**Physics 113** **Spring 2009**

# Quantum Theory Seminar #5

**Readings:**

**Zettili - Chapter - 4**
**Boccio - Chapter(s) - 7,8(pages 1-8,22-28,33-49,52-59)**

**Lots of reading but you have seen most of the stuff in Zettili chapter 4 and Boccio Chapter 8 in Physics 14. So, that portion of the readings will be review and go fast!**

**Topics:**

**How does QM really work**
**Photons, K-Mesons and Stern Gerlach**
**Schrodinger Equation and 1-dimensional Systems**

**Presentations:**

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| **Photon Polarization (Chapter 7 - pages 14-33)** | **_Anna____** |
| **Evolution of Photon States (Chapter 7 - pages 34-38)** | **_Erik____** |
| **The World of K-Mesons (Chapter 7 - pages 43-52)** | **_Markus_______** |
| **x- and p-representations and the Schrodinger Equation (Chapter 8 - pages 1-8)** | **_Dogus____** |

**Problems:**

**1. Change the Basis(Karan) -** In examining light polarization, we have been working in the $(|x\rangle,|y\rangle)$ basis.

(a) Just to show how easy it is to work in other bases, express $|x\rangle$ and $|y\rangle$ in the $(|R\rangle,|L\rangle)$ and $(|45°\rangle,|135°\rangle)$ bases.

(b) If you are working in the $(|R\rangle,|L\rangle)$ basis, what would the operator representing a vertical polaroid look like?

**2. Polaroids(Elizabeth) -** Imagine a situation in which a photon in the $|x\rangle$ state strikes a vertical polaroid. Clearly the probability of the photon getting through the vertical polaroid is 0. Now consider the case of two polaroids with the photon in the $|x\rangle$ state striking a polaroid at $45^\circ$ and then striking a vertical polaroid.

(a) Show that the probability of the photon getting through both polaroids is 1/4.

Consider now the case of three polaroids with the photon in the $|x\rangle$ state striking a polaroid at $30^\circ$ first, then a polaroid at $60^\circ$ and finally a vertical polaroid.

(b) Show that the probability of the photon getting through all three polaroids is 27/64.

**3. Calcite(Bevan) -** A photon is polarized at an angle $\theta$ to the optic axis is sent in the z direction through a slab of calcite $10^{-2}\ cm$ thick in the z direction. Assume that the optic axis lies in the x-y plane. Calculate, as a function of $\theta$, the transition probability for the photon to emerge left circularly polarized. Sketch the result. Let the frequency of the light be given by $c/\omega = 5000\,\mathring{A}$, and let $n_e = 1.50$ and $n_o = 1.65$ for the calcite.

**4. Turpentine(Matt) -** Turpentine is an "optically active" substance. If we send plane polarized light into turpentine then it emerges with its plane of polarization rotated. Specifically, turpentine induces a left-hand rotation of about $5^\circ$ per cm of turpentine that the light traverses. Write down the transition matrix that relates the incident polarization state to the emergent polarization state. Show that this matrix is unitary. Why is that important? Find its eigenvectors and eigenvalues, as a function of the length of turpentine traversed.

**5. What QM is all about(Emily) -** Photons polarized at $30^\circ$ to the x-axis are sent through a y-polaroid. An attempt is made to determine how frequently the photons that pass through the polaroid, pass through as "right circularly polarized photons" and how frequently they pass through as "left circularly polarized photons". This attempt is made as follows:

First, a prism that passes only right circularly polarized light is placed between the source of the $30^\circ$ polarized photons and the y-polaroid, and it is determined how frequently the $30^\circ$ photons pass through the y-polaroid. Then this experiment is repeated with a prism that passes only left circularly polarized photons instead of the one that passes only right.

Show by explicit calculation that the sum of the probabilities for passing through the y-polaroid measured in these two experiments is different from the probability that one would measure if there were no prism in the path of the photon and only the y-polaroid.

Relate this experiment to the two-slit diffraction experiment.

Also do this problem using density matrix methods.

**6. Photons and polarizers(EVERYONE) -** A photon polarization state for a photon propagating in the z-direction is given by

$$|\psi\rangle = \sqrt{\frac{2}{3}}|x\rangle + \frac{i}{\sqrt{3}}|y\rangle$$

(a) What is the probability that a photon in this state will pass through a polaroid with its transmission axis oriented in the y-direction?

(b) What is the probability that a photon in this state will pass through a polaroid with its transmission axis y' making an angle $\phi$ with the y-axis ?

(c) A beam carrying N photons per second, each in the state $|\psi\rangle$, is totally absorbed by a black disk with its normal to the surface in the z-direction. How large is the torque exerted on the disk? In which direction does the disk rotate? REMINDER: The photon states $|R\rangle$ and $|L\rangle$ each carry a unit $\hbar$ of angular momentum parallel and antiparallel, respectively, to the direction of propagation of the photon.

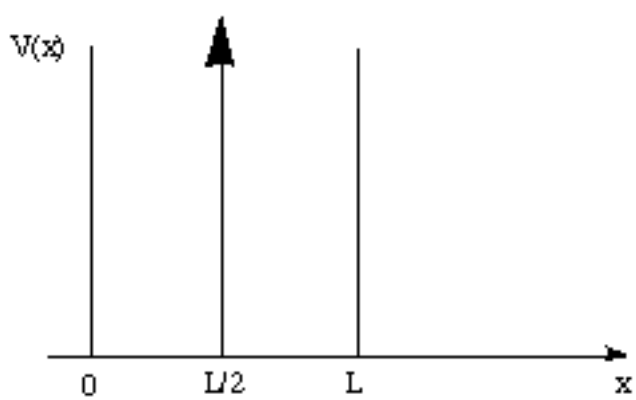
**7. As a function of angle(Nick) -** A photon is polarized at an angle $\theta$ to the optic axis is sent in the z direction through a slab of calcite $10^{-2}\, cm$ thick in the z direction. Assume that the optic axis lies in the x-y plane. Calculate, as a function of $\theta$, the transition probability for the photon to emerge left circularly polarized. Sketch the result. Let the frequency of the light be given by $c/\omega = 5000\,\mathring{A}$ and let $n_e = 1.50$ and $n_o = 1.65$ for the calcite.

**8. Time evolution(Dogus) -** The matrix representation of the Hamiltonian for a photon propagating along the optic axis (taken to be the z-axis) of a quartz crystal using the linear polarization states $|x\rangle$ and $|y\rangle$ as a basis is given by

$$\hat{H} = \begin{pmatrix} 0 & -iE_0 \\ iE_0 & 0 \end{pmatrix}$$

(a) What are the eigenstates and eigenvalues of the Hamiltonian?
(b) A photon enters the crystal linearly polarized in the x direction, that is, $|\psi(0)\rangle = |x\rangle$. What is $|\psi(t)\rangle$, the state of the photon at time t? Express your answer in the $\{|x\rangle, |y\rangle\}$ basis.
(c) What is happening to the polarization of the photon as it travels through the crystal?

**9. Delta function in a well(Markus) -** A particle of mass m moving in one dimension is confined to a space $0 < x < L$ by an infinite well potential. In addition, the particle experiences a delta function potential of strength $\lambda$ ($\lambda\delta(x - L/2)$) located at the center of the well as shown in the figure.




Find a transcendental equation for the energy eigenvalues E in terms of the mass m, the potential strength $\lambda$, and the size of the well L.

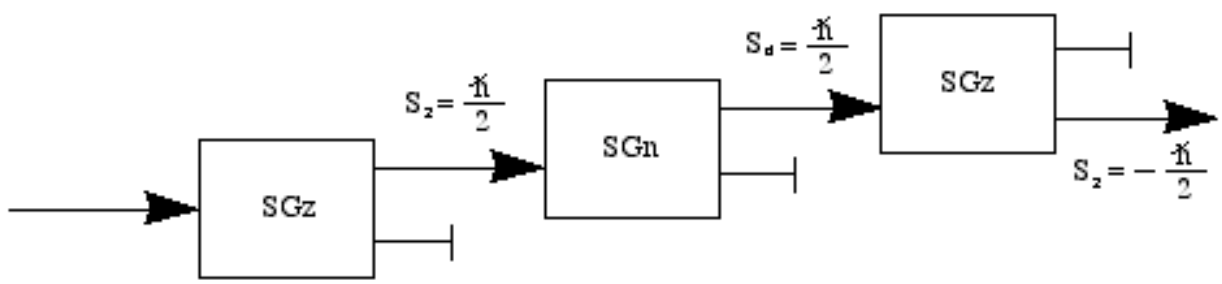
**10. 1/x potential(EVERYONE) -** An electron moves in one dimension and is confined to the right
half-space (x > 0) where it has potential energy

$$V(x) = -\frac{e^2}{4x}$$

where e is the charge on an electron.

(a) What is the solution of the Schrodinger equation at large x ?
(b) What is the boundary condition at x = 0 ?
(c) Use the results of (a) and (b) to guess the ground state solution of the equation. Remember the ground state wave function has no zeros except at the boundaries.
(d) Find the ground state energy.
(e) Find the expectation value $\langle \hat{x} \rangle$ in the ground state.

**11. What comes out?(Erik) -** A beam of spin 1/2 particles is sent through series of three Stern-Gerlach measuring devices as shown below:




The first SGz device transmits particles with $\hat{S}_z = \hbar/2$ and filters outs particles with $\hat{S}_z = -\hbar/2$. The second device, an SGn device transmits particles with $\hat{S}_n = \hbar/2$ and filters outs particles with $\hat{S}_n = -\hbar/2$, where the axis $\hat{n}$ makes an angle $\theta$ in the x-z plane with respect to the z-axis. Thus the particles passing through this SGn device are in the state

$$|+\hat{n}\rangle = \cos\frac{\theta}{2}|+\hat{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\hat{z}\rangle$$

with the angle $\phi = 0$. A last SGz device transmits particles with $\hat{S}_z = -\hbar/2$ and filters outs particles with $\hat{S}_z = \hbar/2$.

(a) What fraction of the particles transmitted through the first SGz device will survive the third measurement?
(b) How must the angle $\theta$ of the SGn device be oriented so as to maximize the number of particles the at are transmitted by

the final SGz device? What fraction of the particles survive the third measurement for this value of $\theta$?

(c) What fraction of the particles survive the last measurement if the SGn device is simply removed from the experiment?

**12. Using the commutator(Elizabeth) -** Using the coordinate-momentum commutation relation prove that

$$\sum_n (E_n - E_0)\left|\langle E_n|\hat{x}|E_0\rangle\right|^2 = \text{constant}$$

where $E_n$ is the energy corresponding to the eigenstate $|E_n\rangle$.

Determine the value of the constant. Assume the Hamiltonian has the general form

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

**13. Properties of the wave function(Anna) -** A particle of mass m is confined to a one-dimensional region $0 \le x \le a$ (an infinite square well potential). At t = 0 its normalized wave function is

$$\psi(x,t=0) = \sqrt{\frac{8}{5a}}\left(1+\cos\left(\frac{\pi x}{a}\right)\right)\sin\left(\frac{\pi x}{a}\right)$$

(a) What is the wave function at a later time $t = t_0$?

(b) What is the average energy of the system at t = 0 and $t = t_0$?

(c) What is the probability that the particle is found in the left half of the box(i.e., the region ($0 \le x \le a/2$) at $t = t_0$?

**14. Step and Delta Functions(EVERYONE)** - Consider a one-dimensional potential with a step-function component and an attractive delta function component just at the edge of the step, namely,

$$V(x) = V\Theta(x) - \frac{\hbar^2 g}{2m}\delta(x)$$

(a) For E > V, compute the reflection coefficient for particle incident from the left. How does this result differ from that of the step barrier alone at high energy?

(b) For E < 0 determine the energy eigenvalues and eigenfunctions of any bound-state solutions.

**15. Repulsive Potential(Emily) -** A repulsive short-range potential with a strongly attractive core can be approximated by a square barrier with a delta function at its center, namely

$$V(x) = V_0\Theta(x-|a|) - \frac{\hbar^2 g}{2m}\delta(x)$$

(a) Show that there is a negative energy eigenstate (the ground-state).

(b) If $E_0$ is the ground-state energy of the delta-function potential in the absence of the positive potential barrier, the ground-state energy of the present system satisfies the relation

$$E \geq E_0 + V_0$$

What is the particular value of $V_0$ for which we have the limiting case of a ground-state with zero energy.

**16. Measurement on a Particle in a Box(Nick) -** Consider a particle in a box of width a, prepared in the ground state.

(a) What are then possible values one can measure for:
(1) energy, (2) position, (3) momentum

(b) What are the probabilities for the possible outcomes you found in part (a)?

(c) At some (call it t = 0) we perform a measurement of position. However, our detector has only finite resolution. We find that the particle is in the middle of the box (call it the origin) with an uncertainty $\Delta x = a/2$, that is, we know the position is, for sure, in the range $-a/4 < x < a/4$, but we are completely uncertain where it is within this range. What is the (normalized) post-measurement state?

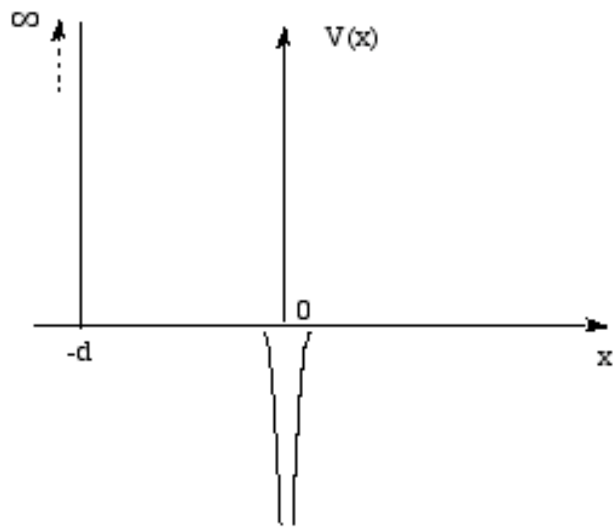
(d) Immediately after the position measurement what are the possible values for
(1) energy, (2) position, (3) momentum
and with what probabilities?

(e) At a later time, what are the possible values for
(1) energy, (2) position, (3) momentum
and with what probabilities? Comment.

**17. Atomic Model(Bevan) -** An approximate model for an atom near a wall is to consider a particle moving under the influence of the one-dimensional potential given by

$$V(x) = \begin{cases} -V_0\delta(x) & \text{x > -d} \\ \infty & \text{x < -d} \end{cases}$$

as shown below:




(a) Find the modification of the bound-state energy caused by the wall when it is "far away". Define what you mean by "far away".

(b) What is the exact condition on $V_0$ and d for the existence of at least one bound state?