Physics 113 Spring 2010

Quantum Theory Seminar #1

This week we start to learn the **mathematical language of quantum mechanics.** We continue learning the mathematical language in the second seminar also. Much of this work should be review of mathematics you learned in linear algebra. **Learning the language of quantum mechanics BEFORE studying quantum mechanics is important.** The only way to do this is **LOTS of problems.**

Readings:

Zettili - Chapter 2
Sections 2.1-2.4.4;2.4.6-2.4.8; 2.5-2.5.3
+ relevant solved problems
(available on website as PDF if not in bookstore yet)

Boccio - Chapter(s) - 4(pages 1-38),5(pages 1-12)

Website:

http://chaos.swarthmore.edu/courses/Phys113_2010/index.html

Break: John

Topics:

Mathematics of Quantum Mechanics
Linear Vector Spaces and Linear Operators
Probability(Basic Ideas)

Presentations:

Discuss content of readings emphasizing whatever you think are the important concepts. These presentations should, at least, cover the basic concepts of the readings; also see if you can find new material from any other sources. Include examples where appropriate.

Linear Functionals and Dirac "bra" Vectors
Boccio - Chapter 4 Sections 4 & 5 **__Ed________**

Probability Concepts
Boccio - Chapter 5 (pp 1-12) **__Max________**

Basic Problems:

These problems are **all short and straightforward** and are meant to get you **thinking again** and start the process of **reviewing and recalling** your knowledge of linear algebra.

In addition, we begin to learn **DIRAC LANGUAGE,** which will be the mathematical language of quantum mechanics in this seminar.

The name **NEXT** to a problem is the person who **MUST** have solved it and will **PRESENT** it in seminar.

If the name is **EVERYONE**, then I will **ask for volunteers** to present the problem(**extra credit**).

There are 5 EVERYONE problems in this assignment. You must hand in your solutions to the EVERYONE problems at the BEGINNING of seminar.

It is OK to talk with others about the EVERYONE problems, but the solutions and write-ups should be your own work.

If you did not solve an EVERYONE problem before seminar, you can hand in a solution after seminar (for partial credit 70%).

Solutions handed in after seminar must be turned in by next morning at 10 AM for credit.

Thus, each person will be responsible for the EVERYONE problems + 2-3 other problems each week and sometimes a short presentation.

You should always attempt as many of the problems not assigned to you as possible since this will allow you to better understand another student's presentation.

Quantum mechanics is learned by doing problems! There is no other way!!

Zettili Exercises

2.1 Dirac Algebra(Ed)
2.6 Properties of a Ket Vector(Max)
2.9 Linear Independence(Sandy)
2.14 2-Dimensional Space(James)
2.22 Matrix Properties(Rachael)
2.29 Common Eigenvectors(EVERYONE)
2.39 Unitary and/or Hermitian?(Elizabeth)
2.48 Dirac Algebra and Matrix Properties(EVERYONE)

Boccio Problems

1. Simple Basis Vectors(Dougal) - Given two vectors

$$\vec{A} = 7\hat{e}_1 + 6\hat{e}_2 \quad , \quad \vec{B} = -2\hat{e}_1 + 16\hat{e}_2$$

written in the $\{\hat{e}_1, \hat{e}_2\}$ basis set and given another basis set

$$\hat{e}_q = \frac{1}{2}\hat{e}_1 + \frac{\sqrt{3}}{2}\hat{e}_2 \quad , \quad \hat{e}_p = -\frac{\sqrt{3}}{2}\hat{e}_1 + \frac{1}{2}\hat{e}_2$$

(a) Show that $\hat{e}_q$ and $\hat{e}_p$ are orthonormal

(b) Determine the new components of $\vec{A}$, $\vec{B}$ in the $\{\hat{e}_q, \hat{e}_p\}$ basis set

2. Eigenvalues and Eigenvectors(Jen) - Find the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ 5 & 0 & 3 \end{pmatrix}$$

Are the eigenvectors orthogonal? Comment on this.

3. Orthogonal Basis Vectors(Ben) - Determine the eigenvalues and eigenstates of the following matrix

$$\begin{pmatrix} 2 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

Using Gram-Schmidt, construct an orthonormal basis set from the eigenvectors of this operator.

4. Operator Matrix Representation(Ed) - If the states $\{|1\rangle,|2\rangle|3\rangle\}$ form an orthonormal basis and if the operator $\hat{G}$ has the properties

$$\hat{G}|1\rangle = 2|1\rangle - 4|2\rangle + 7|3\rangle$$
$$\hat{G}|2\rangle = -2|1\rangle + 3|3\rangle$$
$$\hat{G}|3\rangle = 11|1\rangle + 2|2\rangle - 6|3\rangle$$

What is the matrix representation of $\hat{G}$ in the $\{|1\rangle,|2\rangle|3\rangle\}$ basis?

5. A Matrix Element(Max) - If the states $\{|1\rangle,|2\rangle|3\rangle\}$ form an orthonormal basis and if the operator $\hat{K}$ has the properties

$$\hat{K}|1\rangle = 2|1\rangle$$
$$\hat{K}|2\rangle = 3|2\rangle$$
$$\hat{K}|3\rangle = -6|3\rangle$$

(a) Write an expression for $\hat{K}$ in terms of its eigenvalues and eigenvectors (projection operators). Use this expression to derive the matrix representing $\hat{K}$ in the $\{|1\rangle,|2\rangle|3\rangle\}$ basis.

(b) What is the "expectation value" of $\hat{K}$, defined as $\langle\alpha|\hat{K}|\alpha\rangle$, in the state

$$|\alpha\rangle = \frac{1}{\sqrt{83}}\left(-3|1\rangle + 5|2\rangle + 7|3\rangle\right)$$

6. Projection Operator Representation(Sandy) - Let the states $\{|1\rangle,|2\rangle|3\rangle\}$ form an orthonormal basis. We consider the operator given by $\hat{P}_2 = |2\rangle\langle 2|$. What is the matrix representation of this operator? What are its eigenvalues and eigenvectors. For the arbitrary state

$$|A\rangle = \frac{1}{\sqrt{83}}\left(-3|1\rangle + 5|2\rangle + 7|3\rangle\right)$$

what is the result of $\hat{P}_2|A\rangle$?

7. Operator Algebra(EVERYONE) - An operator for a two-state system is given by

$$\hat{H} = a\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

where a is a number. Find the eigenvalues and the corresponding eigenkets.

8. Functions of Operators(James) - Suppose that we have some operator $\hat{Q}$ such that $\hat{Q}|q\rangle = q|q\rangle$ i.e., $|q\rangle$ is an eigenvector of $\hat{Q}$ with eigenvalue q.

Show that $|q\rangle$ is also an eigenvector of the operators

$\hat{Q}^2, \hat{Q}^n$ and $e^{\hat{Q}}$ and determine the corresponding eigenvalues.

Simple Probability Concepts

There are 14 short problems here(9 & 10). If you have not studied any probability ideas, these are all new to you and doing them **should enable** you to learn the basic ideas of probability methods.

9. Rules of Probability

(a)(Ed) Two dice are rolled, one after the other. Let A be the event that the second number if greater than the first. Find P(A).

(b)(Max) Three dice are rolled and their scores added. Are you more likely to get 9 than 10, or vice versa?

(c)(Sandy) Which of these two events is more likely?
(1) four rolls of a die yield at least one six
(2) twenty-four rolls of two dice yield at least one double six

(d)(James) From meteorological records it is known that for a certain island at its winter solstice, it is wet with probability 30%, windy with probability 40% and both wet and windy with probability 20%. Find
(1) Prob(dry)
(2) Prob(dry and windy)
(3) Prob(wet or windy)

(e)(Rachael) A kitchen contains two fire alarms; one is activated by smoke and the other by heat. Experiment has shown that the probability of the smoke alarm sounding within one minute of a fire starting is o,95, the probability of the heat alarm sounding within one minute of a fire starting is 0.91, and the probability of both alarms sounding within one minute is 0.88. What is the probability of at least one alarm sounding within a minute?

(f)(Elizabeth) Suppose you are about to roll two dice, one from each hand. What is the probability that your right-hand die shows a larger number than your left-hand die?

Now suppose you roll the left-hand die first and it shows 5. What is the probability that the right-hand die shows a larger number?

(g)(Dougal) A coin is flipped three times. Let A be the event that the first flip gives a head and B be the event that there are exactly two heads overall. Determine

(1) $P(A|B)$
(2) $P(B|A)$

(h)(Jen) A box contains a double-headed coin, a double-tailed coin and a conventional coin. A coin is picked at random and flipped. It shows a head. What is the probability that it is the double-headed coin?

(i)(Ben) A box contains 5 red socks and 3 blue socks. If you remove 2 socks at random, what is the probability that you are holding a blue pair?

(j)(Sandy) An inexpensive electronic toy made by Acme Gadgets Inc. is defective with probability 0.001. These toys are so popular that they are copied and sold illegally but cheaply. Pirate versions capture 10% of the market and any pirated copy is defective with probability 0.5. If you buy a toy, what is the chance that it is defective?

(k)(James) Patients may be treated with any one of a number of drugs, each of which may give rise to side effects. A certain drug C has a 99% success rate in the absence of side effects and side effects only arise in 5% of cases. However, if they do arise, then C only has a 30% success rate. If C is used, what is the probability of the event A that a cure is effected?

(l)(Rachael) Suppose a multiple choice question has c available choices. A student either knows the answer with probability p or guesses at random with probability 1-p. Given that the answer selected is correct, what is the probability that the student knew the answer?

10. Counting and Gambling

(a)(Elizabeth) Common PINs do not begin with zero. They have four digits. A computer assigns you a PIN at random. What is the probability that all four digits are different?

(b)(Dougal) You are dealt a hand of 5 cards from a conventional deck(52 cards). A **full house** comprises 3 cards of one value and 2 of another value. If that hand has 4 cards of one value, this is called **four of a kind.** Which is more likely?

11. A Symmetric Matrix(Jen) - Let A be a 4×4 symmetric matrix. Assume that the eigenvalues are given by 0, 1, 2, and 3 with the corresponding normalized eigenvectors

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

Find the matrix A.

12. Determinants and Traces(Ben) - Let A be an $n \times n$ matrix. Show that $\det(\exp(A)) = \exp(tr(A))$.

13. Function of a Matrix(EVERYONE) - Let

$$A = \begin{pmatrix}-1 & 2\\2 & -1\end{pmatrix}$$

Calculate $\exp(\alpha A)$, α real.

14. More Gram-Schmidt(EVERYONE) - Let A be the symmetric matrix

$$A = \begin{pmatrix}5 & -2 & -4\\-2 & 2 & 2\\-4 & 2 & 5\end{pmatrix}$$

Determine the eigenvalues and eigenvectors of A. Are the eigenvectors orthogonal to each other? If not, find an orthogonal set using the Gram-Schmidt process.

15. Infinite Dimensions(Ed) - Let A be a square finite-dimensional matrix (real elements) such that $AA^T = I$.

(a) Show that

$$A^T A = I$$

(b) Does this result hold for infinite dimensional matrices?