

Physics 113

Spring 2010

Quantum Theory Seminar #4

Readings:

Zettili - Chapter(s) - 3

Boccio - Chapter(s) - 6.9-6.15 (new book versions)

Topics: Formulation of Quantum Mechanics

**Presentations: 8 min \leq TIME \leq 12 min (graded this week)
EVERYONE must read chapter before presentations.**

Symmetries of Space and Time _James____
Generators of Transformations
(Chapter 6-Sections 6.9 and 6.10)

Commutators and Identities _Max_____
(Chapter 6-Section 6.11)

Identification of Operators with Observables _Ed_____
(Chapter 6-Section 6.12 pages 38-44)

Examples 1-3 _Sandy_____
(Chapter 6-Section 6.12 pages 44-51)

Multi-Particle States (Chapter 6-Section 6.13) _Dougal____

Equations of Motion (Chapter 6-Section 6.14) _Elizabeth_

Symmetry and Conservation Laws _Ben_____
(Chapter 6-Section 6.15)

Collapse (Chapter 6-Section 6.16) _Jen_____

Composite Quantum Systems and the Tensor _Rachael____
Quantum Entanglement and the EPR "Paradox"
Product(Chapter 6-Section 6.16.1-6.16.2)

Problems:

1. Scale Transformation (Ed) - Space is invariant under the scale transformation

$$x \rightarrow x' = e^c x$$

where c is a parameter. The corresponding unitary operator may be written as

$$\hat{U} = e^{-ic\hat{D}}$$

where \hat{D} is the **dilation** generator. Determine the commutators $[\hat{D}, \hat{x}]$ and $[\hat{D}, \hat{p}_x]$ between the generators of dilation and space displacements. Determine the operator \hat{D} . Not all the laws of physics are invariant under dilation, so the symmetry is less common than displacements or rotations. You will need to use the identity in Problem 4.

2. Operator Properties (Jen)

- (a) Prove that if H is a Hermitian operator, then $U = e^{iH}$ is a unitary operator.
(b) Show that $\det U = e^{i\text{tr}H}$

3. An Instantaneous Boost - MIDTERM PROBLEM #1 (Do this one on your own; Can ask instructor questions)

The unitary operator

$$\hat{U}(\vec{v}) = e^{i\vec{v} \cdot \hat{G}}$$

describes the instantaneous ($t=0$) effect of a transformation to a frame of reference moving at the velocity \vec{v} with respect to the original reference frame. Its effects on the velocity and position operators are:

$$\hat{U}\hat{V}\hat{U}^{-1} = \hat{V} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{Q}\hat{U}^{-1} = \hat{Q}$$

Find an operator \hat{G}_i such that the unitary operator $\hat{U}(\vec{v}, t) = e^{i\vec{v} \cdot \hat{G}_i}$ will yield the full Galilean transformation

$$\hat{U}\hat{V}\hat{U}^{-1} = \hat{V} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{Q}\hat{U}^{-1} = \hat{Q} - \vec{v}t\hat{I}$$

Verify that \hat{G}_i satisfies the same commutation relation with \hat{P}, \hat{J} and \hat{H} as does \hat{G} .

4. A Very Useful Identity(Rachael) - Prove the following identity, in which \hat{A} and \hat{B} are operators, and x is a parameter.

$$e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + [\hat{A},\hat{B}]x + [\hat{A},[\hat{A},\hat{B}]]\frac{x^2}{2} + [\hat{A},[\hat{A},[\hat{A},\hat{B}]]]\frac{x^3}{6} + \dots$$

There is a clever way(see Section 8.12) to do this problem(not just multiplying everything out).

5. Another Very Useful Identity(Sandy) - Prove that

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}[\hat{A},\hat{B}]}$$

provided that the operators \hat{A} and \hat{B} satisfy

$$[\hat{A},[\hat{A},\hat{B}]] = [\hat{B},[\hat{A},\hat{B}]] = 0$$

Clever solution uses problem 4 result.

6. Pure to Nonpure?(James) - Use the equation of motion for the density operator $\hat{\rho}(t)$ to show that a pure state cannot evolve into a nonpure state and vice versa.