PARTICLE PHYSICS

2003

PART 3 (Lectures 5-6)

CONSERVATION LAWS AND SYMMETRY

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Some conservation laws and their associated symmetry

QUANTITY	STRONG	EM	WEAK	SYMMETRY
Charge	YES	YES	YES	Phase of Wave Function
Momentum	YES	YES	YES	Spatial Translation
Energy	YES	YES	YES	Time Translation
Angular Momentum	YES	YES	YES	Spatial Rotation
Baryon Number	YES	YES	YES(?)	Rotation in Baryon Space
Lepton Number	-	YES	YES(?)	Rotation in Lepton Space

QUANTITY	STRONG	EM	WEAK	SYMMETRY
Strong Isospin	YES	-	NO	Rotation in Isospin Space
Strangeness	YES	YES	NO	Rotation in Strangeness Space
Parity $\hat{P}$	YES	YES	NO	Reflection in Space
Charge Conjugation $\hat{C}$	YES	YES	NO	Particle-Antiparticle Inversion
$\hat{C}\hat{P}$	YES	YES	Almost	Simultaneous P and C
Time Reversal $\hat{T}$	YES	YES	Almost	Reverse direction of time
$\hat{C}\hat{P}\hat{T}$	YES	YES	YES	Simultaneous C, P and T

NOETHER'S THEOREM

A very deep principle in physics is due to Emily Noether (1882-1935)

For every fundamental symmetry there is a conserved quantity

As an example, consider spatial translation

Define an operator $\hat{D}$ such that

$$\psi' = \psi(x + \Delta x) = \hat{D}\psi(x) = \psi(x)\left[1 + \frac{\partial}{\partial x}\Delta x\right]$$

This is linked to the momentum operator $\hat{p_x} = -i\frac{\partial}{\partial x}$ or $i\hat{p_x} = \frac{\partial}{\partial x}$

For a macroscopic translation through $x = n\Delta x$, the relevant operator is approximately

$$\hat{D^n} = [1 + i\hat{p_x}\Delta x]^n = \left[1 + i\hat{p_x}\frac{x}{n}\right]^n$$

In the limit, this becomes exact

$$\hat{D} = \lim_{n \to \infty} \left[1 + i\hat{p_x}\frac{x}{n}\right]^n = e^{i\hat{p_x}x}$$

If the Hamiltonian is independent of $\hat{D}$, i.e. $\left[\hat{H}, \hat{D}\right] = 0$

then $\frac{\partial \hat{D}}{\partial t} = 0$, $\hat{D}$ is constant, and therefore so is $\hat{p_x}$

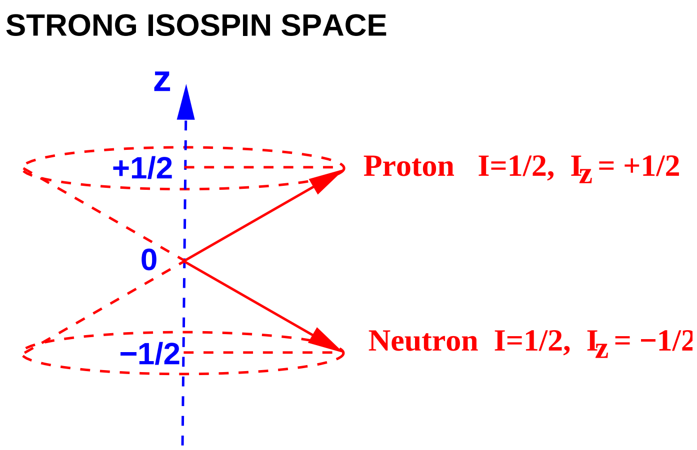
Therefore the expectation value of momentum

$$\langle p_x \rangle = \iiint \psi^* \hat{p_x} \psi d\tau$$

is constant, and momentum in the x direction is conserved.

STRONG ISOSPIN

Strong isospin is different from the weak isospin of the Weinberg Salam model that we will meet later in the course. Strong isospin does not differentiate between states such as p and n, they can be viewed as different orientations (projections) of the same vector in strong isospin space, representing a nucleon.



The strong interaction conserves isospin, which means that it is a good quantum number for labelling strongly interacting states.

Here are some examples of strong isospin families

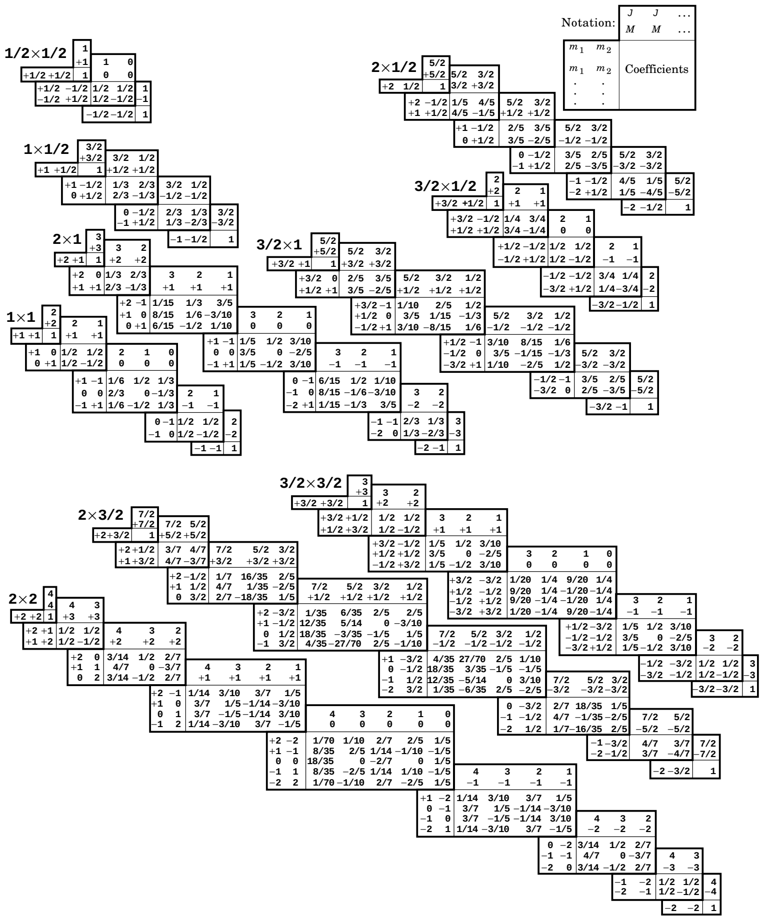
$$\begin{pmatrix} p \\ n \end{pmatrix} \text{ with } I = \frac{1}{2} \qquad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \text{ with } I = 1 \qquad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \text{ with } I = \frac{1}{2}$$

$$\begin{pmatrix} \overline{K^0} \\ K^- \end{pmatrix} \text{ with } I = \frac{1}{2} \qquad \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \text{ with } I = 1 \qquad \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \text{ with } I = \frac{1}{2}$$

$$\Lambda^0 \text{ with } I = 0 \qquad \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix} \text{ with } I = \frac{3}{2}$$

We can carry over the whole mathematics of spin combinations to isospin and use the **Clebsch-Gordan Coefficients**

CLEBSCH-GORDAN COEFFICIENTS

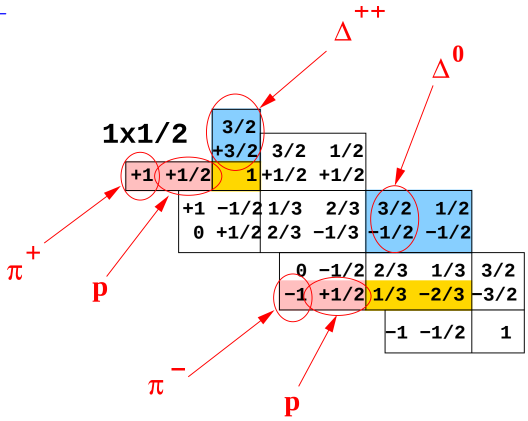


Each coefficient is understood to have a square-root sign over it

Example of use of Clebsch-Gordan tables

Compare the production rates

$$\pi^+ + p \rightarrow \Delta^{++}$$
$$\pi^- + p \rightarrow \Delta^0$$



Since pions have $I = 1$ and nucleons have $I = 1/2$, we use the $1 \times 1/2$ table.

In terms of (I, I_3) values for the final state

$$\pi^+ + p \equiv \left(\frac{3}{2}, +\frac{3}{2}\right)$$
$$\pi^- + p \equiv \frac{1}{\sqrt{3}}\left(\frac{3}{2}, -\frac{1}{2}\right) - \sqrt{\frac{2}{3}}\left(\frac{1}{2}, -\frac{1}{2}\right)$$

The rate for the first process is therefore 3 times that for the second (compare $(1)^2$ to $(1/\sqrt{3})^2$).

Isospin governs why some strong interactions are allowed and some disallowed

e.g.

	Λ^0	$\rightarrow$	p	$+$	π^-	
I	0	$\rightarrow$	$\frac{1}{2}$	$+$	1	hence DISALLOWED
I_3	0	$\rightarrow$	$+\frac{1}{2}$	$-$	1	hence DISALLOWED

Cannot proceed as a strong interaction. It does occur as a weak decay.

	π^-	$+$	p	$\rightarrow$	Λ^0	$+$	K^0	
I	1	$+$	$\frac{1}{2}$	$\rightarrow$	0	$+$	$\frac{1}{2}$	hence OK
I_3	-1	$+$	$\frac{1}{2}$	$\rightarrow$	0	$-$	$\frac{1}{2}$	hence OK

Proceeds fast via the strong interaction.

	d	$+$	d	$\rightarrow$	^4He	$+$	π^0	
I	0	$+$	0	$\rightarrow$	0	$+$	1	hence DISALLOWED
I_3	0	$+$	0	$\rightarrow$	0	$+$	0	hence OK

Disallowed because isospin is not vectorially conserved (even though z components look OK)

EXAMPLE OF USEFULNESS OF STRONG ISOSPIN

We wish to compare the rates of the strong interactions

$$\pi^+ + p \rightarrow \pi^+ + p \qquad \text{(Elastic Scattering)}$$
$$\pi^- + p \rightarrow \pi^- + p \qquad \text{(Elastic Scattering)}$$
$$\pi^- + p \rightarrow \pi^0 + n \qquad \text{(Charge Exchange Scattering)}$$

where everything else is unchanged - just the species have changed.

Historically it was assumed that these were interactions occurring at a single vertex. In this case we have

$$\text{Rate} \propto |\langle\psi_f|V|\psi_i\rangle|^2$$

Suppose that $V = \hat{V_{3/2}} + \hat{V_{1/2}}$ where $\hat{V_{3/2}}$ is an operator only applicable to the isospin 3/2 states (i.e. it doesn't "see" isospin 1/2 states) and $\hat{V_{1/2}}$ is the opposite.

For $\quad \pi^+ + p \rightarrow \pi^+ + p, \quad \psi_i = \psi_f = \left(\frac{3}{2}, +\frac{3}{2}\right)$

$$\langle\psi_f|\hat{V_{3/2}} + \hat{V_{1/2}}|\psi_i\rangle = \langle\psi_f|v_{3/2}|\psi_i\rangle = v_{3/2}\langle\psi_f|\psi_i\rangle$$
$$= v_{3/2}\langle\left(\tfrac{3}{2}, +\tfrac{3}{2}\right) \mid \left(\tfrac{3}{2}, +\tfrac{3}{2}\right)\rangle = v_{3/2}$$

which means the rate is proportional to ${v_{3/2}}^2$ where $v_{3/2}$ is the eigenvalue of the $\hat{V_{3/2}}$ operator.

For $\quad \pi^- + p \rightarrow \pi^0 + n$

$$\psi_i = \sqrt{\tfrac{1}{3}}\left(\tfrac{3}{2}, -\tfrac{1}{2}\right) - \sqrt{\tfrac{2}{3}}\left(\tfrac{1}{2}, -\tfrac{1}{2}\right)$$
$$\psi_f = \sqrt{\tfrac{2}{3}}\left(\tfrac{3}{2}, -\tfrac{1}{2}\right) + \sqrt{\tfrac{1}{3}}\left(\tfrac{1}{2}, -\tfrac{1}{2}\right)$$

The amplitude is

$$\langle\sqrt{\tfrac{2}{3}}\left(\tfrac{3}{2}, -\tfrac{1}{2}\right) + \sqrt{\tfrac{1}{3}}\left(\tfrac{1}{2}, -\tfrac{1}{2}\right)|\hat{V_{3/2}} + \hat{V_{1/2}}|\sqrt{\tfrac{1}{3}}\left(\tfrac{3}{2}, -\tfrac{1}{2}\right) - \sqrt{\tfrac{2}{3}}\left(\tfrac{1}{2}, -\tfrac{1}{2}\right)\rangle$$
$$= \tfrac{\sqrt{2}}{3}v_{3/2}\langle\left(\tfrac{3}{2}, -\tfrac{1}{2}\right) \mid \left(\tfrac{3}{2}, -\tfrac{1}{2}\right)\rangle - \tfrac{\sqrt{2}}{3}v_{1/2}\langle\left(\tfrac{1}{2}, -\tfrac{1}{2}\right) \mid \left(\tfrac{1}{2}, -\tfrac{1}{2}\right)\rangle$$

and the rate is proportional to $\frac{2}{9}\left[v_{3/2} - v_{1/2}\right]^2$

We can make predictions regarding the relative rates for the following scenarios

INTERACTION	RATE	$\hat{V_{3/2}} \neq 0$, $\hat{V_{1/2}} = 0$	$\hat{V_{3/2}} = 0$, $\hat{V_{1/2}} \neq 0$	$\hat{V_{3/2}} = \hat{V_{1/2}}$ both non-zero
$\pi^+ + p \rightarrow \pi^+ + p$	${v_{3/2}}^2$	9	0	1
$\pi^- + p \rightarrow \pi^- + p$	$\frac{1}{9}\left[v_{3/2} + 2v_{1/2}\right]^2$	1	2	1
$\pi^- + p \rightarrow \pi^0 + n$	$\frac{2}{9}\left[v_{3/2} - v_{1/2}\right]^2$	2	1	0

In the energy region where $\sqrt{s} = 1232$ MeV the experimental ratio is $91 : 11 : 22$, fairly unambiguously showing that the reaction proceeds via a channel for which strong isospin is $3/2$. ($m_\Delta = 1232$ MeV)

STRANGENESS

The concept of strangeness was postulated by Gell-Mann to explain

(a) **Associated Production in Strong Interactions**

Some particles such as hyperons and kaons always seem to be produced in pairs, never singly

i.e. $\pi^- + p \rightarrow \Lambda^0 + K^0$ (never just Λ^0)

(b) **Vast Difference in Some Decay Rates**

For example

$\Delta \rightarrow N + \pi$	1232 MeV	$\tau \approx 10^{-23}$ s	STRONG
$\Lambda^0 \rightarrow N + \pi$	1115 MeV	$\tau \approx 2.5 \times 10^{-10}$ s	WEAK

(c) **Why Kaons form Doublets**

Why the K^+ and K^0 form a distinct strong isospin doublet from the $\overline{K^0}$ and K^-? Why are they not all lumped together into a quartet of states like the Δ resonances?

Particles are proposed to possess a strangeness tag (quantum number) S which is conserved in strong interactions (but not weak)

As a definition we can write (N is baryon number)

$$\boxed{Q = e\left[I_3 + \frac{1}{2}(N+S)\right] = e\left[I_3 + \frac{1}{2}Y\right]}$$

If we average over a family all of which have the same N and S (and hence the same $Y = N + S$, we have $\sum I_3 = 0$

FOR MESONS $\quad \frac{\overline{Q}}{e} = \frac{1}{2}S \quad$ so $\quad S = 2\frac{\overline{Q}}{e}$

FOR BARYONS $\quad \frac{\overline{Q}}{e} = \frac{1}{2}(N+S) \quad$ so $\quad S = 2\frac{\overline{Q}}{e} - N$

For mesons with

$S = 2\frac{\overline{Q}}{e}$

$\begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}, \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$ have $\frac{\overline{Q}}{e} = 0$ so $S = 0$

$\begin{pmatrix} K+ \\ K^0 \end{pmatrix}$ have $\frac{\overline{Q}}{e} = +\frac{1}{2}$ so $S = +1$

$\begin{pmatrix} \overline{K^0} \\ K^- \end{pmatrix}$ have $\frac{\overline{Q}}{e} = -\frac{1}{2}$ so $S = -1$

ω^0, ϕ^0 have $\frac{\overline{Q}}{e} = 0$ so $S = 0$

For baryons with

$S = 2\frac{\overline{Q}}{e} - N$

$(N = 1)$

$\begin{pmatrix} p \\ n \end{pmatrix}$ have $\frac{\overline{Q}}{e} = +\frac{1}{2}$ so $S = 0$

$\Lambda^0, \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}$ have $\frac{\overline{Q}}{e} = 0$ so $S = -1$

$\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$ have $\frac{\overline{Q}}{e} = -\frac{1}{2}$ so $S = -2$

Ω^- has $\frac{\overline{Q}}{e} = -1$ so $S = -3$

$\begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$ have $\frac{\overline{Q}}{e} = +\frac{1}{2}$ so $S = 0$

The strangeness quantum number obviously arises because the state possesses a strange quark (or quarks) - see later in course.

Strangeness makes a big difference to which strong reactions are available for the various particles.

For example, for K^+ we only have elastic scattering (with $S = +1$)

$$K^+ + p \to K^+ + p$$

while for K^- there are many more final states (all having $S = -1$)

e.g.

$K^- + p \to K^- + p$ $\quad$ $K^- + p \to \overline{K^0} + n$ $\quad$ $K^- + p \to \Sigma^+ + \pi^-$

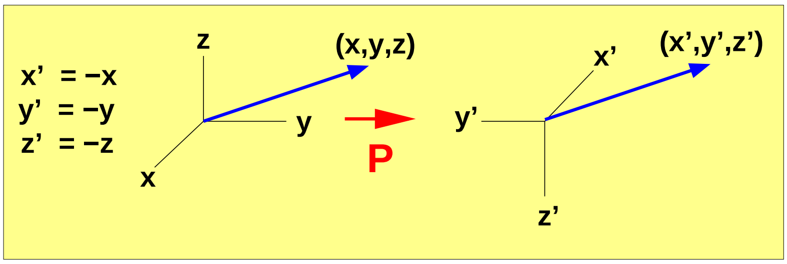
$K^- + p \to \Sigma^0 + \pi^0$ $\quad$ $K^- + p \to \Lambda^0 + n\pi^0$ $\quad$ $K^- + p \to \Lambda^0 + \pi^+ + \pi^-$

As a result the attenuation of a beam of K^- in material will be stronger than for a beam of K^+.

A similar conclusion holds for the K^0 and $\overline{K^0}$. $\overline{K^0}$ mesons with $S = -1$ are much more strongly attenuated than K^0 mesons with $S = +1$.

PARITY

The parity operator is a reflection through the origin



$$\begin{pmatrix} x \to -x \\ y \to -y \\ z \to -z \end{pmatrix} \begin{pmatrix} r \to r \\ \theta \to \pi - \theta \\ \phi \to \pi + \phi \end{pmatrix}$$

$$\hat{P}\psi(\underset{\sim}{r}) = \psi(-\underset{\sim}{r})$$

Note that

$$\hat{P}\psi(\underset{\sim}{r}) = \psi(-\underset{\sim}{r}) = p\psi(\underset{\sim}{r}) \qquad \hat{P}\hat{P}\psi(\underset{\sim}{r}) = p^2\psi(\underset{\sim}{r}) = \psi(\underset{\sim}{r})$$

Hence $p^2 = 1$ and so the eigenvalues are $p = \pm 1$

The parity operation is equivalent to a mirror relection plus a 180^o rotation. The mirror reflection turns left-handed systems into right-handed systems, and vice-versa.

For two particles with orbital angular momentum $L\hbar$, (L in natural units) the parity eigenvalue (coming from the spherical harmonic) is $(-1)^L$.

Additionally, each particle has an *intrinsic* parity. The total parity eigenvalue is $(-1)^L p_1 p_2$.

- BOSONS The intrinsic parities are unambiguously defined (e.g. for the pion and kaon they are negative - their spin is 0, so they are often written as 0^- PSEUDOSCALAR particles.
- FERMIONS The parity of a fermion-antifermion pair is unambiguously negative (e.g. e^+e^-, $p\overline{p}$). The individual fermion wave function has a 720^o symmetry - hence a parity eigenvalue which is imaginary.

SYMMETRY TRANSFORMATIONS

The parity transformation is the spatial subset of the more general symmetry transformation where ALL the parameters of the two particles are exchanged (space, spin, isospin etc.). This transformation acts on the total wavefunction of the system.

An assembly of identical fermions must have a total wave function which is **antisymmetric** with respect to the exchange of any two of the particles. Thus the wave function can be written

$$\psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_a(1) & \psi_b(1) & \ldots & \psi_z(1) \\ \psi_a(2) & \psi_b(2) & \ldots & \psi_z(2) \\ \ldots & \ldots & \ldots & \ldots \\ \psi_a(N) & \psi_b(N) & \ldots & \psi_z(N) \end{vmatrix}$$

SLATER DETERMINANT

Here $1, 2, \ldots N$ labels the N fermions, and $a, b, \ldots$ specifies the set of spatial coordinates and quantum numbers possessed by each particle.

Thus for say, electrons,

$\psi_a(1)$ could indicate that electron 1 is at $\underset{\sim}{r} = (x, y, z)$ with spin up.

$\psi_b(3)$ could indicate that electron 3 is at $\underset{\sim}{r}' = (x', y', z')$ with spin down, etc.

Because of the properties of determinants (any two identical rows cause it to vanish) this wavefunction has the correct symmetry properties.

The **Pauli Exclusion Principle** is a consequence of this. e.g. if electrons 1 and 3 were at the same position in space $\underset{\sim}{r} = \underset{\sim}{r}'$ AND were both spin up, the total wavefunction must vanish.

An assembly of identical bosons has a total wave function which is **symmetric** with respect to any interchange of a pair of particles.

Example 1 : Two Neutrons

We have

$$\psi_{total} = \psi_{space}\psi_{spin}\psi_{isospin}$$

and in terms of eigenvalues under exchange of the two neutrons, we have

$$(-1) = (-1)^{L}(+1)(+1)_{parity}(-1)^{S+1}_{spin}(+1)_{isospin}$$

$$(-1)^{L+S} = 1 \quad \text{so} \quad L + S = 0, 2, 4, 6, 8$$

So the allowed states are

$$^{1}S_0, ^{3}P_0, ^{3}P_1, ^{3}P_2, ^{1}D_2, ^{3}F_2, ^{3}F_3, ^{3}F_4 \quad \text{etc}$$

and the remaining states are disallowed. Note that the only state allowed with total angular momentum $1(\hbar)$ is the $^{3}P_1$ state.

Spectroscopic Notation

For total angular momentum vector $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}$

$$^{2S+1}A_J$$

where A reflects the value of L as follows

L	0	1	2	3	...
A	S	P	D	F	...

Why does the spin part pick up a factor $(-1)^{S+1}$?

The first neutron can be spin up $\uparrow$ or spin down $\downarrow$, and the second likewise. So the orientations of the pair can be $\uparrow\uparrow$ or $\downarrow\downarrow$ or $\uparrow\downarrow$ or $\downarrow\uparrow$.

We can construct three symmetric wavefunctions with total spin 1, and one antisymmetric wavefunction with total spin 0.

Wavefunction	(S, S_z)	Symmetry	$(-1)^{S+1}$
$\vert\uparrow\uparrow\rangle$	$(1, +1)$	S	1
$\frac{1}{\sqrt{2}}(\vert\uparrow\downarrow\rangle + \vert\downarrow\uparrow\rangle)$	$(1, 0)$	S	1
$\vert\downarrow\downarrow\rangle$	$(1, -1)$	S	1
$\frac{1}{\sqrt{2}}(\vert\uparrow\downarrow\rangle - \vert\downarrow\uparrow\rangle)$	$(0, 0)$	A	-1

The factor $(-1)^{S+1}$ summarises the symmetry of this situation.

Example 2 : The Intrinsic Parity of the Pion

The capture of a pion by a deuteron occurs from an S state with $L = 0$ and hence no orbital angular momentum.

$$\pi^- + d \rightarrow n + n$$

Angular momentum is always conserved, and as this is a strong interaction, so is parity.

Initial angular momentum is $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S_\pi} + \underset{\sim}{S_d}$

which gives $J = 1$ since $L = 0$, the pion has $S = 0$ and the deuteron $S = 1$.

The final state must therefore have $J = 1$, which from example 1 means that the two neutrons must be in a ${}^3P_1$ state.

Initial parity is $(-1)^L p_\pi p_d = p_\pi$

Final parity is $p_n p_n (-1)^L = -1$

Hence $p_\pi = -1$

Example 3 : The Two Pion System

For the $\pi^+\pi^-$ system

$$\begin{aligned} \psi_{total} = & \quad \psi_{space}\psi_{spin}\psi_{isospin} \\ +1 = & \quad (-1)^L(-1)^2_{space}(+1)_{spin} \cdot e_I \end{aligned}$$

where e_I is the isospin exchange eigenvalue.

Hence $e_I = (-1)^L$ and is the same as the parity eigenvalue.

For the $\pi^0\pi^0$ system

$$\begin{aligned} \psi_{total} = & \quad \psi_{space}\psi_{spin}\psi_{isospin} \\ +1 = & \quad (-1)^L(-1)^2_{space}(+1)_{spin}(+1)_{isospin} \end{aligned}$$

Hence $(-1)^L = +1$ and L is restricted to even values - all odd values are disallowed.

Example 4 : The e^+e^- System

We wish to know the charge conjugation eigenvalue for the e^+e^- system.

$$\hat{C}\ \psi_{e^+e^-} = c\ \psi_{e^+e^-}$$

We argue slightly differently here as the strong isospin of the electron is not defined because electrons do not see the strong interaction.

The charge conjugation operator reverses the positions of e^+e^- in all the wavefunctions.

In the spatial wavefunction we acquire a factor $(-1)^L(-1)$ due to the intrinsic negative parity of the fermion-antifermion pair.

In the spin wave function we acquire a factor of $(-1)^{S+1}$

So we acquire a total eigenvalue of $(-1)^{L+S}$. We have

$$\hat{C}\ \psi_{e^+e^-} = (-1)^{L+S}\ \psi_{e^+e^-}$$

and this turns out to be a general result for both fermions and bosons.

Example 5 : Positronium

Annihilation of the electron and positron to gammas, from the S $(L=0)$ state (don't confuse this spectroscopic "S" with the spin eigenvalue "S" below). There are two cases:

PARAPOSITRONIUM, opposite spins, so $S=0$ i.e. singlet $^1S_0$ state.

Therefore $\quad c=(-1)^{L+S}=+1$

As $\quad \hat{C}|n\gamma\rangle=(-1)^n|\gamma\rangle \quad$ we must have $n=0,2,4,6,\ldots$

$n=0$ is disallowed from conservation of energy, so the easiest decay is to two gammas with $\tau=1.25\times 10^{-10}$s.

ORTHOPOSITRONIUM, parallel spins, so $S=1$ i.e. triplet $^3S_1$ state.

Therefore $\quad c=(-1)^{L+S}=-1$

We must have $n=1,3,5,7\ldots$

$n=1$ is disallowed from conservation of momentum, so the easiest decay is to three gammas with $\tau=1.4\times 10^{-7}$s.

THE CPT THEOREM

This is perhaps the most general theorem in particle physics, depending on (a) Microcausality and (b) Lorentz Invariance.

It says that any physical system which is valid is the real world is equally valid and viable when operated upon by the $\hat{C}\hat{P}\hat{T}$ operator.

The $\hat{C}\hat{P}\hat{T}$ operator commutes with every possible Hamiltonian, i.e. $\hat{C}\hat{P}\hat{T}$ is strictly conserved.

In terms of eigenvalues

QUANTITY	$\hat{P}$	$\hat{T}$	$\hat{C}$	$\hat{C}\hat{P}\hat{T}$
Position $\underset{\sim}{r}$	-1	+1	+1	-1
Momentum $\underset{\sim}{p}$	-1	-1	+1	+1
Spin $\underset{\sim}{\sigma}$	+1	-1	+1	-1
Helicity $\underset{\sim}{\sigma}\cdot\underset{\sim}{p}/\lvert\underset{\sim}{\sigma}\rvert\lvert\underset{\sim}{p}\rvert$	-1	+1	+1	-1
$\underset{\sim}{B}$	+1	-1	-1	+1
$\underset{\sim}{E}$	-1	+1	-1	+1

Prediction 1: The mass of particle and antiparticle are identical

In general

$$\hat{C}\hat{P}\hat{T}\psi_A(\underset{\sim}{r},\underset{\sim}{\sigma}) = \psi_{\overline{A}}(-\underset{\sim}{r},-\underset{\sim}{\sigma}) = p\psi_{\overline{A}}(\underset{\sim}{r},-\underset{\sim}{\sigma})$$

so in the particle's rest frame (with $\underset{\sim}{r} = 0$)

$$\hat{C}\hat{P}\hat{T}\psi_A(\underset{\sim}{\sigma}) = p\psi_{\overline{A}}(-\underset{\sim}{\sigma})$$

where p is now the intrinsic parity of the particle.

In this frame we have $\hat{H}\psi = m\psi$ so we can write

$$\hat{H}\hat{C}\hat{P}\hat{T}\psi_A(\underset{\sim}{\sigma}) = p\hat{H}\psi_{\overline{A}}(-\underset{\sim}{\sigma}) = pm_{\overline{A}}\psi_{\overline{A}}(-\underset{\sim}{\sigma})$$

$$\hat{C}\hat{P}\hat{T}\hat{H}\psi_A(\underset{\sim}{\sigma}) = pm_{\overline{A}}\psi_{\overline{A}}(-\underset{\sim}{\sigma})$$

$$\hat{C}\hat{P}\hat{T}m_A\psi_A(\underset{\sim}{\sigma}) = pm_{\overline{A}}\psi_{\overline{A}}(-\underset{\sim}{\sigma})$$

$$m_A p\psi_{\overline{A}}(-\underset{\sim}{\sigma}) = m_{\overline{A}}p\psi_{\overline{A}}(-\underset{\sim}{\sigma})$$

Hence $m_{\overline{A}} = m_A$

Prediction 2: Unstable particles have the same mean lifetime as their antiparticles

This follows from prediction 1 if we allow the mass to have an imaginary part to describe the fact that the particle decays

$$m \rightarrow m - i\frac{\Gamma}{2}$$

We have

$$\psi = \psi_0 e^{-imt} e^{-\Gamma t/2}$$

Hence

$$|\psi|^2 = |\psi_0|^2 e^{-\Gamma t}$$

where $\Gamma = 1/\tau$ with τ the mean lifetime of the state.

Equating real and imaginary parts ensures that the mean lifetimes of particle and antiparticle must be identical.

<u>Prediction 3</u>: The magnetic moments of particle and antiparticle are equal and opposite

The magnetic moment μ of a particle can be defined through the relationship for the potential energy due to placing the particle in a magnetic field

$$U = -\underset{\sim}{\mu} \cdot \underset{\sim}{B} = -\frac{e}{2m} g \underset{\sim}{J} \cdot \underset{\sim}{B}$$

Under a CPT transformation, $\underset{\sim}{J} \rightarrow -\underset{\sim}{J}$ and $\underset{\sim}{B} \rightarrow +\underset{\sim}{B}$ (refer to table)

So for the potential energy to be the same, as it needs to be for complete symmetry, we must have $g \rightarrow -g$.

So the g factors (gyromagnetic ratios) for particle and antiparticle are equal in magnitude but opposite in sign. The same therefore applies for magnetic moments.

<u>SOME CURRENT LIMITS FROM TESTS OF CPT SYMMETRY</u>

	QUANTITY	VALUE OR LIMIT
Mass	$(m_{\pi^+} - m_{\pi^-})/m_{avg}$	$(2 \pm 5) \times 10^{-4}$
	$(m_{K^+} - m_{K^-})/m_{avg}$	$(-0.6 \pm 1.8) \times 10^{-4}$
	$(m_p - m_{\overline{p}})/m_p$	$< 6 \times 10^{-8}$
	$(m_{e^+} - m_{e^-})/m_{avg}$	$< 8 \times 10^{-9}$
	$(m_{K^0} - m_{\overline{K^0}})/m_{avg}$	$< 10^{-18}$
Lifetime	$(\tau_{\pi^+} - \tau_{\pi^-})/\tau_{avg}$	$(6 \pm 7) \times 10^{-4}$
	$(\tau_{\mu^+} - \tau_{\mu^-})/\tau_{avg}$	$(2 \pm 8) \times 10^{-5}$
	$(\tau_{K^+} - \tau_{K^-})/\tau_{avg}$	$(1.1 \pm 0.9) \times 10^{-3}$
Magnetic Moment	$(g_{e^+} - g_{e^-})/g_{avg}$	$(-0.5 \pm 2.1) \times 10^{-12}$
	$(g_{\mu^+} - g_{\mu^-})/g_{avg}$	$(-2.6 \pm 1.6) \times 10^{-8}$