RC and RL Circuits

P$_{RE}$-L$_{AB}$
Prelab assignment: (show your work to the instructor when you arrive at lab)
In class we discussed the frequency dependence of the amplitudes of the voltages across the capacitor and the resistor in a driven RC circuit, and found expressions for V_C and V_R in terms of R, C, and the amplitude ε_0 and angular frequency ω of the driving emf. Find an expression for the "crossover angular frequency," the angular frequency at which $V_C = V_R$, in terms of R and C. If R = 1 kΩ and C = 0.02 μF, what is the value of the crossover angular frequency ω_{CO}? What frequency ν_{CO} does this correspond to?

I$_{NTRODUCTION}$
This week, you will first investigate the charging and discharging of a capacitor. Then you will observe the steady-state behavior of resistor-capacitor and resistor-inductor combinations driven by a function generator. You will measure the frequency dependence of the amplitudes of the voltages in the circuit and also the relative phases of the voltages.

E$_{XPERIMENTS}$

Experiment 1: Resistor-Capacitor Circuits

In this experiment, you'll use the two input channels of a device known as an *oscilloscope* to simultaneously measure the voltage across the resistor-capacitor combination and the voltage across just the capacitor. You will also use the pulse from a piece of equipment called a pulse generator to charge a capacitor, and you will then monitor the potential difference across the capacitor as it discharges.

Measure the capacitance of your capacitor and the resistance of your resistor with the precision meter. Construct the circuit shown in Figure 1. The negative side of the capacitor should be connected to the ground terminal of the *pulse generator*. Measure the potential difference across the capacitor on channel 1 of the oscilloscope by connecting the positive (red) lead from the oscilloscope to the positive side of the capacitor and the negative (black) lead to the negative side, as shown in Figure 1. Measure the potential difference across the resistor-capacitor combination on channel 2 of the scope in the same manner (red lead at the top of the resistor, black lead at the bottom of the capacitor). NOTE: The oscilloscope will not operate properly unless its ground (black) lead is connected to the ground of the circuit.

Turn on the pulse generator, and provide a voltage pulse to the circuit. On the oscilloscope display, you should see both the voltage from the pulse generator and the voltage across the capacitor displayed as a function of time. Adjust the pulse duration to be sufficiently long that the capacitor is completely charged by the pulse (how will you know if this is the case?). Estimate the time constant for the resistor-capacitor combination by measuring the time it takes the voltage across the capacitor to decrease by a factor of 1/e.

Download the capacitor response data from the oscilloscope to a computer (using the software supplied in lab) and read the data into Kaleidagraph. Fit the decaying part of the data to a decaying exponential, $V(t) = V_0 \exp(-t/t_0)$. Compare the fit parameter t_0 to RC (the product of the resistance and capacitance values measured with the meter).
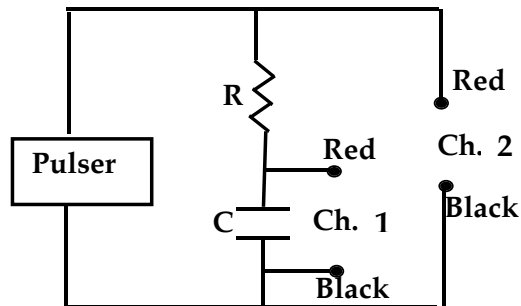
Figure 1. RC Circuit.

Experiment 2: Driven RC circuit

The term AC ("alternating current") refers generally to electrical potential differences (voltages) and currents that vary with time. (As mentioned in last week's lab: Although the term "voltage" is less precise than the term "potential difference," it is so entrenched in the language of electronics, and makes discussion of electronics so much easier, that we will use it in these labs. Keep in mind that a voltage is a potential *difference*, meaning that it is always measured with respect to some reference point. Generally the ground of the circuit is taken as the reference point.)

Figure 2. A potential difference, or voltage, that depends sinusoidally on time. A source that delivers such a sinusoidal voltage is called an AC (alternating current) source.
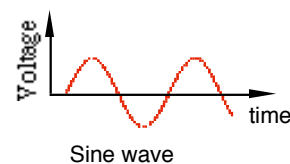
Although any periodic voltage is referred to as AC, typically AC voltages vary *sinusoidally*, as in Figure 2. The voltage supplied by household wiring, colloquially known as the "line voltage," is sinusoidal with a frequency of 60 Hz. In today's lab you will drive circuits with AC voltages obtained from a source called a *function generator*, which allows us to control the frequency and amplitude of the voltage.

Alternating voltages and currents are characterized by both *amplitude* and *phase*. The function generator is taken to provide the reference point for the phase, so the voltage across the function generator is given by

$$\varepsilon(t) = \varepsilon_0 \cos\omega t = \text{Re}\{\varepsilon_0 e^{i\omega t}\}$$

and the current in the circuit is given by

$$I(t) = I_0 \cos(\omega t + \phi) = \text{Re}\left\{I_0 e^{i(\omega t + \phi)}\right\}.$$

In this lab, we'll use the oscilloscope to monitor time-dependent quantities, but we will analyze just the amplitudes of those time-dependent data, so you will only need to measure the amplitudes from the scope display, and will not need to download the scope traces.

Figure 3 shows a resistor and a capacitor connected in series (a "series RC circuit") with an AC generator supplying a time-dependent emf $\varepsilon(t)$ with amplitude ε_0. The emf $\varepsilon(t)$ is measured from the circuit ground (the horizontal line at the bottom of the circuit) to the upper side of the source.
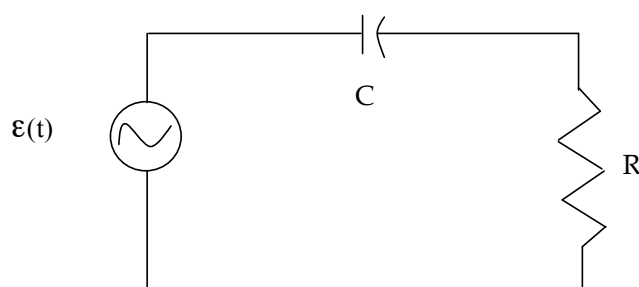

Figure 3. The RC circuit with an AC voltage source.

Consider how current will flow in this circuit if the driving emf changes *very slowly*. If the driving emf changes slowly enough, then at any instant in time, the capacitor will have exactly the right amount of charge on its plates so that the potential difference across the capacitor will equal the driving emf, so little current will flow through the circuit. It is colloquially said that the driving emf all "drops" across the capacitor, with little "voltage drop" across the resistor. (How do you know that there is little current if the potential difference, or "voltage drop," across the resistor is zero?)

Next, consider what happens if the driving emf changes *very rapidly*. If the driving emf oscillates very rapidly, the charge on the capacitor can't change quickly enough for the voltage across the capacitor to match the applied voltage. If the time period of the driving emf is orders of magnitude shorter than the time required for the capacitor to charge, this situation, the capacitor doesn't have time to charge from the positive voltage before the voltage drops to zero, and likewise with the negative part of the voltage oscillation. Consequently, the voltage (potential difference) across the capacitor is very small.

Today you will measure the frequency dependence of the amplitude of the current and the voltage across the capacitor, and also examine the frequency dependence of the phases of the current and the voltage across the capacitor relative to the driving emf. Instead of measuring the current directly, you'll measure the time-dependent voltage across the resistor, and determine the amplitude and the phase of the current from that.

Using a capacitor and a resistor supplied to you, construct the circuit shown in Figure 3. Use the function generator to provide a driving emf of a few volts in amplitude (3 V amplitude/6 V peak-to-peak works well), and connect both the ground lead of the function generator and the ground lead of the oscilloscope to the same side of the resistor. Monitor the voltage across the resistor, $v_R(t)$, on channel 1 of the oscilloscope; monitor the driving emf on channel 2. Measure the amplitudes V_R and $\mathcal{E}_0$ (using either the voltage cursors or the oscilloscope "Measure" button). Then measure the amplitude of the voltage across the capacitor V_C **without changing any of the circuit connections**, by making use of the "Math" menu on the oscilloscope (ask your instructor if you need help figuring this out). Also measure the time between the peak of $v_R(t)$ and the peak of $\mathcal{E}(t)$, and likewise between the peak of $v_C(t)$ and the peak of $\mathcal{E}(t)$. Be sure to note the *sign* of the time difference (which voltage peaks first?).

Now change the frequency of the driving emf, **and adjust the amplitude of the function generator output to keep the amplitude the same as in your previous measurement**. Note that the function generator and oscilloscope display frequency, with units of Hertz (cycles per second). Frequency is usually denoted ν, while angular frequency, with units of radians per second, is denoted ω and equals $2\pi\nu$. Determine the amplitudes V_R, $\mathcal{E}_0$, V_C, and the time shifts of the traces relative to the driving emf for at least seven different frequencies ranging over four to five orders of magnitude, with logarithmic spacing. One of those frequencies should be the crossover frequency expected for your particular values of R and C (alternatively, you could experimentally determine the crossover frequency and use that as one of your frequencies), and the other frequencies should be divided evenly between frequencies less than and frequencies greater than the crossover frequency. Plot V_R and V_C as a function of angular frequency ω on the same set of axes, and fit them to the functional forms you expect based on the analysis performed in class. (The more data you have, the better chance you have of getting a decent fit.) Using a logarithmic scale for the frequency axis makes it easiest to see the full range of the behavior of both quantities.

The "resistor output" of an RC circuit (the amplitude of the voltage across the resistor) is also called a "high pass" filter, and the "capacitor output" of an RC circuit (the amplitude of the voltage across the capacitor) is also called a "low pass" filter. Does your graph reflect this behavior? Are your plots consistent with your expected value of the crossover angular frequency? If not, why not? If you measured the crossover angular frequency experimentally, compare it to your expected value and discuss any discrepancies. If you measured the voltage amplitudes at the expected value of the crossover angular frequency, comment on any discrepancies.

From the time shifts, calculate the phases ϕ_R and ϕ_C of the resistor and capacitor voltages. Recall the sign convention for the phases is

$$v_R(t) = V_R \cos(\omega t + \phi_R) = \text{Re}\left\{ V_R e^{i(\omega t + \phi_R)} \right\}$$

$$v_C(t) = V_C \cos(\omega t + \phi_C) = \text{Re}\left\{ V_C e^{i(\omega t + \phi_R)} \right\}$$

Use the cursors on the oscilloscope to find the time between and the period of the various waveforms. These are all you need to compute the phases. Then plot each of these phases vs. angular frequency. If time permits, fit your phase vs. angular frequency data.

Discuss your measured frequency dependence of the phase qualitatively — for each of $v_R(t)$ and $v_C(t)$ does the voltage lead or lag the driving emf, and how does the phase depend on frequency? Does the voltage lead the driving emf in one frequency range and lag in another, or does it always have the same qualitative relationship with the driving emf? Is there a limit in which the voltage is in phase with the driving emf?

Experiment 3: The driven RL circuit

In the third experiment, you will characterize a driven RL circuit the same way you characterized the RC circuit. Construct the circuit shown in Figure 4 using the components supplied to you.
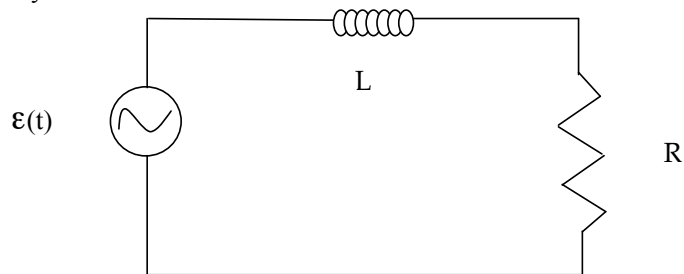

Figure 4. The RL Circuit.

Make the same set of measurements and perform the same analysis for this circuit as you did for the driven RC circuit. Compare your measurements for the frequency response of the current (i.e. the frequency response of V_R) in the RL circuit to the frequency response of the current in the RC circuit. Describe which voltage output in this circuit serves as a high-pass filter and which serves as a low-pass filter.

LAB BOOK CHECKLIST
You should have:
- values of circuit elements used
- calculated values of the time constant and crossover angular frequency
- theoretical relationships important to the analysis
- plot of V(t) vs. t for the discharging capacitor with fit
- plots of V_R and V_C vs. ω for the driven RC circuit, with fits
- phase vs. angular frequency plots for the RC circuit
- plots of V_R and V_L vs. ω for the driven RL circuit with fits
- phase vs. angular frequency plots for the RL circuit
- discussion of results along the lines suggested in the manual