# Physics 113 Spring 2009

## Quantum Theory Seminar #4

**Readings:**

Zettili - Chapter(s) - 3
Boccio - Chapter(s) - 6(pages 30-94)

**Topics:**

Formulation of Quantum Mechanics

**Presentations: 12 minutes maximum (graded this week)**
**EVERYONE must read chapter before presentations.**

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| **Symmetries of Space and Time**<br>**Generators of Transformations**<br>**(Chapter 6 - pages 30-36)** | **_Bevan____** |
| **Commutators and Identities**<br>**(Chapter 6 - pages 36-42)** | **_Nick_____** |
| **Identification of Operators with Observables**<br>**(Chapter 6 - pages 42-48)** | **_Anna_____** |
| **Examples 1-3**<br>**(Chapter 6 - pages 48-56)** | **_Markus __** |
| **Multi-particle States (Chapter 6 - pages 56-63)** | **_Erik_____** |
| **Equations of motion (Chapter 6 - pages 63-67)** | **_Matt_____** |
| **Symmetry and conservation laws**<br>**(Chapter 6 - pages 67-70)** | **_Karan____** |
| **Collapse (Chapter 6 - pages 70-87)** | **_Emily____** |
| **Composite Quantum Systems and the Tensor**<br>**Product(Chapter 6 - pages 77-84)** | **_Dogus____** |
| **Quantum Entanglement and the EPR "Paradox"**<br>**Entanglement and Communication**<br>**Nonlocality and Tests of Quantum Entanglement**<br>**(Chapter 6 - pages 84-94)** | **_Elizabeth_** |

**Problems:**

**1. Scale Transformation(Nick) -** Space is invariant under the scale transformation

$$x \to x' = e^c x$$

where c is a parameter. The corresponding unitary operator may be written as

$$\hat{U} = e^{-ic\hat{D}}$$

where $\hat{D}$ is the **dilation** generator. Determine the commutators $\left[\hat{D},\hat{x}\right]$ and $\left[\hat{D},\hat{p}_x\right]$ between the generators of dilation and space displacements. Determine the operator $\hat{D}$. Not all the laws of physics are invariant under dilation, so the symmetry is less common than displacements or rotations. You will need the identity in Problem 7.

**2. Operator Properties(Markus)**

(a) Prove that if $H$ is a Hermitian operator, then $U = e^{iH}$ is a unitary operator.
(b) Show that $\det U = e^{iTrH}$

**3. An Instantaneous Boost(EVERYONE) -** The unitary operator

$$\hat{U}(\vec{v}) = e^{i\vec{v}\cdot\hat{\vec{G}}}$$

describes the instantaneous (t=0) effect of a transformation to a frame of reference moving at the velocity $\vec{v}$ with respect to the original reference frame. Its effects on the velocity and position operators are:

$$\hat{U}\hat{\vec{V}}\hat{U}^{-1} = \hat{\vec{V}} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{\vec{Q}}\hat{U}^{-1} = \hat{\vec{Q}}$$

Find an operator $\hat{G}_t$ such that the unitary operator $\hat{U}(\vec{v},t) = e^{i\vec{v}\cdot\hat{\vec{G}}_t}$ will yield the full Galilean transformation

$$\hat{U}\hat{\vec{V}}\hat{U}^{-1} = \hat{\vec{V}} - \vec{v}\hat{I} \quad , \quad \hat{U}\hat{\vec{Q}}\hat{U}^{-1} = \hat{\vec{Q}} - \vec{v}t\hat{I}$$

Verify that $\hat{G}_t$ satisfies the same commutation relation with $\hat{\vec{P}},\hat{\vec{J}}$ and $\hat{H}$ as does $\hat{G}$.

**4. A Very Useful Identity(EVERYONE) -** Prove the following identity, in which $\hat{A}$ and $\hat{B}$ are operators, and $x$ is a parameter.

$$e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + \left[\hat{A},\hat{B}\right]x + \left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\frac{x^2}{2} + \left[\hat{A},\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\right]\frac{x^3}{6} + ......$$

**There is a clever way to do this problem(not just multiplying everything out).**

**5. Another Very Useful Identity(EVERYONE)** - Prove that

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}\left[\hat{A},\hat{B}\right]}$$

provided that the operators $\hat{A}$ and $\hat{B}$ satisfy

$$\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] = \left[\hat{B},\left[\hat{A},\hat{B}\right]\right] = 0$$

**Clever solution uses problem 4 result.**

**6. Pure to Nonpure?(Dogus) -** Use the equation of motion for the density operator $\hat{\rho}(t)$ to show that a pure state cannot evolve into a nonpure state and vice versa.