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Physics 113
Boccio 7.5.2.

For this problem we have a photon in the $|x\rangle$ state striking a polaroid oriented at 45^o and then striking a vertically oriented polaroid for part (a). Then, in part (b) we have 3 polaroids; the first is oriented at 30^o, the second at 60^o, and the third is vertically oriented.

In part (a) we want to show that the probability of the photon getting through both polaroids is $\frac{1}{4}$. So, we want to find $|\langle y|45^o\rangle\langle 45^o|x\rangle|^2$, where $|45^o\rangle = \cos(45^o)|x\rangle + \sin(45^o)|y\rangle$. Thus,

$$\begin{aligned}
|\langle y|45^o\rangle\langle 45^o|x\rangle|^2 &= (\sin(45^o))^2 \cdot (\cos(45^o))^2 \\
&= \left(\frac{1}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 \\
&= \frac{1}{2} \cdot \frac{1}{2} \\
&= \frac{1}{4}.
\end{aligned}$$

Then, for part (b) we want to show that the probability of passing through all of the polaroids is $\frac{27}{64}$. We have that $|30^o\rangle = \cos(30^o)|x\rangle + \sin(30^o)|y\rangle$ and $|60^o\rangle = \cos(60^o)|x\rangle + \sin(60^o)|y\rangle$. So, finding $|\langle y|60^o\rangle\langle 60^o|30^o\rangle\langle 30^o|x\rangle|^2$, we have

$$\begin{aligned}
|\langle y|60^o\rangle\langle 60^o|30^o\rangle\langle 30^o|x\rangle|^2 &= [\sin(60^o) \cdot (\cos(30^o)\cos(60^o) + \sin(60^o)\sin(30^o)) \cdot \cos(30^o)]^2 \\
&= \left[\left(\frac{\sqrt{3}}{2}\right) \cdot \left(\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right) \cdot \left(\frac{\sqrt{3}}{2}\right)\right]^2 \\
&= \left[\left(\frac{\sqrt{3}}{2}\right) \cdot \left(\frac{2\sqrt{3}}{4}\right) \cdot \left(\frac{\sqrt{3}}{2}\right)\right]^2 \\
&= \left[\left(\frac{3\sqrt{3}}{8}\right)\right]^2 \\
&= \frac{27}{64}.
\end{aligned}$$

Let $z = z[cm]$ be the length of turpentine. The transition matrix is given by the rotation matrix with a rotation by the angle $5z$. So, it is

$$U_{trans} = \begin{pmatrix} \cos(5z) & -\sin(5z) \\ \sin(5z) & \cos(5z) \end{pmatrix} \tag{1}$$

Then we can verify that

$$UU^\dagger = I \tag{2}$$

so U is unitary. Next we set the determinant equal to zero to get the eigenvalues. Let

$$s = \sin(5z) \tag{3}$$

$$c = \cos(5z) \tag{4}$$

Then

$$det(U - \lambda I) = \lambda^2 - 2c\lambda + 1 = 0 \tag{5}$$

So

$$\lambda = c \pm is \tag{6}$$

so

$$\lambda = e^{\pm i5z} \tag{7}$$

Then Mathematica gives the eigenvectors as

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{8}$$

$$\vec{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{9}$$

which correspond to right and left circularly polarized light.

First, we are presented with two experiments:

1. A prism which passes only right circularly polarized light is placed between a source of 30° polarized photons and a y-polaroid.

2. A prism which passes only left circularly polarized light is placed between a source of 30° polarized photons and a y-polaroid.

(a) Show by explicit calculation using standard amplitude mechanics that the sum of the probabilities for passing through the y-polaroid measured in these two experiments is different from the probability that one would measure if there were no prism in the path of the photon and only the y-polaroid.

$$|R\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} \quad |L\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix} \quad |y\rangle = \begin{pmatrix}0\\ 1\end{pmatrix} \quad |\psi\rangle = \begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}$$

$$P(30^\circ \to R \to y) = |\langle y|\, R\rangle\,|^2 |\,\langle R|\,\psi\rangle\,|^2 = \tfrac{1}{4}$$

$$P(30^\circ \to L \to y) = |\langle y|\, L\rangle\,|^2 |\,\langle L|\,\psi\rangle\,|^2 = \tfrac{1}{4}$$

$$P(30^\circ \to R \to y) + P(30^\circ \to L \to y) = \tfrac{1}{2}$$

$$P(30^\circ \to y) = |\,\langle y|\,\psi\rangle\,|^2 = \tfrac{1}{4}$$

$$P(30^\circ \to R \to y) + P(30^\circ \to L \to y) \neq P(30^\circ \to y)$$

When we make a measurement of whether the photon is right or left circularly polarized we gain information and therefore interference cross term disapears from the equation. This is a different experiment and result from not having a polarizer in the middle. Collapsing the intereference pattern by taking a measurement is similar to the Stern Gerlach two-slit diffraction experiment discussed in the textbook.

(b) Repeat calculation using density matrix methods instead of amplitude mechanics.

$$|R\rangle\langle R| = \tfrac{1}{2}\begin{pmatrix}1 & -i\\ i & 1\end{pmatrix} \quad |L\rangle\langle L| = \tfrac{1}{2}\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix} \quad |y\rangle\langle y| = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

$$P(30^\circ \to R \to y) = |\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\tfrac{1}{2}\begin{pmatrix}1 & -i\\ i & 1\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}|^2 = \tfrac{1}{4}$$

$$P(30^\circ \to L \to y) = |\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\tfrac{1}{2}\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}|^2 = \tfrac{1}{4}$$

$$P(30^\circ \to y) = |\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}|^2 = \tfrac{1}{4}$$

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For this problem we have a photon propagating along the z-axis of a quartz crystal. In this situation we have the matrix representation of the Hamiltonian, with $|x\rangle$ and $|y\rangle$ as a basis, given by

$$\hat{H} = \begin{bmatrix} 0 & -iE_0 \\ iE_0 & 0 \end{bmatrix}$$

.

In part (a) we find the eigenstates and eigenvalues of the Hamiltonian. Doing this, we get $\lambda_1 = E_0$ and $\lambda_2 = -E_0$. These eigenvalues give us eigenvectors of

$$\vec{v_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ 1 \end{pmatrix}$$

, and

$$\vec{v_2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

.

Then, in part (b) we have that a photon enters the crystal linearly polarized in the x-direction. So, $|\psi(0)\rangle = |x\rangle$. Using part (a) we have that $\langle E_0|x\rangle = \frac{1}{\sqrt{2}}(i \;\; 1) \cdot \vec{x} = \frac{i}{\sqrt{2}}$. Also, $\langle -E_0|x\rangle = \frac{1}{\sqrt{2}}(1 \;\; i) \cdot \vec{x} = \frac{1}{\sqrt{2}}$. Here

$$\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

We also have from part (a) that $\{|E_0\rangle, |-E_0\rangle\}$ is a basis. So,

$$|E_{0,t}\rangle = e^{\frac{-i\hat{H}t}{\hbar}}|E_0\rangle = e^{\frac{-iE_0 t}{\hbar}}|E_0\rangle$$
$$|-E_{0,t}\rangle = e^{\frac{-i\hat{H}t}{\hbar}}|-E_0\rangle = e^{\frac{iE_0 t}{\hbar}}|-E_0\rangle.$$

We can use this basis to write a time-dependent expression for $|\psi(t)\rangle$. So, $|\psi(t)\rangle = a|E_{0,t}\rangle + b|-E_{0,t}\rangle$. Then,

$$\begin{aligned} |\psi(t)\rangle &= ae^{\frac{-iE_0 t}{\hbar}}|E_0\rangle + be^{\frac{iE_0 t}{\hbar}}|-E_0\rangle \\ &= \frac{i}{\sqrt{2}}e^{\frac{-iE_0 t}{\hbar}}|E_0\rangle + \frac{1}{\sqrt{2}}e^{\frac{iE_0 t}{\hbar}}|-E_0\rangle \end{aligned}$$

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In this problem we are dealing with K-meson states and the fact that $|K_L\rangle$ and $|K_S\rangle$ decay with time. Therefore, we have that these states have the time dependence of $|K_L(t)\rangle = e^{-i\omega_L t - t/2\tau_L}|K_L\rangle$ and $|K_S(t)\rangle = e^{-i\omega_S t - t/2\tau_S}|K_S\rangle$, where $\omega_{L(S)} = \frac{E_{L(S)}}{\hbar}$, $E_{L(S)} = (p^2c^2 + m_{L(S)}^2 c^4)^{\frac{1}{2}}, \tau_S \approx 0.9 \times 10^{-10}$ sec, and $\tau_L \approx 560 \times 10^{-19}$ sec.

We first suppose that a pure K_L beam is sent through a thin absorber whose only effect is to change the relative phase of K_0 and $\bar{K}_0$ amplitudes by 10^o. Remember, that $|K_L\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle - |\bar{K}_0\rangle$ and $|K_S\rangle = \frac{1}{\sqrt{2}}(|K_0\rangle + |\bar{K}_0\rangle$.

Given these relations, we have that $|\psi_{after}\rangle = (1 - e^{i\pi/18})|K_S\rangle + (1 + e^{i\pi/18})|K_L\rangle$. Then, $|\psi_{after,t}\rangle = (1 - e^{i\pi/18})e^{-i\omega_S t - t/2\tau_S}|K_S\rangle + (1 + e^{i\pi/18})e^{-i\omega_L t - t/2\tau_L}|K_L\rangle$. To calculate the number of K_S decays, relative to the incident number of particles, that will be observed in the first 5 cm after the absorber, we want to calculate the probability of K_S being observed from ψ. So,

$$\begin{aligned} |\langle K_S|\psi_{after,t}\rangle|^2 &= |(1 - e^{i\pi/18})|^2 e^{-t/\tau_S} \\ &= \frac{1}{2}(1 - \cos(\frac{\pi}{18}))e^{-t/\tau_S}. \end{aligned}$$

Then, approxiamately, given τ_S and the fact that the distance traveled/observed is 5 cm, which means that the time is $5/c = 1.6 \times 10^{-10}$ sec., we have that $\frac{t_d}{\tau_S} = \frac{16}{9}$. Therefore, $|\langle K_S|\psi_{after,t}\rangle|^2 = \frac{1}{2}(1 - \cos(\frac{\pi}{18}))e^{-16/9}$. Then, the number of K_S decays that will be observed is approxiamately $N \cdot \frac{1}{2}(1 - \cos(\frac{\pi}{18}))e^{-16/9}$, with N being the number of incident particles.

The initial state is given by

$$|\phi_{in}\rangle = \frac{1}{2}\begin{pmatrix}\sqrt{3}\\ 1\end{pmatrix} \tag{1}$$

since the incident light is linearly polarized at 30° from the optic axis. Then it passes through the quarter wave plate and becomes

$$|\phi\rangle = \frac{1}{2}\begin{pmatrix}\sqrt{3}\\ e^{i\frac{\pi}{2}}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\sqrt{3}\\ i\end{pmatrix} \tag{2}$$

We know that the right and left circularly polarized Jones vectors are given by

$$|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} \tag{3}$$

$$|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix} \tag{4}$$

Since the rate of change of angular momentum is the torque, we can compute

$$\frac{dL}{dt} = \tau = N(\hbar|\langle R|\ \phi\rangle|^2 - \hbar|\langle L|\ \phi\rangle|^2) \tag{5}$$

This gives

$$\frac{dL}{dt} = N\hbar\frac{1}{2}\left((\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2\right) = N\hbar\frac{1}{4}(1+\sqrt{3}-(1-\sqrt{3})) = N\hbar\frac{\sqrt{3}}{2} \tag{6}$$

a

Let x be the distance from the left source to the point x, which means that the distance from the right source to the point x is $L-x$. Applying the time evolution equation gives that the superposition state $|\phi(t)\rangle$ is given by

$$|\phi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-i2\pi\frac{x}{\lambda}} + e^{-i2\pi\frac{L-x}{\lambda}})\,|\phi(0)\rangle = \frac{1}{\sqrt{2}}e^{-i2\pi\frac{x}{\lambda}}(1 + e^{-i2\pi\frac{L-2x}{\lambda}})\,|\phi(0)\rangle \tag{1}$$

b

To find the probability we evalute the following expression

$$|\langle\phi(t)|\,\phi(t)\rangle|^2 = \frac{1}{2}(1 + 1 + e^{-i\frac{2\pi}{\lambda}(2x-L)} + e^{-i\frac{2\pi}{\lambda}(L-2x)} = \frac{1}{2}(2 + e^{-ik(2x-L)} + e^{ik(2x-L)}) \tag{2}$$

$$= \frac{1}{2}(2 + 2\cos(k(2x-L)) = 1 + \cos(k(2x-L)) \tag{3}$$

where

$$k = \frac{2\pi}{\lambda} \tag{4}$$

c

This means that at certain values of x it is possible to get $|\langle\phi(t)|\,\phi(t)\rangle|^2 = 2$, which corresponds to constructive interference, and at other values it is possible to get $|\langle\phi(t)|\,\phi(t)\rangle|^2 = 0$, which corresponds to destructive interference. When light is described as a wave it exhibits this constructive and destructive interference as well.

Let's take a loot at the two slit experiment with photons being shoit at the slits one at a time. Given the path length difference of the set up:

$$PLD = d\sin\theta = d\left[\frac{y}{[L^2+y^2]^{\frac{1}{2}}}\right] \approx \frac{yd}{L}$$

We can easily obtain the wave equation in terms of $|\psi(0)\rangle$ because S_1 and S_2 have a phase difference $k\frac{yd}{L}$ when $k = \frac{2\pi}{\lambda}$.

$$|\psi\rangle = |\psi(0)\rangle + e^{-ik\frac{yd}{L}}\,|\psi(0)\rangle$$

Therefore, the probability of approximating photons at various points along the screen is

$$\begin{aligned}\langle\psi|\,\psi\rangle &= \langle\psi(0)|\,(1+e^{-ik\frac{yd}{L}})(1+e^{ik\frac{yd}{L}})\,|\psi(0)\rangle \\ &= 1+1+e^{-ik\frac{yd}{L}}+e^{ik\frac{yd}{L}} \\ &= 2(1+\cos(\tfrac{kyd}{L})) = 4\cos^2(\tfrac{\pi yd}{\lambda L})\end{aligned}$$