

Tests of general relativity in the solar system

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Summary. — Tests of gravity performed in the solar system show a good agreement with general relativity. The latter is however challenged by observations at larger, galactic and cosmic, scales which are presently cured by introducing “dark matter” or “dark energy”. A few measurements in the solar system, particularly the so-called “Pioneer anomaly”, might also be pointing at a modification of gravity law at ranges of the order of the size of the solar system.

The present lecture notes discuss the current status of tests of general relativity in the solar system. They describe metric extensions of general relativity which have the capability to preserve compatibility with existing gravity tests while opening free space for new phenomena. They present arguments for new mission designs and new space technologies as well as for having a new look on data of existing or future experiments.

Introduction

These notes correspond to the first lecture in a series of three [1] given during the INTERNATIONAL SCHOOL OF PHYSICS “ENRICO FERMI” on ATOM OPTICS AND SPACE PHYSICS held at Varenna in July 2007.

They present experimental tests of gravity performed in the solar system which show a good agreement with general relativity (GR). To this aim, they first recall the basic features of the theoretical description and briefly review the experimental evidences supporting it as well as the challenges standing ahead of it. They then discuss some perspectives in terms of theoretical ideas, new mission designs or technological developments and data analysis for existing or future experiments.

These notes cannot cover completely general relativity and its impressive applications in various domains, which have been the topic of a number of books or reviews (see for example [2-8]). They are focused on the discussion of topics selected for their direct connection with the question of gravity tests performed with tools such as atomic clocks, accelerometers, radioscience or laser links (see for example [9-17]).

1. – The basic ideas in general relativity

General relativity is built up on two basic ideas which should not be confused. The first one is the metrical (geometrical) interpretation of gravitation, which was proposed soon after 1905 [18]. This identification of gravitation field with the space-time metric is the very core of GR, but it is not sufficient to fix the latter theory. In order to select GR out of the variety of metric theories of gravitation, it is necessary to fix the relation between the geometry of space-time and its matter content. In GR, this relation is given by the Einstein-Hilbert equation which was written in 1915 [19].

The geometrical interpretation of gravity, often presented under the generic name of the *equivalence principle*, is one of the most accurately verified properties of nature [6]. Freely falling test masses follow the geodesics of the Riemannian space-time, that is also the curves which extremize the integral

$$(1) \quad \int ds \quad , \quad ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

$g_{\mu\nu}$ is the metric tensor characterizing the space-time and dx^μ the displacements in this space-time. As free motions obey a geometrical definition, they are independent of the compositions of the test masses. This “universality of free fall” property has been verified with an extreme accuracy. Its potential violations are parametrized by a relative difference η in the accelerations a_1 and a_2 undergone by two test bodies in free fall from the same location with the same velocity. Modern experiments constrain the parameter η to stay below 10^{-12} , this accuracy being attained in laboratory experiments [20] as well as in space tests using lunar laser ranging [21] or planetary probe tracking [22]. These results do not preclude the possibility of violations of the equivalence principle and such violations are indeed predicted by unification models [23]. Large improvements

of this accuracy are expected in the future thanks to the existence of space projects MICROSCOPE [24] or, on a longer term, STEP [25].

As another consequence of the geometrical interpretation of gravity, ideal atomic clocks working on different transitions measure the same time, because it is also a geometrical quantity, namely the proper time $\int ds$ along the trajectory [26, 27]. This “universality of clock rates” property has also been verified with an extreme accuracy. Its potential variations are measured as a constancy of relative frequency ratios between different clocks at a level of the order of a few 10^{-16} per year [28]. These results can also be interpreted in terms of a potential “variation of fundamental constants” [29]. In this domain also, large improvements of the accuracy can be expected in the future thanks to the existence of the space project PHARAO-ACES [30] and, on a longer term, of ambitious projects using optical clocks [31] or atomic sensors [32-34].

As already stated, GR is selected out of the large family of metric theories of gravity by the Einstein-Hilbert equation which fixes the coupling between curvature on one hand, matter on the other one. In order to write this equation, let us recall that there exists one tensor in Riemannian geometry, the Einstein curvature tensor $G_{\mu\nu}$, which has a null covariant divergence

$$(2) \quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \quad , \quad D^\mu G_{\mu\nu} \equiv 0$$

($R_{\mu\nu}$ is the Ricci tensor and R the scalar curvature). This geometrical identity has to be compared with the physical property which expresses conservation of energy and momentum as the condition of null divergence of the stress tensor $T_{\mu\nu}$

$$(3) \quad D^\mu T_{\mu\nu} \equiv 0$$

This relation is a necessary and sufficient condition for free motions to follow geodesics. The Einstein-Hilbert equation sets $G_{\mu\nu}$ and $T_{\mu\nu}$ to be proportional to each other

$$(4) \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The coefficient is fixed by the Newtonian limit and determined by the Newton constant G and the velocity of light c .

Though relation (4) can be used to deduce (3) from the identity (2), it is not possible to deduce its specific form only from the geometrical interpretation of gravity. In other words, there exists a variety of metric theories of gravity and GR has to be selected out of it by comparing predictions drawn from the Einstein-Hilbert equation to the results of observations or experiments. When performed in the solar system, the tests effectively show a good agreement with GR, which means that the metric tensor has a form close to its prediction. This statement is made more specific in the next sections.

2. – Post-Newtonian gravity tests in the solar system

The most common approach to gravity tests in the solar system is the so-called “parametrized post-Newtonian” (PPN) approach introduced by Eddington [2] and then developed by a number of physicists [35].

The main idea can be described by coming back to the GR solution in the solar system. For simplicity, we write this solution with a simple model of the solar system where the gravity sources are reduced to the Sun treated as a point-like motion-less mass M , and we use the Eddington gauge convention

$$(5) \quad ds^2 = g_{00}c^2dt^2 + g_{rr} (dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2))$$

Equation (5) is the general form of an isotropic and stationary metric, written with a specific gauge convention where spatial coordinates are isotropic ; spherical coordinates are used with the time t , the radius r (in this specific gauge), the colatitude and azimuth angles θ and φ ; other gauge choices, such as the one leading to the Schwartzschild solution [36], would lead to the same results for the physical observables.

With these simplifications, an exact solution can be found for the Einstein-Hilbert equation (4) in our simple model of the solar system. It may be expressed through rational forms of the reduced Newton potential ϕ

$$(6) \quad g_{00} = \left(\frac{1 + \phi/2}{1 - \phi/2} \right)^2, \quad g_{rr} = -(1 - \phi/2)^4, \quad \phi \equiv -\frac{GM}{rc^2}$$

Note that the dimensionless ϕ is much smaller than unity everywhere in the solar system. The length scale GM/c^2 has indeed a value of the order of 1.5km (with M the mass of the Sun), so that the value of ϕ created by Sun on Earth is close to 1×10^{-8} . It is therefore convenient to rewrite (6) as a power series expansion in the small ϕ

$$(7) \quad g_{00} = 1 + 2\phi + 2\phi^2 + \dots, \quad g_{rr} = -1 + 2\phi + \dots$$

The family of PPN metrics can then be introduced by inserting constants in front of the terms of the power expansion

$$(8) \quad g_{00} = 1 + 2\alpha\phi + 2\beta\phi^2 + \dots, \quad g_{rr} = -1 + 2\gamma\phi + \dots$$

The first parameter α can be set to unity by fixing Newton constant G to its effective value in the solar system. The two other parameters β and γ then characterize various members of the PPN family, with GR corresponding to $\beta = \gamma = 1$. The values of these Eddington parameters affect the predicted motions, *i.e.* the geodesics associated with the metric (8), and can therefore be confronted to the observations. The results are conveniently presented in terms of likelihood plots for the anomalies $\gamma - 1$ and $\beta - 1$ which show that tests favor GR as the description of gravity in the solar system.

Doppler ranging on probes in the vicinity of Mars [22] and deflection measurements using VLBI astrometry [37] have led to more and more stringent bounds on $|\gamma - 1|$. The current bound is essentially given by the experiment performed through radar ranging of the Cassini probe during its 2002 solar occultation [38]

$$(9) \quad \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

In the meantime, analysis of the precession of planet perihelions [39] and of lunar laser ranging data [40] have led to determinations of linear superpositions of β and γ , resulting in bounds on the anomaly $|\beta - 1|$. The current bound is essentially given by lunar laser ranging

$$(10) \quad 4(\beta - 1) - (\gamma - 1) = (4.4 \pm 4.5) \times 10^{-4}$$

When using (9), this leads to [40]

$$(11) \quad \beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$$

Equations (9,11) constitute a quantitative statement of the agreement with GR of gravity tests in the solar system. Let us emphasize that their common presentation under the form “general relativity is confirmed by the tests” is a bit too loose. The tests discussed so far do not answer by a final “yes” answer to a mere “yes/no” question. They rather select a vicinity of GR, defined by (9,11), as the best current description of gravity within the family of PPN metrics. This warning is not a mere precaution, it is rather a pointer to possible future progress in the domain. There indeed exist theoretical models [41] which deviate from GR while staying within the bounds (9,11). Furthermore, as discussed below, extensions of GR do not necessarily belong to the PPN family (8).

This last point is in particular emphasized by the so-called “fifth force” tests which are focused on a possible scale-dependent deviation from the gravity force law. Such deviations, predicted by some unification models, remain however unobserved to date [42].

3. – Scale-dependent tests in the solar system

The main idea is here to check the r -dependence of the gravity potential, that is also of the component g_{00} . Hypothetical modifications of its standard expression, predicted by unification models, are often parametrized in terms of an additional Yukawa potential

$$(12) \quad g_{00} = [g_{00}]_{\text{GR}} + \delta g_{00}, \quad \delta g_{00} = 2\phi(r) \alpha \exp\left(-\frac{r}{\lambda}\right)$$

This potential depends on two parameters, an amplitude α measured with respect to Newton potential and a range λ related through a Yukawa-like relation to the mass scale of the hypothetical new particle which would mediate the “fifth force”.

The presence of such a correction has been looked for on a large range of distances. When the whole set of results is reported on a global diagram (printed as Fig. 1 in

Ref. [43] where it was reproduced thanks to a courtesy of Coy *et al.* [44]), it appears that the Yukawa term is excluded with a high accuracy at some ranges

$$(13) \quad \alpha < 10^{-10} \text{ at } \lambda \simeq \text{Earth} - \text{Moon distance}$$

$$(14) \quad \alpha < 10^{-9} \text{ at } \lambda \simeq \text{Sun} - \text{Mars distance}$$

These bounds, again deduced from lunar laser ranging [21] and tracking of planetary probes [22], correspond to a remarkable result which approaches the accuracy of equivalence principle tests. It is also clear that windows remain open for large Yukawa corrections ($\alpha > 1$) at short ranges as well as long ranges [43].

These two windows are being actively explored. The accuracy of short range tests has been recently improved, as gravity experiments were pushed to smaller and smaller distances [45]. The strongest constraints at sub-millimeter ranges come from experiments with torsion pendula and have recently reached an impressive performance [46]

$$(15) \quad \alpha < 1 \text{ for } \lambda > 56\mu\text{m}$$

At even shorter ranges, tests of the gravity force law are pursued as careful comparisons between theoretical predictions and experimental measurements of the Casimir force, which becomes dominant at micrometric ranges [47].

In the long range window, a test of the gravity force was initiated by NASA as the extension of Pioneer 10/11 missions and it led to the observation of the Pioneer anomaly, the most notable exception of a signal in conflict with the predictions of GR. Before embarking in the discussion of this signal (in the next section), let us emphasize that the apparent contradiction between Pioneer observations and other gravity tests may be cured in an extended framework, where deviations from GR may show a scale dependence. This corresponds to a situation where the accurate tests on the Sun-Earth-Moon and Sun-Mars systems would reproduce GR predictions whereas tests at different scales could show deviations.

In fact, this idea has to be considered with great attention in the current context of fundamental physics where questions naturally arise about the validity of GR at galactic or cosmic scales. It is well known that the rotation of the outer regions of galaxies cannot be reproduced by using GR for gravity and fixing the matter content as we see it from other observation channels. This gravitational anomaly is commonly accounted for by introducing “dark matter” tuned to reproduce the rotation curves [48]. A further anomaly has been detected more recently as an acceleration of cosmic expansion and it is interpreted as due to the presence of some “dark energy” component [49]. Both components of the “dark side” of the universe have no universally accepted interpretation nor are they observed through other means than the gravitational anomalies they have been designed to cure [50]. As long as this situation is lasting, these anomalies may as well be interpreted as long range deviations from GR [51].

It is therefore important to make any possible effort to test gravity at the largest scales attainable by man made instruments, in an attempt to bridge the gap between

gravity experiments in the solar system and observations at the much larger galactic and cosmic scales.

4. – The Pioneer anomaly

The best example of such a strategy to date was the NASA decision to extend Pioneer 10 & 11 missions after their primary planetary objectives had been met [52]. The idea was from the beginning to use the two precisely navigated deep space vehicles to test the laws of gravity at large heliocentric distances and to search for low-frequency gravitational waves. The latter objective failed to detect gravitational waves [53] but the former one led to the largest scaled test of gravity ever carried out. The most notable output of this experiment is certainly that it failed to confirm the known laws of gravity [54, 55].

The anomaly was recorded on tracking data [56] of the Pioneer 10 & 11 probes by the NASA deep space network (DSN). An up-link radio signal is emitted from Earth at a DSN station, it is then transponded back by the probe, and the down-link radio signal is finally received on Earth at the same or another DSN station. For Pioneer 10 & 11 probes, the observable was the Doppler shift, defined as the ratio of cycle counting rates at the down and up events, as measured by reference clocks located at reception and emission stations. This observable can equivalently be interpreted as a Doppler velocity v measuring a relative velocity of the probe with respect to the station

$$(16) \quad \frac{f_d}{f_u} \equiv \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}$$

Note that this is just an interpretation of the observable, which contains not only the effect of motion but also relativistic and gravitational effects. It is also worth keeping in mind that perturbations due to transmission media effects have to be accounted for [57].

These Doppler tracking data were analyzed during the cruises of Pioneer 10 & 11. When the probes had flown by the giant planets (Jupiter for Pioneer 10, Jupiter and Saturn for Pioneer 11) and reached a quieter environment, it was noticed that tracking data were deviating from the predictions of GR. Precisely, the observed Doppler velocity was departing from the modelled Doppler velocity (see the Fig. 8 of Ref. [55]). As the deviation δv was varying linearly with elapsed time, it was natural to introduce an anomalous acceleration to describe it

$$(17) \quad \delta v \equiv v_{\text{observed}} - v_{\text{modelled}} \simeq -a_P (t - t_{\text{in}})$$

Note that the anomalous acceleration a_P does not necessarily reflect a force acting on the probe. But, when considered as such, it appears to be directed towards the Sun with an approximately constant amplitude over a large range of heliocentric distances (AU \equiv astronomical unit)

$$(18) \quad a_P = (0.87 \pm 0.13) \text{ nm s}^{-2} \quad , \quad 20 \text{ AU} \lesssim r_P \lesssim 70 \text{ AU}$$

The Pioneer anomaly has been registered on the two deep space probes showing the best navigation accuracy. Anomalous observations have also been reported for Ulysses and Galileo probes, but not with the same reliability as for Pioneer probes. For other probes like Voyager 1 & 2, the navigation accuracy was not sufficient. This means that the Pioneer gravity test has been performed twice with identical probes on similar trajectories - escape directions opposite in the solar system - and the same result - similar Sunward acceleration. This is not an impressive statistics when compared to the large number of tests confirming GR. In particular, when the possibility of an artefact onboard the probe is considered, this artefact could be the same on the two probes.

The extensive analysis of the JPL team has been published after years of cross checks [55] and the presence of the anomaly has been confirmed by several independent analysis [58]. A number of mechanisms have been considered as attempts of explanations of the anomaly as a systematic effect generated by the spacecraft itself or its environment ([59] and references therein) but they have not led to a satisfactory understanding to date. Further informations are expected to come out of the re-analysis process now going on with recently recovered Pioneer data [60,61] (more details below).

The Pioneer anomaly constitutes an intriguing piece of information in a context where the status of gravity theory is challenged by the puzzles of dark matter and dark energy. If confirmed, this signal might already reveal an anomalous behaviour of gravity at scales of the order of the size of the solar system and thus have a strong impact on fundamental physics, astrophysics and cosmology. It is therefore important to investigate what could be the signatures of the Pioneer anomaly as a long range modification of gravity (see for example [62-66] and references therein).

5. – Can the Pioneer signal be a metric anomaly ?

If there exist gravity theories where a Pioneer signal can take a natural place, they must be considered with great care. If on the contrary one can prove that there exist no such theories, this result is also important since it allows the range of validity of GR to be extended to the whole solar system. Anyway, the relevance of the Pioneer anomaly for space navigation is already sufficient to deserve a close scrutiny.

As we know that tests of the equivalence principle (EP) have shown it to be preserved at the level of a few 10^{-13} in lunar laser ranging, it appears quite unlikely that an EP violation could account for the Pioneer anomaly. To make this statement more precise, let us say that the standard Newton acceleration is of the order of $1 \mu\text{m s}^{-2}$ at 70 UA so that the Pioneer anomaly is only one thousand times smaller. This remark does not contradict the possibility of EP violations which are predicted by unification models and looked for in space experiments such as MICROSCOPE and STEP, but however suggests that such violations are expected to occur at a too small level to produce the Pioneer anomaly. In the following, we therefore restrict our attention to the confrontation of GR with alternative metric theories of gravity.

In this well understood metric framework, the form of the coupling between curvature and matter can still be discussed [62]. Like the other fundamental interactions, gravita-

tion may be treated as a field theory [67] and radiative corrections due to its coupling to other fields thus naturally lead to consider GR within a larger class of theories [68]. Modifications are thus expected to appear in particular at large length scales [69]. Note that the embedding of GR in the larger family of fourth order theories cures its unrenormalizability while providing asymptotic freedom [70]. This is a strong argument for extending the gravitation theory at scales not constrained by experiments, for instance using renormalization group trajectories [71]. Renormalizability of these theories however comes with a counterpart, that is the problem of ghosts. It has however been argued that this problem does not constitute a deadend for an effective field theory and, in particular, that the departure from unitarity is expected to be negligible at ordinary scales tested in present day universe [72].

The main message at this point is that GR, as other current theories of fundamental interactions, has to be considered as an effective theory of gravity, certainly valid at the length scales for which it has been accurately tested, but still to be tested at other scales. Within the metric framework, it is possible to develop a well founded phenomenological point of view for analyzing the potential meaning of the Pioneer anomaly [63]. This framework covers the whole spectrum of metric extensions of GR which remain in the vicinity of GR and it includes as particular cases the PPN extensions as well as the “fifth force” extensions of GR. It can be considered as a natural consequence of radiative corrections due to the coupling of gravity with other fields [68] and has also recently been shown to be reproduced by field theoretical extensions of GR [73].

6. – Post-Einsteinian metric extensions of GR

We refer the interested readers to [63] for a complete description of this framework. Here we rapidly present some of its salient features and phenomenological consequences.

As we consider an isotropic and stationary model of the solar system with gravity field created by a single point-like and motion-less source, the Sun, the extended metric can be written in terms of two potentials

$$(19) \quad ds^2 = g_{00}(r)c^2 dt^2 + g_{rr}(r) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2) \\ g_{00}(r) \equiv [g_{00}(r)]_{\text{GR}} + \delta g_{00}(r) \quad , \quad g_{rr}(r) \equiv [g_{rr}(r)]_{\text{GR}} + \delta g_{rr}(r)$$

This expression has been written in the Eddington isotropic gauge (it could have been written as well with other gauge choices). The GR metric $[g_{\mu\nu}]_{\text{GR}}$ may be treated exactly (see eq.6), or at a sufficient order in the power expansion (7). Finally the anomalous part $\delta g_{\mu\nu}$ of the metric may be treated at first order, since we consider the solution to remain in the vicinity of GR. The extension of GR is thus parametrized by two functions, say $\delta g_{00}(r)$ and $\delta(g_{00}g_{rr})(r)$, which describe a much wider phenomenology than the PPN framework which was parametrized by two constants.

It is also possible to consider that the post-Einsteinian extensions generalize the PPN metrics by promoting the two constants β, γ to the status of functions of r . In fact, the two functions accomodate the freedom coming with the extension of the coupling

between curvature and matter. This extension is characterized by two running couplings which depend on scale and cover the two sectors of the gravity field corresponding to traceless (conformal weight 0) and traced components (conformal weight 1) of the metric tensor [68]. Equivalently, it may be characterized by the variation with r of two components of the Einstein curvatures which no longer vanish outside the source, as they did in GR. Anew, the PPN family is a particular case which already shows such an anomalous behaviour of Einstein curvatures, with however a specific r -dependence.

In phenomenological terms, the function δg_{00} represents an anomaly of the gravity force analogous to a “fifth force”. It has to remain small enough to preserve the good agreement of gravity tests performed on planetary orbits with GR (see the discussion of “fifth force” tests above). Meanwhile, the second function $\delta(g_{00}g_{rr})$ represents an extension of PPN phenomenology with a r -dependent Eddington parameter γ . It opens an additional freedom with respect to the mere modification of the gravity potential g_{00} as well as with respect to the PPN metrics where γ is constant. This additional phenomenological freedom makes it possible to accommodate the Pioneer signal without raising conflicts with other gravity tests [62].

If one supposes for simplicity that δg_{00} follows a Yukawa law, it is easy to show that it cannot explain the Pioneer signal and simultaneously have escaped detection in the constraining tests performed with martian probes [43]. But δg_{00} could appear only after Saturn (this is suggested by Fig. 7 in [55]) while being cut at smaller distances [64]. Some authors have argued that the ephemeris of outer planets were accurate enough to discard the presence of an anomaly at distances explored by the Pioneer probes [74]. They have pushed their claim one step farther by objecting to the very possibility of accounting for the Pioneer anomaly in any metric theory of gravity. This claim is clearly untenable at the moment. It is only after a careful comparison of observations in the outer solar system with the post-Einsteinian phenomenological framework that it will be possible to know whether or not the latter is able to fit all gravity tests, including the Pioneer one.

This comparison will have to account for the presence of the two potentials as well as for their r -dependences. The results of the calculations in [63] (after the correction of a mistake in previous papers [62]), show that a Pioneer-like signal can be produced either by an anomaly proportional to r in δg_{00} or by an anomaly proportional to r^2 in $\delta(g_{00}g_{rr})$. A linear superposition of the two possibilities can also be considered. As the two possibilities produce different signatures, they can in principle be distinguished in the comparison with tracking data. This situation certainly pleads for pushing the study farther and comparing the theoretical expectations with the Pioneer data.

Conclusion

In order to address the challenge raised by the existence of the Pioneer anomaly, one has different strategies at one’s disposal and should use all of them :

- the reanalysis of the Pioneer data is an important tool with the ability to confirm (or not) the existence of the anomaly and learn more about its properties; this idea

is today more relevant than ever thanks to the recent recovery of data covering the whole period of Pioneer 10 & 11 missions from launch to the last data point [60]; not only Doppler tracking data, but also telemetry data, are available and the latter contain valuable information on the history of the probes.

- the Pioneer data re-analysis program is an ongoing international joint effort with teams at work in USA, Canada, Germany, France, Italy and Norway [61]; this effort should lead to an improved control of systematics and produce new information of importance on several properties of the force - its direction, long-term variation as well as annual or diurnal modulations, spin dependence...; of particular interest would be a clear answer to the question of the onset of the anomaly [75].
- several proposals [76-80] have been made for new missions designed to study the anomaly and understand its origin; two missions have been proposed to the Cosmic Vision selection process at ESA, the first one as a medium size mission [79] using accelerometer and radioscience instruments upgraded from existing technology, the second one as a large size mission using new quantum sensors to map the two components of the gravity field with high accuracy [80].
- these projects are proposing new technologies of importance for future fundamental physics and deep space exploration; some of the proposed instruments may be flown as passengers on planetary missions to Mars, Jupiter or Saturne, with the aim of meeting some of the objectives of deep space gravity tests; an accelerometer on-board would measure non gravitational forces of any origin and solve ambiguities on thermal radiation force, or drag force; better radio tracking techniques or laser techniques would reduce uncertainties and improve control systematics; such passenger instruments could provide new information on the anomalous behaviours observed on several probes at planetary flybys [81].
- in the meantime, it would be wise to develop and validate enabling technologies, such as accelerometers for measuring non gravitational forces of any origin (thermal radiation budget on the probe, drag force...), radio and laser techniques for ranging/tracking, accurate clocks for measuring the gravity potential on board.
- without waiting for new programs to be set up, the theoretical studies must be pushed farther and their predictions compared with the Pioneer data; this study can lead to Pioneer-related signals, produced by the same causes, but to be looked for in the data of other experiments; in this respect, the GAIA project [82] is of particular interest, firstly because it will produce an accurate and global mapping of deflection over the sky; such a mapping could reveal the presence of an anomalous metric as a dependence of the parameter γ versus the angular distance to the Sun (the results being reduced to PPN ones when γ is constant); then GAIA will also accurately track the motions of small bodies in the solar system and thus significantly improve the coverage of dynamical behaviours with the possibility of confirming or discarding the existence of metric anomalies.

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