

My Lectures from : Purcell Chapter 9

Web Notes : Lecture Notes #8 and #9

Other Notes: EM_Waves

Purcell Problems:

- 9.08 An EM wave in a metal box
- 9.09 Energy density
- 9.10 Field inside capacitor
- 9.11 Beam energy down from orbit
- 9.12 Voltage standing wave ratio
- 9.13 Invariants

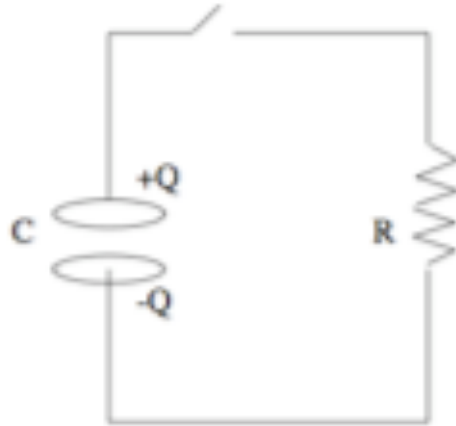
1. The Poynting vector in a capacitor - A current $I=dQ/dt$ delivers charge to a parallel-plate capacitor. This capacitor has circular plates of radius a , and the plates are separated by a distance $s \ll a$ (so you can ignore fringing).

- (a) Find the Poynting vector due to the electric field and the magnetic field between the capacitor plates. Give both the magnitude and the direction.
- (b) Calculate the total power, $P = \int \vec{S} \cdot d\vec{a}$, flowing into the capacitor. Given the Poynting vector found in (a), what is the correct surface to use for the integral?
- (c) Integrate the power over time. Assuming that the capacitor has charge 0 at $t=0$ and has some charge level Q at a later time t , show that the total energy that flows into the capacitor is given by $U = Q^2/2C$.

2. The Poynting vector and a coaxial cable - A coaxial cable transmits DC power from a battery to a load. The cable consists of two concentric, long, hollow cylinders of zero resistance. The inner cylinder has radius a , the outer has radius b , and the length of both is l . The battery provides an emf \mathcal{E} between the two conductors at one end of the cable and the load is a resistance R connected between the two conductors at the other end.

- (a) How much power is dissipated in the resistor?
- (b) What are \vec{E} and \vec{B} in the cable?
- (c) What is the Poynting vector \vec{S} in the cable?
- (d) Show that $\int \vec{S} \cdot d\vec{a} = P$ (from part(a)).

3. Displacement current - Consider the circuit below:



A circular parallel-plate capacitor with radius b and spacing $s \ll b$ (so you can ignore fringing) is charged to a voltage V_0 . At time $t=0$, the switch is closed and the capacitor begins to discharge through the resistor R .

- (a) Give an expression for the charge $Q(t)$ as a function of time of the positively charged plate (the upper one in the figure) of the capacitor.
- (b) Find an electric field $\vec{E}(t)$ between the two capacitor plates.
- (c) Find the displacement current density $\vec{J}(t)$ between the two capacitor plates.
- (d) Find the magnetic field $\vec{B}(t)$ between the two plates.

4. Polarized light - For this problem, an electric field for a wave will be given. Find the associated magnetic field, the direction of propagation and the direction of polarization for the light wave.

(a) $\vec{E} = E_0 \hat{y} \cos(kx - \omega t) + E_0 \hat{z} \cos(kx - \omega t)$

(b) $\vec{E} = E_0 \hat{x} \cos(ky - \omega t) + E_0 \hat{z} \sin(ky - \omega t)$

(c) $\vec{E} = E_{01} \hat{x} \cos(kz - \omega t) + E_{02} \hat{y} \cos(kz - \omega t + \pi/6)$ where $E_{01} = \frac{3 + \sqrt{13}}{2} E_{02}$