Physics 14 Quantum Physics Assignment #10 Spring 2005

Readings for week #6 of Quantum Mechanics

Pages 141-168 in Boccio Wave Mechanics Notes
Chapter 3 - pp 124-174 in French

Exercises #32, #33, #35, and #41

[32] 1/x potential

An electron moves in one dimension and is confined to the right half-space (x > 0) where it has potential energy

$$V(x) = -\frac{e^2}{4x}$$

where e is the charge on an electron.

(a) What is the solution of the Schrodinger equation at large x ?
(b) What is the boundary condition at x = 0 ?
(c) Use the results of (a) and (b) to guess the ground state solution of the equation. Remember the ground state wave function has no zeros except at the boundaries.
(d) Find the ground state energy.
(e) Find the expectation value $\langle \hat{x} \rangle$ in the ground state.

[33] Particle in a box with a membrane (DIFFICULT)

A box, containing a particle, is divided into a right and a left compartment by a thin partition. Suppose that the amplitude for the particle being on the left side of the box is ψ_1 and the amplitude for the particle being on the right side of the box is ψ_2 . Neglect spatial variations of these amplitudes within the halves of the box. Suppose that the particle can tunnel through the partition and that the rate of change of the amplitude on the right is given by

$$i\hbar \frac{\partial \psi_2}{\partial t} = K\psi_1$$

where K is real. Assume that in the absence of tunneling, i.e., an impermeable membrane, that $\frac{\partial \psi_1}{\partial t} = 0$.
Use the orthonormal basis

$$\left\{ u_1(x), u_2(x) \right\}$$

where $u_1(x)$ and $u_2(x)$ are uniform in space where they are non-zero with $u_1(x) = 0$ on the right side of the box and $u_2(x) = 0$ on the left side of the box so that $\langle u_1 | u_2 \rangle = 0$ and

$$\psi_1(t) = \langle u_1 | \psi \rangle \quad \text{and} \quad \psi_2(t) = \langle u_2 | \psi \rangle$$

where $\psi(x,t) = \psi_1(t)u_1(x) + \psi_2(t)u_2(x)$ is the wave function of the box.

(a) Determine the equation that determines the rate of change of the amplitude on the left?
(b) Find the normalized energy eigenstates of the particle in the box.
(c) Suppose that at time t = 0, the amplitude on the right equals $e^{i\delta}$ time the amplitude on the left. Calculate, as a function of time, the probability of observing the particle on the right.

[35] Given the wave function ......

A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose it is in an energy eigenstate

$$\psi(x) = \left(\frac{\gamma^2}{\pi}\right)^{1/4} \exp\left(-\gamma^2 x^2/2\right)$$

with energy $E = \hbar^2\gamma^2/2m$.

(a) Find the mean position of the particle.
(b) Find the mean momentum of the particle.
(c) Find $V(x)$.
(d) Find the probability $P(p)dp$ that the particle's momentum is between p and $p+dp$.

[41] What is the state?

A particle of mass m in a one dimensional harmonic oscillator potential is in a state for which a measurement of the energy yields the values $\hbar\omega/2$ or $3\hbar\omega/2$, each with a probability of one-half. The average value of the momentum $\langle \hat{p}_x \rangle$ at time $t=0$ is $(m\omega\hbar/2)^{1/2}$. This information specifies the state of the particle completely. What is this state and what is $\langle \hat{p}_x \rangle$ at time t? This is easier if you use creation/annihilation operators in stead of Hermite polynomials.