**Physics 131** **Spring 2009**

# Particle Physics Seminar #12

**Textbook: Griffiths - Introduction to Elementary Particles**
**Website: (all notes referred to below are on web site)**

http://chaos.swarthmore.edu/courses/Phys131_2007/index.html

## Readings:

**REQUIRED: Griffiths - Chapter - 7**
**renormalization**

## Topic(s):

**(1) QED and Renormalization**

## Professor Lecture Topic(s): Renormalization

## Problems:

1. **Griffiths 7-38 Electron-muon scattering**
2. **Griffiths 7-39 Electron-electron scattering in CM**
3. **Griffiths 7-41 Pair annihilation to photons**
4. **Griffiths 7-43 Electron-muon scattering**
5. **Griffiths 7-46 Photon decay**
6. **Griffiths 7-48 Heavy photons**
   **Griffiths 7-49 Heavy photons**
   **Griffiths 7-50 Heavy photons**
7. **Griffith 7-51 Majorana particles**

**EP-13 Pair annihilation to muons**
**EP-14 Chiral symmetry**

**EP-13 Pair annihilation to muons**

(a) Construct, to lowest order in $\alpha$, the amplitude for the process $e^{+}+e^{-}\rightarrow\mu^{+}+\mu^{-}$, due to the electromagnetic interaction

$$L_I = -eA_\mu j^\mu$$
$$j^\alpha =:\bar{\psi}_e\gamma^\alpha\psi_e :+:\bar{\psi}_\mu\gamma^\alpha\psi_\mu :$$

(b) Calculate the cross-section, averaged over initial spins and summed over final spins, as a function of the CM energy Ignore the lepton masses.

**Extra Problem 14 - Chiral Symmetry**

We consider the Dirac equation

$$(i\partial\!\!\!/ - m)\psi = 0$$

but now use the so-called Chiral representation for the Dirac $\gamma$-matrices, i.e.,

$$\gamma^0 = -\sigma^1 \otimes I = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad , \quad \vec{\gamma} = i\sigma^2 \otimes \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$\gamma_5 = \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\sigma^{0i} = i\begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix} \quad , \quad \sigma^{ij} = \varepsilon^{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

where $\sigma^i$ are the three Pauli matrices and $I$ is the 2 x 2 identity matrix. Recall the definition of the matrix $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

(1) Using the Dirac matrices in the Chiral representation, write down the Dirac equation in terms of 2-spinors $\phi$ and $\chi$, where

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

(2) Show that if the excitations have zero mass (i.e., m = 0) the Dirac equation, written in the Chiral basis, decouples into two 2 x 2 equations. Find the plane wave solutions of these equations and calculate their dispersion law(i.e., energy-momentum relation). Assign a chirality ($\gamma_5$) quantum number to each solution.

(3) Consider now the **chiral** transformation (CT)

$$\psi' = e^{i\gamma_5\theta}\psi$$

(a) Find how the 2-spinors $\phi$ and $\chi$ transform under a CT.

(b) Find how $\bar{\psi}$ transforms under a CT.

(c) Find the transformation laws under a CT of the bilinears $\bar{\psi}\psi$ and $\bar{\psi}\gamma^\mu\psi$.

(d) Is the Dirac equation covariant under a CT if $m \neq 0$? Find the form of the Dirac equation, in terms of 4-spinors $\psi$, after a CT with angle $\theta$ has been carried out. What new terms do you find?