solutions of the field equations.

Because there is no event horizon in the static spherically symmetric solution, we can resolve the problem of information loss[10] associated with black holes at a classical level.

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