Physics 113 Spring 2010

Quantum Theory Seminar #12

Readings:

Zettili - Chapter - 10
Boccio - Chapter - 11

Presentations:

Time-Dependent Perturbation Theory **_Sandy___**
Fermi Golden Rule
(Chapter 11 - Section 11.1(ALL))

Atomic Radiation and Selection Rules **_Jen_____**
(Chapter 11 - Section 11.2(ALL))

Ionization of Hydrogen Example **_Max_____**
(Chapter 11 - Section 11.3(ALL))

Zettili Problems:

1. Z10-12 Hydrogen in a crystal field (EVERYONE)

2. Z10-15 Oscillator in harmonic perturbation (EVERYONE)

3. Z10-22 ∞ square well perturbed by delta function (Sandy)

4. Z10-24 3-dim box perturbed by xz potential (Jen)

Boccio Problems:

1. Square Well Perturbed by an Electric Field(Ben) - At time t = 0, an electron is known to be in the n = 1 eigenstate of a 1-dimensional infinite square well potential

$$V(x) = \begin{cases} \infty & |x| > a/2 \\ 0 & |x| < a/2 \end{cases}$$

At time t = 0, a uniform electric field of magnitude ε is applied in the direction of increasing x. This electric field is left on for a short time τ and then removed. Use time-dependent perturbation theory to calculate the probability that the electron will be in the n = 2,3 eigenstates at some time $t > \tau$.

2. 3-Dimensional Oscillator in an electric field(Max) - A particle of mass M, charge e, and spin zero moves in an attractive potential

$$V(x,y,z) = k\left(x^2 + y^2 + z^2\right)$$

(a) Find the three lowest energy levels E_0, E_1, E_2 and their associated degeneracy.

(b) Suppose a small perturbing potential $Ax\cos\bar{\omega}t$ causes transitions among the various states in (a). Using a convenient basis for degenerate states, specify in detail the allowed transitions neglecting effects proportional to A^2 or higher.

(c) In (b) suppose the particle is in the ground state at time t = 0. Find the probability that the energy is E_1 at time t.

3. Hydrogen in decaying potential(FINAL #2) - A hydrogen atom (assume spinless electron and proton) in its ground state is placed between parallel plates and subjected to a uniform weak electric field

$$\vec{\varepsilon} = \begin{cases} 0 & t < 0 \\ \vec{\varepsilon}_0 e^{-\alpha t} & t > 0 \end{cases}$$

Find the 1st-order probability for the atom to be in any of the n=2 states after a long time.

4. 2 spins in a time-dependent potential(Rachael) - Consider a composite system made up of two spin = 1/2 objects. For t < 0, the Hamiltonian does not depend on spin and can be taken to be zero by suitably adjusting the energy scale. For t > 0, the Hamiltonian is given by

$$\hat{H} = \left(\frac{4\Delta}{\hbar^2}\right)\hat{S}_1 \cdot \hat{S}_2$$

Suppose the system is in the state $|+-\rangle$ for $t \leq 0$. Find, as a function of time, the probability for being found in each of the following states

$$|++\rangle, |+-\rangle, |-+\rangle, \text{ and } |--\rangle$$

(a) by solving the problem exactly.

(b) by solving the problem assuming the validity of 1st-order time-dependent perturbation theory with $\hat{H}$ as a perturbation switched on at t = 0.

Under what conditions does (b) give the correct results?

5. A Variational Calculation of the Deuteron Ground State Energy(FINAL #3 - This is a big, messy problem which will tell me whether you know all aspects of variational method theory - I expect all details to be carried out, however messy they are!)) - Use the same empirical potential energy function

$$V(r) = -Ae^{-r/a}$$

where $A = 32.7\ MeV$, $a = 2.18 \times 10^{-13} cm$, to obtain a variational approximation to the energy of the ground state ($\ell = 0$).

Try a simple variational function of the form

$$\varphi(r) = e^{-\alpha r/2a}$$

where α is the variational parameter to be determined. Calculate the energy in terms of α and minimize it. Give your results for α and E in MeV. The experimental value of E is $-2.23\,MeV$ (your answer should be VERY close! Is your answer above this? [HINT: do not forget about the "reduced mass" in this problem]

6. Sudden Changes (Elizabeth)

(a). Don't Sneeze - An experimenter has carefully prepared a particle of mass m in the first excited state of a one dimensional harmonic oscillator, when he sneezes and knocks the center of the potential well a small distance a to one side. It takes him a time T to blow his nose, and when he has done so, he immediately puts the center back where it was. Find, to lowest order in a, the probabilities P_0 and P_2 that the oscillator will now be in its ground state and its second excited state.

(b) Cutting the spring - A particle is allowed to move in one dimension. It is initially coupled to two identical harmonic springs, each with spring constant K. The springs are symmetrically fixed to the points $\pm a$ so that when the particle is at $x = 0$ the classical force on it is zero.

(a) What are the energy eigenvalues of the particle when it is connected to both springs?
(b) What is the wave function in the ground state?
(c) One spring is suddenly cut, leaving the particle bound to only the other one. If the particle is in the ground state before the spring is cut, what is the probability that it is still in the ground state after the spring is cut?

7. Another perturbed oscillator(Ed) - Consider a particle bound in a simple harmonic oscillator potential. Initially(t < 0), it is in the ground state. At t = 0 a perturbation of the form

$$H'(x,t) = Ax^2 e^{-t/\tau}$$

is switched on. Using time-dependent perturbation theory, calculate the probability that, after a sufficiently long time ($t >> \tau$), the system will have made a transition to a given excited state. Consider all final states.

8. Nuclear Decay(James) - Nuclei sometimes decay from excited states to the ground state by internal conversion, a process in which an atomic electron is emitted instead of a photon. Let the initial and final nuclear states have wave functions $\phi_i(\vec{r}_1,\vec{r}_2,...,\vec{r}_Z)$ and $\phi_f(\vec{r}_1,\vec{r}_2,...,\vec{r}_Z)$, respectively, where $\vec{r}_i$ describes the protons. The perturbation giving rise to the transition is the proton-electron interaction,

$$W = -\sum_{j=1}^{Z} \frac{e^2}{\left|\vec{r} - \vec{r}_j\right|}$$

where $\vec{r}$ is the electron coordinate.

(a) Write down the matrix element for the process in lowest-order perturbation theory, assuming that the electron is initially in a state characterized by the quantum number $(n\ell m)$, and that its energy, after it is emitted, is large enough so that its final state may be described by a plane wave, Neglect spin.

(b) Write down an expression for the internal conversion rate.

(c) For light nuclei, the nuclear radius is much smaller than the Bohr radius for a give Z, and we can use the expansion

$$\frac{1}{\left|\vec{r} - \vec{r}_j\right|} \cong \frac{1}{r} + \frac{\vec{r} \cdot \vec{r}_j}{r^3}$$

Use this expression to express the transition rate in terms of the dipole matrix element

$$\vec{d} = \left\langle \phi_f \right| \sum_{j=1}^{Z} \vec{r}_j \left| \phi_i \right\rangle$$