Physics 113 Spring 2010

Quantum Theory Seminar #4

Readings:

Zettili - Chapter(s) - 3
Boccio - Chapter(s) - 6.9-6.15 (new book versions)

Topics: Formulation of Quantum Mechanics

Presentations: 8 min ≤ TIME ≤ 12 min (graded this week)
EVERYONE must read chapter before presentations.

Symmetries of Space and Time Generators of Transformations (Chapter 6-Sections 6.9 and 6.10)	_Jonah____
Commutators and Identities (Chapter 6-Section 6.11)	_Sarah____
Identification of Operators with Observables (Chapter 6-Section 6.12 pages 38-44)	_Kevin____
Examples 1-2 (Chapter 6-Section 6.12 pages 44-48)	_Karen____
Example 3 (Chapter 6-Section 6.12 pages 48-51)	_Jono_____
Multi-Particle States (Chapter 6-Section 6.13)	_Andrew Z_
Equations of Motion (Chapter 6-Section 6.14)	_Jean_____
Symmetry and Conservation Laws (Chapter 6-Section 6.15)	_Andrew K_
Collapse (Chapter 6-Section 6.16)	_Ari______
Composite Quantum Systems and the Tensor	_Orion____
Quantum Entanglement and the EPR "Paradox" Entanglement and Communication Product(Chapter 6-Sections 6.16.2-6.16.3)	_Dan______
Nonlocality and Tests of Quantum Entanglement (Chapter 6-Section 6.16.4)	_Robert___

Problems:

1. Scale Transformation(Orion) - Space is invariant under the scale transformation

$$x \to x' = e^c x$$

where c is a parameter. The corresponding unitary operator may be written as

$$\hat{U} = e^{-ic\hat{D}}$$

where $\hat{D}$ is the **dilation** generator. Determine the commutators $\left[\hat{D},\hat{x}\right]$ and $\left[\hat{D},\hat{p}_x\right]$ between the generators of dilation and space displacements. Determine the operator $\hat{D}$. Not all the laws of physics are invariant under dilation, so the symmetry is less common than displacements or rotations. You will need to use the identity in Problem 4.

2. Operator Properties(Dan)

(a) Prove that if H is a Hermitian operator, then $U = e^{iH}$ is a unitary operator.
(b) Show that $\det U = e^{iTrH}$

3. An Instantaneous Boost - MIDTERM PROBLEM #1
(Do this one on your own; Can ask instructor questions)

The unitary operator

$$\hat{U}(\vec{v}) = e^{i\vec{v}\cdot\hat{\vec{G}}}$$

describes the instantaneous (t=0) effect of a transformation to a frame of reference moving at the velocity $\vec{v}$ with respect to the original reference frame. Its effects on the velocity and position operators are:

$$\hat{U}\hat{\vec{V}}\hat{U}^{-1} = \hat{\vec{V}} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{\vec{Q}}\hat{U}^{-1} = \hat{\vec{Q}}$$

Find an operator $\hat{G}_t$ such that the unitary operator $\hat{U}(\vec{v},t) = e^{i\vec{v}\cdot\hat{\vec{G}}_t}$ will yield the full Galilean transformation

$$\hat{U}\hat{\vec{V}}\hat{U}^{-1} = \hat{\vec{V}} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{\vec{Q}}\hat{U}^{-1} = \hat{\vec{Q}} - \vec{v}t\hat{I}$$

Verify that $\hat{G}_t$ satisfies the same commutation relation with $\hat{\vec{P}},\hat{\vec{J}}$ and $\hat{H}$ as does $\hat{G}$.

4. A Very Useful Identity(Kevin) - Prove the following identity, in which $\hat{A}$ and $\hat{B}$ are operators, and x is a parameter.

$$e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + \left[\hat{A},\hat{B}\right]x + \left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\frac{x^2}{2} + \left[\hat{A},\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\right]\frac{x^3}{6} + ...$$

There is a clever way(see Section 8.12) to do this problem(not just multiplying everything out).

5. Another Very Useful Identity(Ari) - Prove that

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}\left[\hat{A},\hat{B}\right]}$$

provided that the operators $\hat{A}$ and $\hat{B}$ satisfy

$$\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] = \left[\hat{B},\left[\hat{A},\hat{B}\right]\right] = 0$$

Clever solution uses problem 4 result.

6. Pure to Nonpure?(Andrew Z) - Use the equation of motion for the density operator $\hat{\rho}(t)$ to show that a pure state cannot evolve into a nonpure state and vice versa.