# RLC Circuits

**PRE-LAB**

Before coming to lab, review your lecture notes and readings on RLC circuits, and review the procedure for measuring the frequency response of the resistor output of the *RC* circuit (found in the lab handout for *RC and RL Circuits*).

**Pre-lab Question:** Verify the result provided in Eq. 8 of this handout (and immediately after) for the amplitude and phase of the voltage across the L-C combination in a driven series RLC circuit. *You do not need to turn these calculations in before lab; instead, do them in your notebook and show your calculations to your instructor as you arrive at lab.*

**INTRODUCTION TO THE RLC CIRCUIT**

Now we explore a circuit that exhibits damped harmonic oscillations: the series *RLC* circuit, shown in Fig. 1. Each element in this circuit has an analogous element in the mechanical oscillator you may recall (hopefully!) from Physics 7:

| <u>**Mechanical Oscillator**</u> | <u>**AC Circuit**</u> |
|---|---|
| **x** (displacement) | **Q** (charge) |
| **m** (mass) | **L** (inductance) |
| **k** (spring constant) | **1/C** (capacitance$^{-1}$) |
| **b** (damping) | **R** (resistance) |
| $\mathbf{F_0(\omega)}$ (driving force) | $\boldsymbol{\varepsilon_0(\omega)}$ (driving signal) |

If the capacitor is charged and then the circuit disconnected from the source of emf, then equating the sum of the voltage drops around the circuit to zero gives us

$$v_C(t) - L\frac{dI(t)}{dt} - RI(t) = 0 \qquad (2)$$

Expressing the current in terms of the time-dependent voltage across the capacitor, $v_C$, yields the following differential equation for $v_C$:

$$\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = 0 \qquad (3)$$

As in the case of the mechanical system, the solutions to this equation are classified as underdamped, critically-damped, or overdamped, depending on the relative magnitudes of R, L, and C. In the underdamped case, $v_C(t)$ oscillates at a frequency $\omega$ which is shifted from the natural frequency $\omega_0$, and the oscillation amplitude decays with decay time $(R/2L)^{-1}$:

$$v_C(t) = \mathrm{Re}(V_0 e^{-\alpha t} e^{i\omega t}) \quad \text{with } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ and } \alpha = \frac{R}{2L}. \qquad (4)$$

In the absence of damping (R = 0), the natural frequency of oscillation is $\omega_0 = \sqrt{\frac{1}{LC}}$.

When the RLC circuit is driven by an AC generator providing a sinusoidal emf $\varepsilon_0(t)$ of amplitude $\mathcal{E}_0$, the circuit displays the same resonant behavior observed in a driven mechanical oscillator. Applying the loop rule gives:

$$\mathcal{E}(t) = L\frac{dI(t)}{dt} + \frac{Q(t)}{C} + RI(t) \tag{5}$$

Rewriting Q(t) in terms of I(t) and solving the resulting differential equation for I(t), gives a solution of the form

$$I(t) = I_0 e^{i(\omega t + \phi)} \quad \text{with} \quad I_0 = \frac{V_0}{\sqrt{R^2 + X_{LC}^2}} \quad \text{and} \quad \tan\phi = -\frac{X_{LC}}{R} \tag{6}$$

where we define $X_{LC}$, the reactance of the inductor-capacitor (LC) combination, as the imaginary part of the impedance of the LC combination:

$$X_{LC} \equiv \omega L - 1/\omega C = \mathrm{Im}(Z_{LC}) \tag{7}$$

As with the freely oscillating circuit, the natural frequency of the circuit is $\omega_0 = 1/\sqrt{LC}$. At this frequency, called the resonant frequency, $X_{LC} = 0$.

In this lab, we will not measure the current directly. Rather, we will measure the time-dependent voltage across the resistor, $v_R(t)$, which is proportional to and in phase with the current. Eq. 6 implies that the current is maximum on resonance (when $\omega = \omega_0$), and the current leads the driving emf by a frequency-dependent angle $\phi$. In particular, for very low frequencies, $\phi \cong 90°$; as frequency increases, $\phi$ becomes a smaller number until on resonance, $\phi = 0$. Consequently, at resonance, the current is in phase with the driving emf. As $\omega$ increases beyond the resonant frequency, $\phi$ continues to decrease, approaching -90° as $\omega$ approaches infinity.

The phase angle $\phi$ gives the relative phase of the current and the driving emf; this phase is not the same as the phase between the current and other voltages in the circuit, such as the voltage across the capacitor or the resistor. As discussed in class, the current is in phase with the resistor, and the current always leads $v_C(t)$ by 90º and lags $v_L(t)$ by 90º.

Of particular interest is the voltage across the L-C combination, $v_{LC}(t)$, and its phase relationship with the current. Analyzing this in the same fashion used to analyze AC circuits in class gives us that the amplitude of $v_{LC}(t)$ is

$$V_{LC} = X_{LC} I_0 = \frac{V_0 X_{LC}}{\sqrt{R^2 + X_{LC}^2}} \tag{8}$$

and that $v_{LC}(t)$ leads the current by 90° if $X_{LC} > 0$ and lags the current by 90° if $X_{LC} < 0$.

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**EXPERIMENTS — RLC CIRCUIT**

Construct an RLC circuit as shown in Fig. 1 using the appropriate resistor for each experiment.

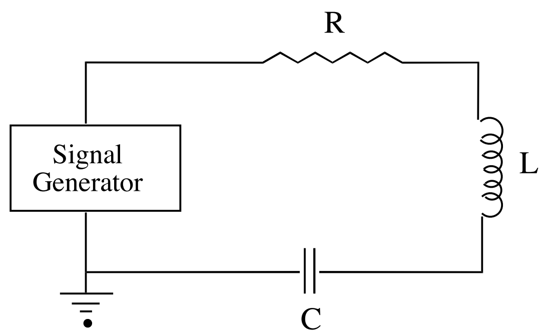


**Figure 1:** Series *RLC* circuit. For part 1, "signal generator" is the pulse generator. For parts 2 and 3, "signal generator" is the function generator. In part 2, the positions of R and C are switched.

**1) Pulsed circuit: a freely oscillating damped circuit.** For this circuit use the resistor with the least resistance (think about why you would choose this one). Using the pulse generator as the signal generator, apply a few-microsecond, several-volt pulse across the RLC combination. Monitor the voltage across the capacitor $v_C(t)$ as a function of time. Download the data to a file and use Kaleidagraph to fit your data to a sinusoid with exponentially decaying amplitude (Eq. 4). Note that Kaleidagraph can't handle more than about 500 points; if you acquire data from before the pulse is applied, you will want to delete points that precede the pulse, so that your data start at the time that the pulse is applied. Also note that even deleting points preceding the pulse, your data may not begin at exactly the time the pulse was applied, so you may need to add a phase term to the argument of the cosine in your fit: $v_C(t) = V_0 e^{-\alpha t}\cos(\omega t+\phi)$

From the fit parameters, extract the constants R/2L and $\omega$, with uncertainties, and calculate $\omega_0$ from these constants. Do these values agree with what you would predict from your measured values of R, L, and C? If not, how might you account for the observed differences?

**2) Driven circuit: current vs. frequency.** Begin by qualitatively examining the frequency dependence of oscillations in a mechanical oscillator. At your station, you have a mass on a spring. Holding the top of the spring in your hand, shake the spring so that the mass oscillates, shaking with very low frequency at first. Observe the amplitude of the oscillations. Now gradually increase the driving frequency (shake it faster), and observe how the amplitude of the oscillations changes with frequency. What did you observe? By analogy, how do you expect the amplitude of the current in a driven RLC circuit to depend on the frequency of the driving AC voltage? (Answer these questions in your lab notebook.)

**Now return to the circuit you used for part (1), but replace the resistor with one of the other two resistors of higher resistance, and use the function generator as your voltage source. Also, switch the positions of R and C in the circuit.** As discussed earlier, the current I(t) in the circuit is directly proportional to (and in phase with) $v_R(t)$. Monitor both $v_R(t)$ and the driving emf $\varepsilon_0(t)$ on the oscilloscope. For a series of frequencies, measure frequency (carefully, as you will need to duplicate some of the frequencies in a later measurement!), $\varepsilon_0$ (the amplitude of the driving emf), $V_R$ (the amplitude of $v_R(t)$), and $\Delta t_R$ (the time difference between the peak of

$\varepsilon_0(t)$ and the peak of $v_R(t)$; $\Delta t_R$ should be positive if $v_R(t)$ peaks before $\varepsilon_0(t)$, and negative if $v_R(t)$ peaks after $\varepsilon_0(t)$). Take lots of data near resonance, and fewer data points at frequencies far from resonance. Calculate the current flowing in the circuit and the phase angles corresponding to the time differences measured.

From your current amplitude vs. frequency data, determine two quantities:
**(a) Resonance frequency.** Carefully measure $\nu_0 = \omega_0/2\pi$, the frequency for which the amplitude of the current is maximum, and for which the current is in phase with the driving emf. Does your measured resonant frequency agree with the value you would predict using your measured values of L and C? What precision (uncertainty) can you claim for your various measurements in this consistency check?

**(b) Resonance width.** Measure the $FWHM_{power}$ (full width at 0.707 maximum current) of the current amplitude vs. frequency trace. Now change the resistor in the circuit to your third resistor and measure it again. (This time you shouldn't need to measure as many data points — knowing the shape of the curve should allow you to select frequencies wisely!). Does your observation agree with what you would predict?

**(3) Driven circuit: Phasor diagram.** The phase relationships between voltages can be represented by "phasors", which are simply vectors in the complex plane.

**Rebuild your circuit by switching the positions of R and C** so that you can monitor $v_{LC}(t)$ (the voltage across the capacitor and inductor combination) instead of $v_R(t)$ on the oscilloscope, and measure $V_{LC}$ and $\Delta t_{LC}$ for five frequencies at which you previously measured $V_R$ and $\Delta t_R$: two below resonance, on resonance, and two above resonance. Calculate the phase of $v_{LC}(t)$ relative to the driving emf from $\Delta t_{LC}$ and then calculate the phase of $v_{LC}(t)$ relative to the current using the phase of the current relative to the driving emf that you found previously. Check for agreement with the theory provided earlier. With this phase information, construct a phasor diagram including the driving emf, the current, the voltage across the resistor, and the voltage across the capacitor and inductor combination for a frequency below resonance, resonance, and a frequency above resonance.

## LAB NOTEBOOK CHECKLIST

You should have:

- prelab calculation verifying Eq. 8 and sentence immediately after
- measurements of R, L, and C for components
- answers to questions in handout
- graph of potential difference across the capacitor $v_C(t)$ vs. time for pulsed RLC circuit, and values of $\omega$, $\omega_0$, and decay time, with uncertainties
- graph of current amplitude vs. frequency for RLC circuit; resonance frequency and $FWHM_{power}$ of current vs. frequency trace, with uncertainties
- graph of phase of current (relative to driving emf) vs. frequency for RLC circuit
- measurements of phase of $v_{LC}(t)$ relative to the driving emf, and corresponding phasor diagram