Physics 008 **Problem Set #12** **Fall 2008**

My Lectures from : Purcell Chapter 9
Web Notes : Lecture Notes #8 and #9
Other Notes: EM_Waves

Purcell Problems:

9.08 An EM wave in a metal box
9.09 Energy density
9.10 Field inside capacitor
9.11 Beam energy down from orbit
9.12 Voltage standing wave ratio
9.13 Invariants

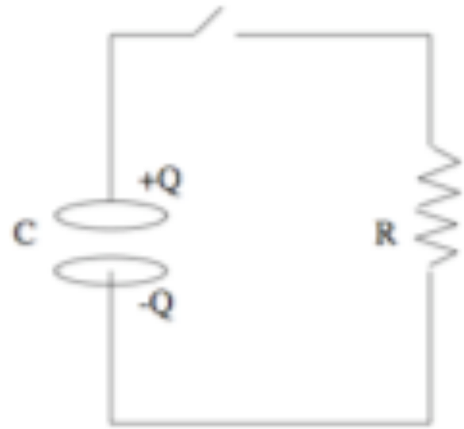
1. The Poynting vector in a capacitor - A current I=dQ/dt delivers charge to a parallel-plate capacitor. This capacitor has circular plates of radius a, and the plates are separated by a distance s << a (so you can ignore fringing).

(a) Find the Poynting vector due to the electric field and the magnetic field between the capacitor plates. Give both the magnitude and the direction.
(b) Calculate the total power, $P=\int \vec{S}\cdot d\vec{a}$, flowing into the capacitor. Given the Poynting vector found in (a), what is the correct surface to use for the integral?
(c) Integrate the power over time. Assuming that the capacitor has charge 0 at t=0 and has some charge level Q at a later time t, show that the total energy that flows into the capacitor is given by $U = Q^2/2C$.

2. The Poynting vector and a coaxial cable - A coaxial cable transmits DC power from a battery to a load. The cable consists of two concentric, long, hollow cylinders of zero resitance. The inner cylinder has radius a, the outer has radius b, and the length of both is l. The battery provides an emf ε between the two conductors at one end of the cable and the load is a resistance R connected between the two conductors at the other end.

(a) How much power is dissapted in the resistor?
(b) What are $\vec{E}$ and $\vec{B}$ in the cable?
(c) What is the Poynting vector $\vec{S}$ in the cable?
(d) Show that $\int \vec{S}\cdot d\vec{a} = P$ (from part(a)).

3. Displacement current - Consider the circuit below:



A circular parallel-plate capacitor with radius b and spacing s << b (so you can ignore fringing) is charged to a voltage V_0. At time t=0, the switch is closed and the capacitor begins to discharge through the resistor R.

(a) Give an expression for the charge Q(t) as a function of time of the positively charged plate (the upper one in the figure) of the capacitor.
(b) Find an electric field $\vec{E}(t)$ between the two capacitor plates.
(c) Find the displacement current density $\vec{J}(t)$ between the two capacitor plates.
(d) Find the magnetic field $\vec{B}(t)$ between the two plates.

4. Polarized light - For this problem, an electric field for a wave will be given. Find the associated magnetic field, the direction of propagation and the direction of polarization for the light wave.

(a) $\vec{E} = E_0\hat{y}\cos(kx-\omega t) + E_0\hat{z}\cos(kx-\omega t)$

(b) $\vec{E} = E_0\hat{x}\cos(ky-\omega t) + E_0\hat{z}\sin(ky-\omega t)$

(c) $\vec{E} = E_{0_1}\hat{x}\cos(kz-\omega t) + E_{0_2}\hat{y}\cos(kz-\omega t+\pi/6)$ *where* $E_{0_1} = \frac{3+\sqrt{13}}{2}E_{0_2}$