Physics 14 Quantum Physics Assignment #9 Spring 2005

Readings for week #5 of Quantum Mechanics

Pages 113-140 in Boccio Wave Mechanics Notes
Chapter 3 - pp 105-123 in French (same as last week)

Exercises #18, #19, #21, #22 and #39

[18] Particle in a 3-dimensional box

$$V(\vec{x}) = \begin{cases} 0 & \text{inside cube} \\ \infty & \text{outside cube} \end{cases}$$

(a) Find the energy eigenfunctions and eigenvalues
(b) Find the degeneracy of the ground state and the first excited state.
(c) $\psi(\vec{x},t_0) = \begin{cases} (1/a)^{3/2} & \text{inside} \\ 0 & \text{outside} \end{cases}$. Find the probability of finding particle in the ground state at t_0.

[19] Given $H\psi = i\hbar\frac{\partial\psi}{\partial t}$ with $H = \frac{\vec{p}\cdot\vec{p}}{2m} + V(\vec{x})$.

(a) Show that $\frac{d}{dt}\langle\psi(t)|\psi(t)\rangle = 0$
(b) Show that $\frac{d}{dt}\langle x\rangle = \left\langle\frac{p_x}{m}\right\rangle$
(c) Show that $\frac{d}{dt}\langle p_x\rangle = \left\langle -\frac{\partial V}{\partial x}\right\rangle$
(d) Find $\frac{d}{dt}\langle H\rangle$
(e) Find $\frac{d}{dt}\langle L_z\rangle$ and compare with the corresponding classical equation $\left(\vec{L} = \vec{x}\times\vec{p}\right)$.

[21] Consider a free particle in one-dimension. Let

$$\psi(x,0) = Ne^{-\frac{(x-x_0)^2}{4\sigma^2}}e^{i\frac{p_0 x}{\hbar}}$$, x_0, p_0, σ = real constants,
N = normalization constant

(a) Find $\tilde{\psi}(p,0)$
(b) Find $\tilde{\psi}(p,t)$
(c) Find $\psi(x,t)$
(d) Show that the "spread" in the spatial probability distribution $\left(\wp(x,t) = \frac{|\psi(x,t)|^2}{\langle\psi(t)|\psi(t)\rangle}\right.$ increases with time.

[22] Free particle in one-dimension $\left(H = \frac{p^2}{2m}\right)$

(a) Show $\langle p_x \rangle = \langle p_x \rangle_{t=0}$

(b) Show $\langle x \rangle = \left[\frac{\langle p_x \rangle_{t=0}}{m}\right] t + \langle x \rangle_{t=0}$

(c) Show $(\Delta p_x)^2 = (\Delta p_x)^2_{t=0}$

(d) Find $(\Delta x)^2$ as a function of time and initial conditions. HINT: Find $\frac{d}{dt}\langle x^2 \rangle$. To solve the resulting differential equation, one needs to know the time dependence of $\langle xp_x + p_x x \rangle$. Find this by considering $\frac{d}{dt}\langle xp_x + p_x x \rangle$.

[39] What happens

A particle of mass m is confined to a space $0 < x < L$ in one dimension by infinitely high walls at $x = 0$ and $x = L$. At $t = 0$ it is in the state

$$\psi(x,0) = \begin{cases} \left(\frac{1+i}{2}\right)\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L} + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L} & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

(a) What is $\psi(x,t)$?

(b) What is $\langle E \rangle$ for the state at time t?

(c) What is the probability that a measurement of the energy will yield the value $\hbar^2\pi^2/2mL^2$?

(d) Show that $\langle x \rangle$ is time dependent.

Extra Problems

EP-5 Measurement on a particle in a box

Consider a particle in a box of width a, prepared in the ground state.

(a) What are then possible values one can measure for :
(1) energy, (2) position, (3) momentum

(b) What are the probabilities for the possible outcomes you found in part (a)?

(c) At some (call it $t = 0$) we perform a measurement of position. However, our detector has only finite resolution. We find that the particle is in the middle of the box (call it the origin) with an uncertainty $\Delta x = a/2$, that is, we know the position is, for sure, in the range $-a/4 < x < a/4$, but we are completely uncertain where it is within this range. What is the (normalized) post-measurement state?

(d) Immediately after the position measurement what are the possible values for
(1) energy, (2) position, (3) momentum
and with what probabilities?

(e) At a later time, what are the possible values for
(1) energy, (2) position, (3) momentum

and with what probabilities? Comment.

EP-6 A particle of mass m moves in a one-dimensional box Infinite well) of length ℓ with the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < \ell \\ \infty & x > \ell \end{cases}$$

At $t = 0$, the wave function of this particle is known to have the form

$$\psi(x,0) = \begin{cases} \sqrt{30/\ell^5}\, x(\ell - x) & 0 < x < \ell \\ 0 & \textit{otherwise} \end{cases}$$

(a) Write this wave function as a linear combination of energy eigenfunctions

$$\psi_n(x) = \sqrt{\frac{2}{\ell}}\sin\left(\frac{\pi n x}{\ell}\right) \quad , E_n = n^2 \frac{\pi^2 \hbar^2}{2m\ell^2} \quad , n = 1,2,3,....$$

(b) What is the probability of measuring E_n at $t = 0$?

(c) What is $\psi(x, t > 0)$?